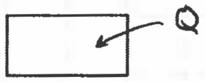
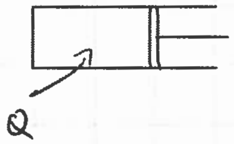
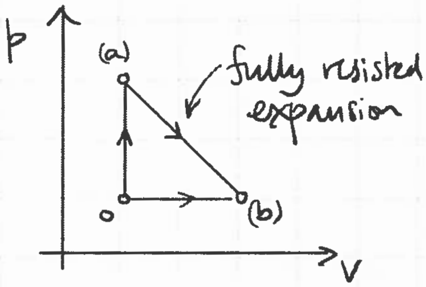
1 (a) 

Closed rigid vessel. $Q = m c_v \Delta T_v \Rightarrow \Delta T_v = Q/mc_v$. 1

(b) 

Piston at const p. $Q = m c_p \Delta T_p \Rightarrow \Delta T_p = Q/mc_p$ 1

(c) If the gas is air, $\Delta T_v > \Delta T_p$. This is because the gas in (b) expands and does work against the piston, while the gas in (a) does not. Therefore the internal energy at the end of the process in (a) is greater than that at the end of the process in (b). [At this stage it is also OK. to say that $c_p > c_v$ for air.] 2

(d) 

Consider doing experiment (b) by first doing experiment (a) and then doing a fully-resisted expansion to (b). We want the work done from (a) to (b)

initial temperature

Internal energy at (a) $= U_a = m c_v (T_0 + \Delta T_v)$

" " " (b) $= U_b = m c_v (T_0 + \Delta T_p)$

1st law: $-W = \Delta U = U_b - U_a = m c_v (\Delta T_p - \Delta T_v)$

$$\Rightarrow -W = m c_v \left(\frac{Q}{m c_p} - \frac{Q}{m c_v} \right)$$

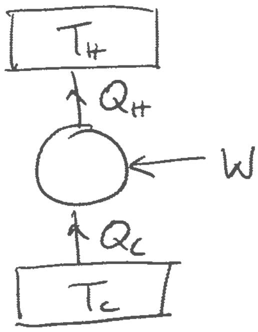
$$\Rightarrow W = Q\left(1 - \frac{c_v}{c_p}\right) = Q\left(\frac{\gamma - 1}{\gamma}\right)$$

$(\gamma-1)/\gamma$ is the proportion of the heat that is converted to work due to gas expansion.

4

(e) We want a material that shrinks when you heat it. Water from 0 celcius to 4 Celcius has this behaviour. This has negative γ. 2

Q2



$$COP_P = \frac{\text{heat to hot space}}{\text{work in}} = \frac{Q_H}{Q_H - Q_C}$$

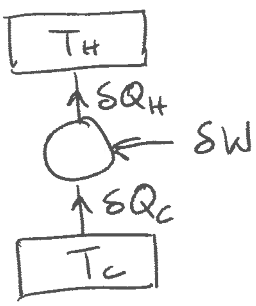
a) COP_P is maximum when reversible: $\frac{Q_H}{T_H} = \frac{Q_C}{T_C}$

$$\Rightarrow COP_P = \frac{T_H}{T_H - T_C}$$

i)	$COP_P = \frac{313}{35} = 8.9$
ii)	$COP_P = \frac{323}{45} = 7.2$
iii)	$COP_P = \frac{523}{245} = 2.1$

Comment: heat pumps are good for heating and washing, but not for cooking. Also note that COP_P will deteriorate when irreversibilities are considered.

b)



instantaneously, $\frac{\delta Q_H}{T_H} = \frac{\delta Q_C}{T_C} \Rightarrow \delta Q_C = \frac{T_C}{T_H} \delta Q_H$

also $\delta W = \delta Q_H - \delta Q_C = \left(1 - \frac{T_C}{T_H}\right) \delta Q_H$

also $\delta Q_H = m c_p \, \delta T_H$

$$\Rightarrow \delta W = \left(1 - \frac{T_C}{T_H}\right) m c_p \, \delta T_H$$

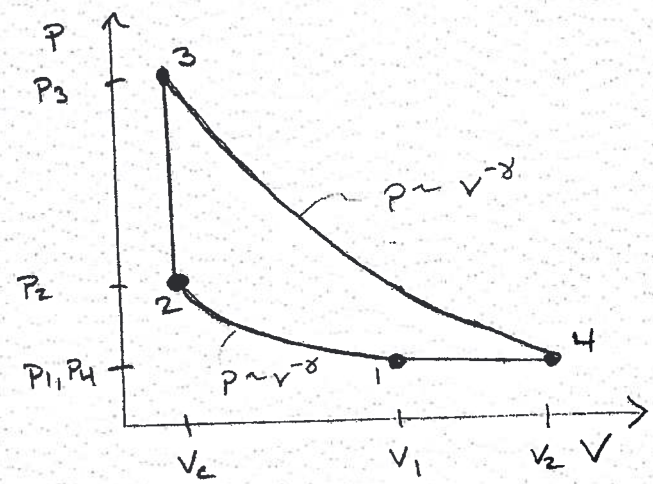
$$\Rightarrow W = \int_0^W dW = m c_p \int_{T_1}^{T_2} \left(1 - \frac{T_C}{T_H}\right) dT_H = m c_p \Big[T_H - T_C \log T_H \Big]_{T_1}^{T_2}$$

