EGT0
ENGINEERING TRIPOS PART IA

Wednesday 7 June 2023 9 to 12.10

Paper 1

MECHANICAL ENGINEERING

*Answer **all** questions.*

*The **approximate** number of marks allocated to each part of a question is indicated in the right margin.*

Answers to questions in each section should be tied together and handed in separately.

*Write your candidate number **not** your name on the top sheet.*

STATIONERY REQUIREMENTS
Single-sided script paper

SPECIAL REQUIREMENTS TO BE SUPPLIED FOR THIS EXAM
CUED approved calculator allowed
Engineering Data Book

10 minutes reading time is allowed for this paper at the start of the exam.

You may not start to read the questions printed on the subsequent pages of this question paper until instructed to do so.

You may not remove any stationery from the Examination Room.

SECTION A

1 (**short**) A reversible heat engine takes in heat Q_H at temperature T_H and rejects heat to the atmosphere at temperature T_0. It produces work, W, which drives a reversible heat pump, which takes in heat at temperature T_0 and rejects heat Q_R to a room at temperature T_R.

(a) Show that

$$\frac{Q_R}{Q_H} = \frac{1-\tau_E}{1-\tau_P}$$

where $\tau_E = T_0/T_H$ and $\tau_P = T_0/T_R$. [5]

(b) Sketch Q_R/Q_H as a function of τ_P for $\tau_E = 1/2$. [3]

(c) Use the result of (b) to state how the performance of domestic heat pumps can be maximized. [2]

2 (**short**) A fire extinguisher contains a perfect gas with c_p = 1000 J kg^{-1} K^{-1} and γ = 1.25 at 300 K and 10 bar. The gas is expanded adiabatically through a valve to 1 bar.

(a) If the process were isentropic, what would the final temperature of the gas be? [2]

(b) The process is not isentropic. Sketch the process on a T-s diagram, labelling all lines clearly. [3]

(c) If the final temperature of the gas is 246 K, calculate the entropy change per unit mass of gas and the ratio of the final volume of the gas to its initial volume. [5]

3 (**long**) Figure 1 shows a diagram of a stationary turbojet engine with no bypass flow. Thermodynamic states are defined at five positions, labelled 1 to 5. It is to be modelled with the Joule cycle adapted for turbojet engines, assuming that the working fluid is a perfect gas, that the fuel mass flowrate can be neglected, that potential energy changes are negligible, and that the compressor, turbine, and exhaust nozzle are isentropic. The compression ratio is defined as $r_p = p_2/p_1$ and the non-dimensional heat addition is $\dot{Q}^\star = \dot{Q}/(\dot{m}c_pT_1)$, where $\dot{m}$ is the air mass flowrate. It is helpful to define $\eta = (\gamma - 1)/\gamma$.

(a) Starting from an appropriate $T\mathrm{d}s$ equation, show that lines of constant p on a T-s diagram for a perfect gas have the form $T/T_0 = \exp\big((s - s_0)/c_p\big)$. [2]

(b) Show that $T_4/T_1 = 1 + \dot{Q}^\star$ for the stationary turbojet engine. [3]

(c) On the same T-s diagram, and with the same values for T_1 and $\dot{Q}^\star$, sketch the thermodynamic cycles of the stationary turbojet for

(i) a high compression ratio (label T_1, T_2, T_3, T_4, T_5);

(ii) a low compression ratio (label $T_1', T_2', T_3', T_4', T_5'$).

Pay particular attention to the sizes of $(T_2 - T_1)$, $(T_3 - T_2)$, and $(T_4 - T_3)$ at both compression ratios, as well as to the locations of T_4 and T_4'. [8]

(d) The jet Mach number is defined as $\mathrm{M} = V_5/\sqrt{\gamma R T_5}$. Show that $(\gamma - 1)\mathrm{M}^2 = 2\big((p_4/p_5)^\eta - 1\big)$. Sketch M as a function of p_4/p_5. [8]

(e) Derive an expression for p_4/p_1 as a function of r_p and $\dot{Q}^\star$. [6]

(f) Using the T-s diagrams in part (c), explain why the jet velocity increases with r_p for a given $\dot{Q}^\star$. [3]

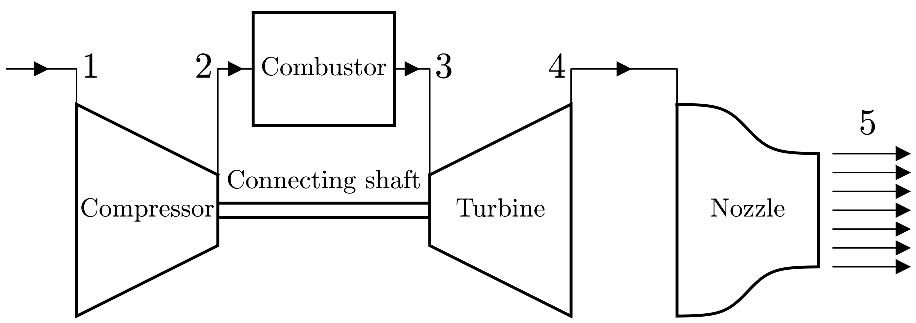

Fig. 1

4 (**short**) A fluid of density ρ is confined in a reservoir by a gate of length L under the effect of gravity g, as shown in Fig. 2. Both the reservoir and the gate are homogeneous in the direction into the page, and the gate is held by a torsional spring. The restoring moment per unit length out of the page from the spring is $M = K\theta$, where K is the spring constant and θ the angle of the gate with the vertical.

(a) Find the moment M produced by the hydrostatic pressure on the gate hinge as a function of ρ, g, h and θ. [5]

(b) Find the value of K in terms of ρ, g and L for which the fluid will spill for $h = \sqrt{3}