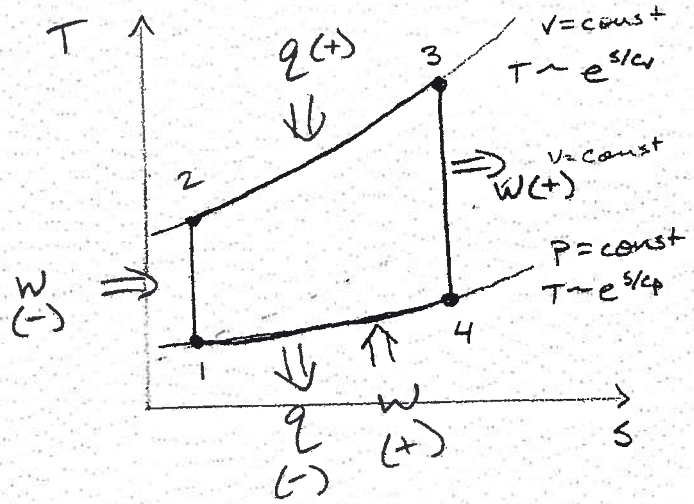
$$= m c_p \left[T_2 - T_1 - T_C \log\left(T_2 / T_1\right) \right]$$

$$= 5 \times 4200 \times \left[70 - 283 \log\left(\frac{363}{293}\right) \right]$$

$$\boxed{W = 1.97 \times 10^5 \text{ J}}$$

a b)


P
3
P3
p ~ v^-γ
P2
2
P1, P4
p ~ v^-γ
1
4
Vc
V1
V2
V


T
q (+)
3
v = const
T ~ e^s/cv
2
v = const
W (+)
W
(–)
p = const
T ~ e^s/cp
4
1
q
(–)
W
(+)
s

b) Show $T_3 = T_1 r_c^{\gamma-1} + q_{2-3}/c_v$

① → ② Isentropic Comp

Perfect Gas $\quad \Delta s_{12} = 0 \quad T_1 v_1^{\gamma-1} = T_2 v_2^{\gamma-1} \quad T_2 = T_1 \underbrace{\left(\frac{v_1}{v_2}\right)}_{r_c}^{\gamma-1}$

② → ③ Isochoric heat addition

$\delta q - \cancel{\delta w}^{\,0} = du \qquad q_{2-3} = c_v (T_3 - T_2)$

$$\boxed{T_3 = q_{2-3}/c_v + T_2 = q_{2-3}/c_v + T_1 r_c^{\gamma-1}}$$

c) Find $r = r_e/r_c$

$$s_3 = s_4$$

$$s_3 = c_v \ln(T_3/T_2) + \cancel{s_1} = c_p \ln(T_4/T_1) + \cancel{s_1} = s_3 \quad (i)$$

Given $T_3 = T_1 r_c^{\gamma-1} + q_{2-3}/c_v$

Note isentropic compression ① → ②

$$T_2 = T_1 r_c^{\gamma-1}$$

Isentropic expansion ③ → ④

$$T_3 = T_4 (V_4/V_3)^{\gamma-1} = T_4 r_e^{\gamma-1}$$

Rearrange (i) and sub in T_3 and T_2

$$\frac{T_3}{T_2} = \left(\frac{T_4}{T_1}\right)^{\gamma} \Rightarrow \frac{T_4 r_e^{\gamma-1}}{T_1 r_c^{\gamma-1}} = \left(\frac{T_4}{T_1}\right)^{\gamma}$$

$r^{\gamma-1}$

$$r^{\cancel{\gamma-1}} = \left(\frac{T_4}{T_1}\right)^{\cancel{\gamma-1}} \qquad r = \frac{T_4}{T_1} = \frac{T_3 r_e^{1-\gamma}}{T_1} = \frac{(T_1 r_c^{\gamma-1} + q_{2-3}/c_v) r_e^{1-\gamma}}{T_1}$$

$$r = r^{1-\gamma} + (q_{2-3}/c_v T_1) r_e^{1-\gamma}$$

$$r \cdot r^{\gamma-1} = r^{1-\gamma} \cdot r^{\gamma-1} + \frac{q_{2-3}}{T_1 c_v} r_e^{1-\gamma} r^{\gamma-1}$$

$$r^{\gamma} = 1 + \frac{q_{2-3}}{T_1 c_v r_c^{\gamma-1}} \Rightarrow \boxed{r = \left(1 + \frac{q_{2-3}}{T_1 c_v r_c^{\gamma-1}}\right)^{1/\gamma}}$$

d) i) $T_3 = T_1 r_c^{\gamma-1} + q_{2-3}/c_v$

$$T_3 = 300 \cdot 8^{0.34} K + \frac{10^3 \, kJ/kg}{0.834 \, kJ/kgK} = \boxed{1807 K}$$

$$T_4 = T_3 \, r_e^{1-\gamma}$$

$$r_e = r \cdot r_c = \left(1 + \frac{q_{2-3}}{T_1 c_v r_c^{\gamma-1}}\right)^{1/\gamma} r_c$$

$$r_e = \left(1 + \frac{10^3 \, kJ/kg}{300K \cdot 0.834 \frac{kJ}{kgK} \, 8^{0.34}}\right)^{1/1.34} 8 = 18$$

$$T_4 = 1807 K \; 18^{.34} = \boxed{676 K}$$

ii) $w_{NET} = q_{41} + q_{23} = w_{12} + w_{34} + w_{41}$

$q_{41} = w_{41} + c_v(T_1 - T_4)$

$V_4 = r_e V_3 = r_e V_2$

$= \frac{r_e}{r_c} V_1 = r V_1$

$r = \frac{r_e}{r_c} = \frac{18}{8} = \frac{9}{4}$

$w_{41} = \frac{p_1}{m}(V_1 - V_4) = \frac{p_1 V_1}{m}(1 - r) = RT_1(1-r)$

$= 287 \, J/kgK \, 300 K \left(1 - \frac{9}{4}\right)$

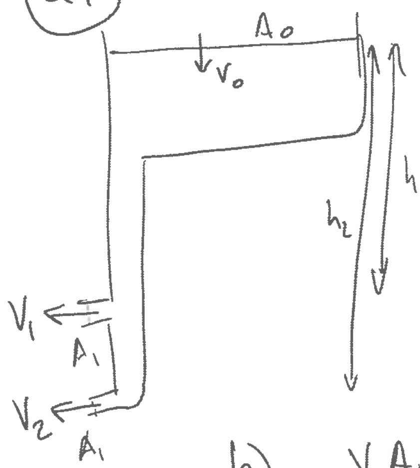
$= -107{,}625 = -108 \; kJ/kg$

$$q_{41} = -108 \frac{kJ}{kg} + \underbrace{0.834 \frac{kJ}{kgK}(300 - 676)K}_{-313 \, kJ/kg} = -421 \, kJ/kg$$

$w_{NET} = 1000 \, kJ/kg - 421 \, kJ/kg$

$$\boxed{w_{NET} = 579 \frac{kJ}{kg}}$$

Q4



a) Bernoulli

$$\tfrac{1}{2}\rho V_0^2 = \tfrac{1}{2}\rho V_1^2 - \rho g h_1 = \tfrac{1}{2}\rho V_2^2 - \rho g h_2$$

$$\boxed{\begin{aligned} V_1^2 &= V_0^2 + 2gh_1 \\ V_2^2 &= V_0^2 + 2gh_2 \end{aligned}}$$

$V_0^2 \ll 2gh_1 \Rightarrow$ (and thus also $2gh_2$)

$$\boxed{\begin{aligned} V_1 &= \sqrt{2gh_1} \\ V_2 &= \sqrt{2gh_2} \end{aligned}}$$

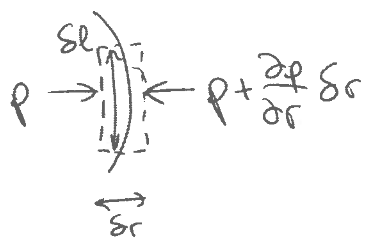
b) $V_1 A_1 + V_2 A_1 = V_0 A_0 \Rightarrow \boxed{V_0 = \frac{A_1}{A_0}\sqrt{2g}\left(\sqrt{h_1}+\sqrt{h_2}\right)}$

$$V_0 = \frac{4\times10^{-4}}{4\times10^{-1}}\sqrt{20}\left(\sqrt{3}+\sqrt{6}\right) = \left(1+\sqrt{2}\right)\sqrt{60}\times10^{-3} \simeq 0.02\ \text{m/s}$$

$$V_1 \simeq \sqrt{2gh_1} = \sqrt{60} \simeq 8$$

$\boxed{V_0 \simeq 5\times10^{-3}\sqrt{2gh_1} \ll \sqrt{2gh_1}}$ assumption is OK

Q5 a)



$$\left(\partial p/\partial r\ \delta r\right)\times\delta\ell = \delta m\, V^2/r$$

$$\partial p/\partial r\ \delta r\,\delta\ell = \rho\,\delta r\,\delta\ell\ V^2/r$$

$$\boxed{\partial p/\partial r = \rho\, V^2/r}$$

b) $p + \tfrac{1}{2}\rho V^2 = \text{constant}$;; $V = V(r) \Rightarrow p = p(r)$

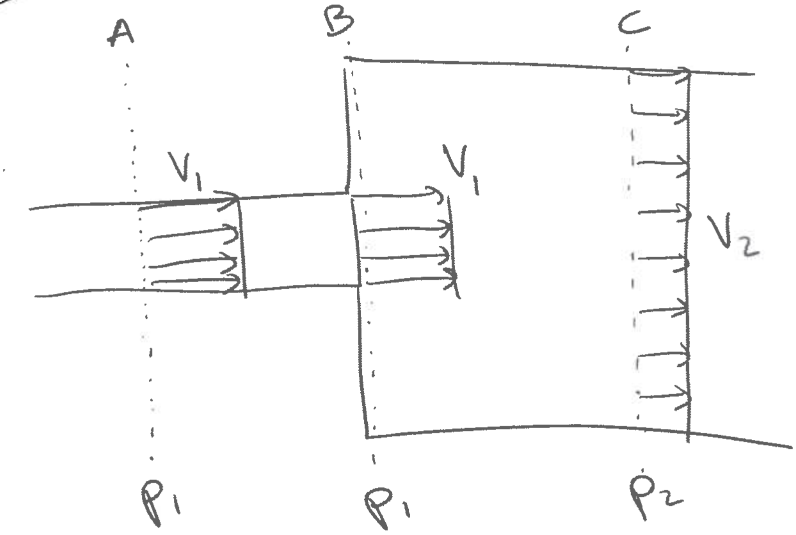
$$\frac{d}{dr}\left(p + \tfrac{1}{2}\rho V^2\right) = 0 = \frac{dp}{dr} + \frac{1}{2}\rho\, 2V\frac{dV}{dr}$$

$$\frac{dp}{dr} = -\rho V\frac{dV}{dr} \ ;; \quad \rho\frac{V^2}{r} = -\rho V\frac{dV}{dr}$$

$$\frac{dr}{r} = -\frac{dV}{V} \ ;; \quad \log r = -\log V + \log C$$

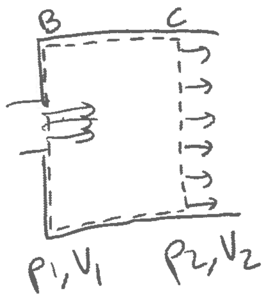
$$V = C/r \qquad \boxed{V(r) = V_0\frac{R_0}{r}} \ \boxed{V_1 = V_0\frac{R_0}{R_1}}$$

Q6



a) From continuity: $\rho A_1 V_1 = \rho A_2 V_2 \Rightarrow \boxed{V_2 = \frac{A_1}{A_2} V_1}$

b)

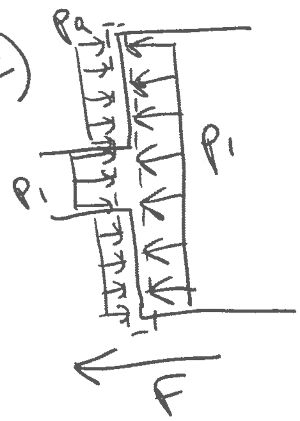


$$p_1 A_2 - p_2 A_2 = \rho A_2 V_2^2 - \rho A_1 V_1^2 = \rho A_1 V_1^2 \left(\frac{A_1}{A_2} - 1\right)$$

$$p_1 - p_2 = \rho V_1^2 \left[\left(\frac{A_1}{A_2}\right)^2 - \frac{A_1}{A_2}\right]$$

$$\boxed{p_2 = p_1 + \rho V_1^2 \left[\frac{A_1}{A_2} - \left(\frac{A_1}{A_2}\right)^2\right]}$$

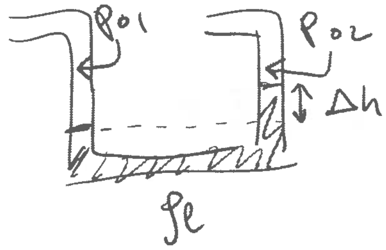
c)



$$\boxed{F = (p_1 - p_a)(A_2 - A_1)}$$

with leftwards taken as positive

d)



$$p_{01} = p_1 + \tfrac{1}{2}\rho V_1^2$$

$$p_{02} = p_2 + \tfrac{1}{2}\rho V_2^2$$
$$= p_1 + \rho V_1^2\left[\frac{A_1}{A_2} - \left(\frac{A_1}{A_2}\right)^2\right] + \frac{1}{2}\rho\left(\frac{A_1}{A_2}\right)^2 V_1^2$$
$$= p_1 + \rho V_1^2\left[\frac{A_1}{A_2} - \frac{1}{2}\left(\frac{A_1}{A_2}\right)^2\right]$$

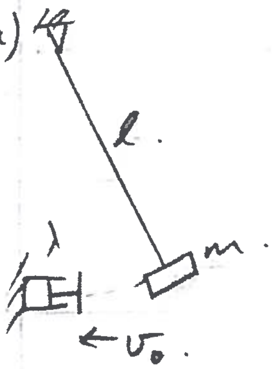
$$p_{02} - p_{01} = \rho V_1^2\left[\frac{A_1}{A_2} - \frac{1}{2}\left(\frac{A_1}{A_2}\right)^2\right] - \frac{1}{2}\rho V_1^2$$
$$= -\frac{1}{2}\rho V_1^2\left[1 - 2\frac{A_1}{A_2} + \left(\frac{A_1}{A_2}\right)^2\right]$$
$$= -\frac{1}{2}\rho V_1^2\left(1 - \frac{A_1}{A_2}\right)^2 \Rightarrow p_{02} < p_{01}$$

$$\rho_l g \Delta h = \frac{1}{2}\rho V_1^2\left(1 - \frac{A_1}{A_2}\right)^2 \;;; \boxed{\Delta h = \frac{1}{2}\frac{\rho}{\rho_l}\frac{V_1^2}{g}\left(1 - \frac{A_1}{A_2}\right)^2}$$

SECTION B.

7

a)



$\cancel{m}gl = \frac{1}{2}\cancel{m}v_0^2 \Rightarrow v_0 = \sqrt{2gl}.$ [1]

b). On contact, dashpot decelerates mass:

$\lambda v = -m\frac{dv}{dt}$, $\frac{m}{\lambda}\frac{dv}{dt} + v = 0$ or $T\frac{dv}{dt} + v = 0$, $T = \frac{m}{\lambda}$

$\Rightarrow v = v_0 e^{-t/T}$. $a_{max} = \left.\frac{dv}{dt}\right)_{max} = \frac{-v_0}{T}$

ie. max. deceleration is $\frac{\lambda}{m}\sqrt{2gl}$. [5]

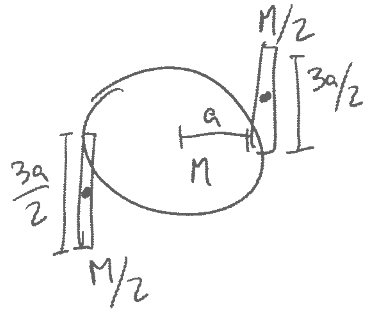
c) $\frac{dx}{dt} = v_0 e^{-t/T} \Rightarrow \int dx = \int v_0 e^{-t/T} dt$

$\therefore x = \left[-v_0 T e^{-t/T}\right]_0^t = v_0 T(1 - e^{-t/T})$

$\Rightarrow x_{max} = v_0 T = \frac{m}{\lambda}\sqrt{2gl}$

Assuming $x_{max} \ll l$ such that the motion of the mass remains approximately horizontal and without reaching the limit of travel of the dashpot [4]

Q8



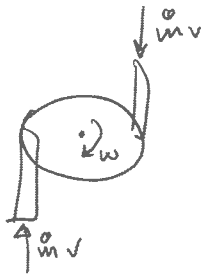
a) $$I_G = \underbrace{\frac{1}{2}Ma^2}_{\text{Disk}} + \underbrace{\cancel{2}\times\left\{\overbrace{\frac{1}{12}\frac{M}{\cancel{2}}\left(\frac{3a}{2}\right)^2}^{\text{w.r.t. their C.O.M.}} + \overbrace{\frac{M}{\cancel{2}}\left[\left(\frac{3a}{4}\right)^2+(a)^2\right]}^{\text{parallel axis}}\right\}}_{\text{Rods}}$$

$$= Ma^2\left[\frac{1}{2}+\left\{\frac{9}{12\times4}+\frac{9}{4\times4}+1\right\}\right]$$

$$= Ma^2\left[\frac{1}{2}+\frac{3}{4}+1\right]$$

$$\boxed{I_G = \frac{9}{4}Ma^2}$$

b)



Moments about G: $2\mathring{m}va = I_G\mathring{\omega}_0$

$$\boxed{\mathring{\omega}_0 = \frac{8\mathring{m}v}{9Ma}}$$

c) • Now $m_{rod} = \frac{M}{2} - \mathring{m}t$

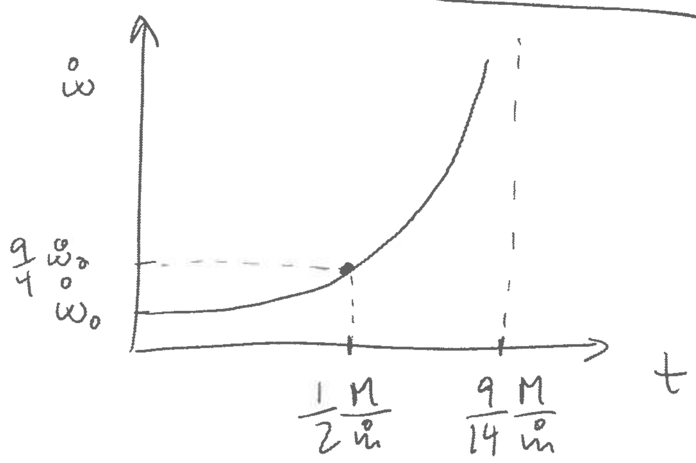
• From result in (a), need to change $\frac{M}{2}$ for $\frac{M}{2}-\mathring{m}t$ for rods:

$$I_G = Ma^2\left[\frac{1}{2}+\left\{\frac{3}{4}+1\right\}\left(1-\frac{2\mathring{m}t}{M}\right)\right] = \frac{9}{4}Ma^2 - \frac{7}{2}\mathring{m}ta^2 = \frac{9M-14\mathring{m}t}{4}a^2.$$

• Moments about G: $$\mathring{\omega} = \frac{2\mathring{m}va}{I_G} = \frac{8\mathring{m}v}{(9M-14\mathring{m}t)a}$$

• Fuel is spent when $\mathring{m}t = M/2$, or $t = M/2\mathring{m}$,

$$\boxed{\mathring{\omega}_{Max} = \frac{8\mathring{m}v}{(9M-7M)a} = \frac{4\mathring{m}v}{2Ma}}$$ Note $\mathring{\omega}_{max} = \frac{9}{4}\mathring{\omega}_0$



Q9

a)

d'Alambert   accelerations

- Moments about A:

$$W\cos\theta\,\frac{\ell}{2} - m\frac{\ell}{2}\ddot\theta\,\frac{\ell}{2} - I_G\ddot\theta = 0\;;\; I_G = \frac{1}{12}m\left(\sqrt{3}\ell\right)^2 = \frac{1}{4}m\ell^2$$

$$\ddot\theta = \frac{mg\ell/2}{m\ell^2/4 + 1/4\,m\ell^2}\cos\theta \qquad \boxed{\ddot\theta = \frac{g}{\ell}\cos\theta}$$

b) • Option 1: integrate $\ddot\theta$ using $d\left(\tfrac{1}{2}\dot\theta^2\right)/d\theta = \ddot\theta$

$$\boxed{\dot\theta^2 = \frac{2g}{\ell}\sin\theta}$$

- Option 2: use energy conservation [(a) not needed]

$$E_K = \frac{1}{2}mv_G^2 + \frac{1}{2}I_G\dot\theta^2 = \frac{1}{2}m\left(\frac{1}{2}\ell\dot\theta\right)^2 + \frac{1}{2}\,\frac{1}{4}m\ell^2\dot\theta^2$$

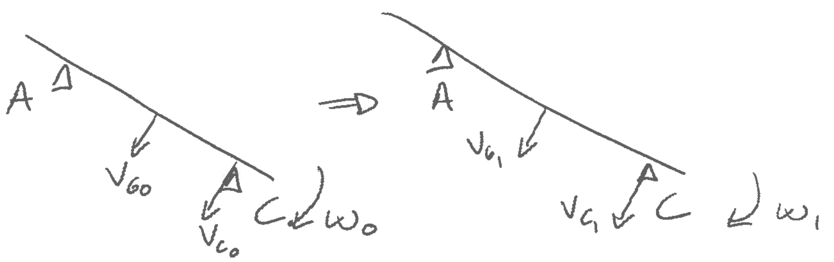
$$= \frac{1}{4}m\ell^2\dot\theta^2$$

$$E_p = -mg\left(\frac{\ell}{2}\sin\theta\right)$$

$$E_K + E_p = \text{constant (initially zero)} \Rightarrow \boxed{\dot\theta^2 = \frac{2g}{\ell}\sin\theta}$$

- $\boxed{\omega_0 = \frac{2g}{\ell}\sin\theta_0}$

c)



Impulsive problem

$$\begin{cases} v_{C0} = (1+c)\dfrac{\ell}{2}\omega_0 \\ v_{C1} = -e\,v_{C0} \end{cases}$$

d'Alambert

"accelerations"

$$\begin{cases} v_{G_0} = \frac{l}{2}\omega_0 \\ v_{G_1} = v_{C_1} - \omega_1 c \frac{l}{2} \end{cases}$$

$$\Delta v_G = v_{G_1} - v_{G_0} = -e(1+c)\frac{l}{2}\omega_0 - \omega_1 c\frac{l}{2} - \frac{l}{2}\omega_0 \;;\; \Delta\omega = \omega_1 - \omega_0$$

- Moments about C: $I_G \Delta\omega - m\Delta v_G c\frac{l}{2} = 0$

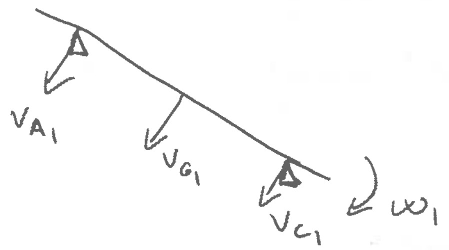
$$\cancel{\tfrac{1}{4}}\cancel{m}\cancel{l^2}(\omega_1 - \omega_0) - \cancel{m}\left[-e(1+c)\frac{\cancel{l}}{\cancel{2}}\omega_0 - c\frac{\cancel{l}}{\cancel{2}}\omega_1 - \frac{\cancel{l}}{\cancel{2}}\omega_0\right]c\frac{\cancel{l}}{\cancel{2}} = 0$$

$$\omega_1 - \omega_0 + \left[c\{1+e(1+c)\}\omega_0 + c^2\omega_1\right] = 0$$

$$[1+c^2]\,\omega_1 = [1-c\{1+e(1+c)\}]\,\omega_0 = [1-c-ec-ec^2]\,\omega_0$$

$$\boxed{\omega_1 = \frac{1-c-ec-ec^2}{1+c^2}\,\omega_0}$$

d) For (c) to hold, $I_A = 0$ implies $v_{A_1} \leq 0$



$$v_{A_1} = v_{C_1} - (1+c)\frac{l}{2}\omega_1$$

$$0 \geq -e(1+c)\frac{l}{2}\omega_0 - (1+c)\frac{l}{2}\omega_1$$

$$0 \geq -e\,\omega_0 - \omega_1 \;;\; 0 \leq e\omega_0 + \omega_1 = \left[e + \frac{1-c-ec-ec^2}{1+c^2}\right]\omega_0$$

$$0 \leq (1+\cancel{c^2})e + 1 - c - ec - \cancel{ec^2} = (1+e) - c(1+e)$$

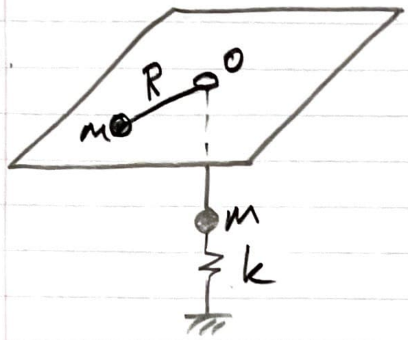
$$0 \leq 1 - c \longrightarrow \boxed{c \leq 1}$$

e) • $c = 1$ is the limit case $\Rightarrow v_{A_1} = 0$, A is pivot also after impact

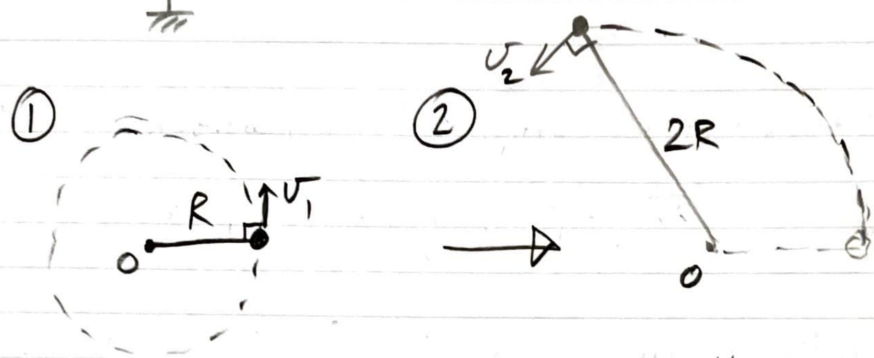
• $e = 1 \Rightarrow v_{C_1} = -v_{C_0}$, $\omega_1 = -\omega_0$

$\Rightarrow$ from energy conservation, plank bounces back all the way to the initial position, $\boxed{\theta = 0}$

(10).

a). 

Steady state.
Initial tension in spring is zero ∴
initial tension in string, $T = mg$. [2]

b). 

v_2 at maximum radius ⊥ string; instantaneously no radial velocity (but accn. ≠ 0)

i). No torque about O ∴ moment of momentum is conserved following impulse:

$$m v_1 R = m v_2 . 2R \quad , \quad v_2 = \frac{v_1}{2}$$

Frictionless ∴ CoE ⇒ $\frac{1}{2} m v_1^2 = \frac{1}{2} m v_2^2 + mgR + \frac{1}{2} k R^2$

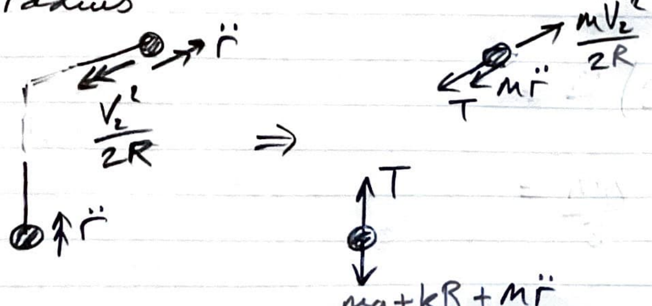
$$= \frac{1}{8} m v_1^2 + mgR + \frac{1}{2} k R^2$$

$$\therefore \frac{3}{8} m v_1^2 = mgR + \frac{1}{2} k R^2$$

$$v_1^2 = \frac{8}{3} g R + \frac{4}{3} \frac{kR^2}{m} = \frac{4}{3} R \left(2g + \frac{kR}{m}\right)$$

$$v_1 = 2\sqrt{\frac{R}{3}\left(2g + \frac{kR}{m}\right)} \quad , \quad v_2 = \sqrt{\frac{R}{3}\left(2g + \frac{kR}{m}\right)}$$ [5]

ii) Motion at ② comprises circular motion superposed with changing radius



$$mg + kR + m\ddot r = \frac{mV_2^2}{2R} - m\ddot r$$

$$\Rightarrow m\ddot r = \frac{mV_2^2}{4R} - \frac{mg}{2} - \frac{kR}{2}$$

$$= \frac{m}{4\cancel{R}} \frac{\cancel{R}}{3}\left(2g + \frac{kR}{m}\right) - \frac{mg}{2} - \frac{kR}{2}$$

$$\therefore T = mg + kR + \frac{m}{12}\left(2g + \frac{kR}{m}\right) - \frac{mg}{2} - \frac{kR}{2}$$

$$= \frac{2mg}{3} + \frac{7kR}{12}$$ [8]

(11).

a). From Fig. 8, $t_1 = \frac{T}{2}$, $T =$ natural period $= \frac{1}{f_n} = \frac{2\pi}{\omega_n}$

$\Rightarrow t_1 = \frac{\pi}{\omega_n}$

b). Initial velocity (at impact) $= U$

Impulse response: $y \approx Y e^{-\zeta\omega_n t} \sin\omega_n t$

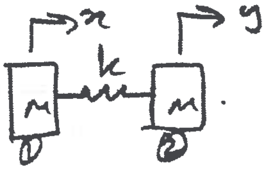
$\Rightarrow \dot{y} = -\zeta\omega_n Y e^{-\zeta\omega_n t} \sin\omega_n t + \omega_n e^{-\zeta\omega_n t} \cos\omega_n t$
$\approx \omega_n Y e^{-\zeta\omega_n t} \cos\omega_n t$

$\therefore \omega_n Y = U$ and $\dot{y} = U e^{-\zeta\omega_n t} \cos\omega_n t$

$\Rightarrow \dot{y}(t_1) = U e^{-\zeta\omega_n t_1} \cos\omega_n t_1 = V$, but $\omega_n t_1 = \pi$

$\therefore -U e^{-\pi\zeta} = V \quad \Rightarrow \frac{|V|}{|U|} = e^{-\pi\zeta} \approx 1 - \pi\zeta.$

(12).



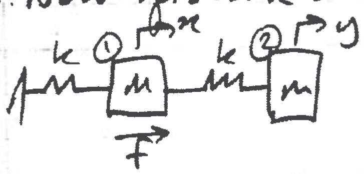
a) i). $k(y-x) = m\ddot{x}$, $m\ddot{x} + kx - ky = 0$

$k(x-y) = m\ddot{y}$, $m\ddot{y} - kx + ky = 0$

$$\therefore \begin{bmatrix} m & 0 \\ 0 & m \end{bmatrix}\begin{bmatrix} \ddot{x} \\ \ddot{y} \end{bmatrix} + \begin{bmatrix} k & -k \\ -k & k \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} = 0$$

ii) Symmetry $\Rightarrow$ $\underline{u}_1 = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$, $\omega_1 = 0$; $\underline{u}_2 = \begin{bmatrix} 1 \\ -1 \end{bmatrix}$, $\omega_2 = \sqrt{\frac{2k}{m}}$

b). Now ground left-hand mass:



i) Spring to ground constrains mass 1, hence eqn. of motion now:

$$\begin{bmatrix} m & 0 \\ 0 & m \end{bmatrix}\begin{bmatrix} \ddot{x} \\ \ddot{y} \end{bmatrix} + \begin{bmatrix} 2k & -k \\ -k & k \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} f \\ 0 \end{bmatrix} \quad \text{ie. } [m]\ddot{\underline{u}} + [k]\underline{u} = \underline{f}$$

Require $|[k] - \omega^2[m]| = 0$ ie. $\begin{vmatrix} 2k-\omega^2 m & -k \\ -k & k-\omega^2 m \end{vmatrix} = 0$

$(2k-\omega^2 m)(k-\omega^2 m) - k^2 = 0$, $m^2\omega^4 - 3km\omega^2 + k^2 = 0$

$$\rightarrow \omega^2 = \frac{3k\cancel{m} \pm \sqrt{9k^2\cancel{m^2} - 4k^2\cancel{m^2}}}{2m^{\cancel{2}}} = \frac{3k \pm \sqrt{5}k}{2m}$$

$$\omega_1^2 = \left(\frac{3-\sqrt{5}}{2}\right)\frac{k}{m} \quad , \quad \omega_2^2 = \left(\frac{3+\sqrt{5}}{2}\right)\frac{k}{m}$$

$$\therefore \omega_1 = 0.618\sqrt{\frac{k}{m}} \quad , \quad \omega_2 = 1.618\sqrt{\frac{k}{m}}$$

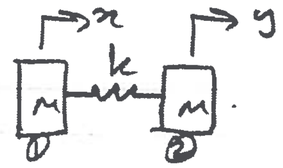
$$X = \frac{A\left(1-\omega^2/\omega_0^2\right)}{\left(1-\omega^2/\omega_1^2\right)\left(1-\omega^2/\omega_2^2\right)}$$

$\omega = 0 \Rightarrow X = \frac{F}{k}$ $\therefore$ $A = \frac{F}{k}$; ω_1 and ω_2 as above.

$\omega = \omega_0 \Rightarrow X = 0$ ie. when cart 2 acts as a TMD for cart 1

$\Rightarrow \omega_0 = \sqrt{\frac{k}{m}}$.

(12).



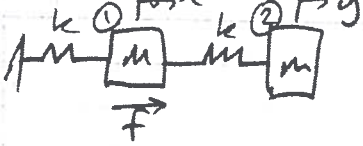
a) $k(y-x) = m\ddot{x}$, $m\ddot{x} + kx - ky = 0$

i). $k(x-y) = m\ddot{y}$, $m\ddot{y} - kx + ky = 0$

$$\therefore \begin{bmatrix} m & 0 \\ 0 & m \end{bmatrix} \begin{bmatrix} \ddot{x} \\ \ddot{y} \end{bmatrix} + \begin{bmatrix} k & -k \\ -k & k \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = 0$$

ii) Symmetry $\Rightarrow \underline{u}_1 = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$, $\omega_1 = 0$; $\underline{u}_2 = \begin{bmatrix} 1 \\ -1 \end{bmatrix}$, $\omega_2 = \sqrt{\frac{2k}{m}}$

b). Now ground left-hand mass:



i) Spring to ground constrains mass (1), hence eqn. of motion now;

$$\begin{bmatrix} m & 0 \\ 0 & m \end{bmatrix} \begin{bmatrix} \ddot{x} \\ \ddot{y} \end{bmatrix} + \begin{bmatrix} 2k & -k \\ -k & k \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} f \\ 0 \end{bmatrix} \quad \text{ie. } [m]\ddot{\underline{u}} + [k]\underline{u} = \underline{f}$$

Require $|[k] - \omega^2[m]| = 0$ ie. $\begin{vmatrix} 2k-\omega^2 m & -k \\ -k & k-\omega^2 m \end{vmatrix} = 0$

$(2k-\omega^2 m)(k-\omega^2 m) - k^2 = 0$, $m^2\omega^4 - 3km\omega^2 + k^2 = 0$

$$\rightarrow \omega^2 = \frac{3km \pm \sqrt{9k^2m^2 - 4k^2m^2}}{2m^2} = \frac{3k \pm \sqrt{5}k}{2m}$$

$$\omega_1^2 = \left(\frac{3-\sqrt{5}}{2}\right)\frac{k}{m} , \quad \omega_2^2 = \left(\frac{3+\sqrt{5}}{2}\right)\frac{k}{m}$$

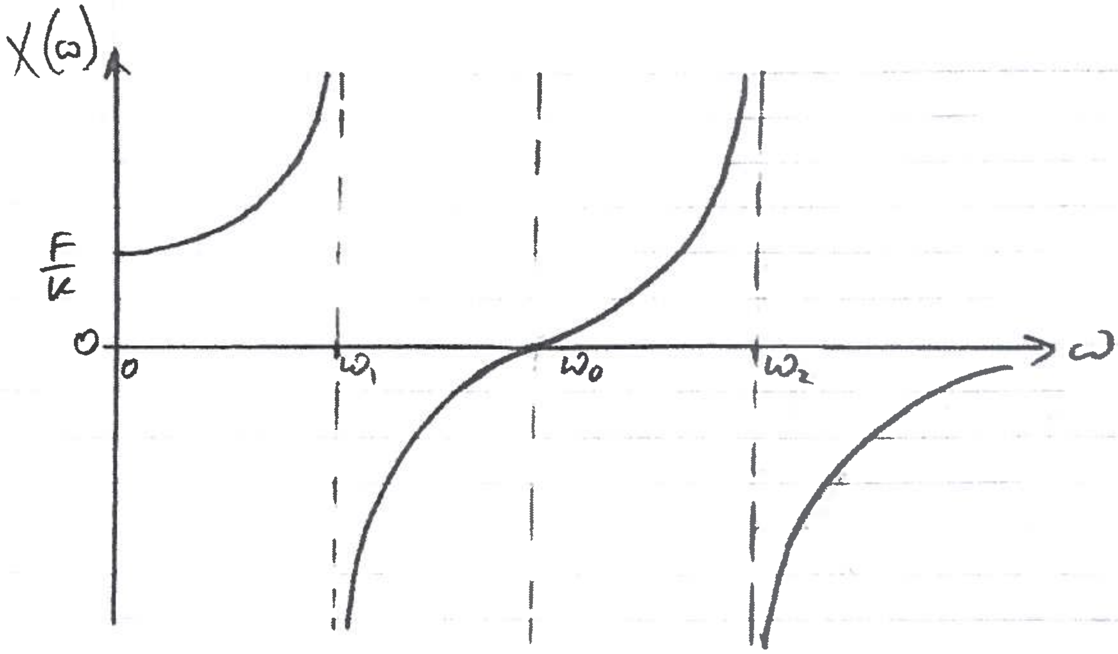
$\therefore \omega_1 = 0.618\sqrt{\frac{k}{m}}$, $\omega_2 = 1.618\sqrt{\frac{k}{m}}$

$$X = \frac{A\left(1-\omega^2/\omega_0^2\right)}{\left(1-\omega^2/\omega_1^2\right)\left(1-\omega^2/\omega_2^2\right)}$$

$\omega = 0 \Rightarrow X = \frac{F}{k}$ $\therefore A = \frac{F}{k}$; ω_1 and ω_2 as above.

$\omega = \omega_0 \Rightarrow X = 0$ ie. when cart (2) acts as a TMD for cart (1)

$\Rightarrow \omega_0 = \sqrt{\frac{k}{m}}$.



X(ω)
F/k
0
0
ω₁
ω₀
ω₂
ω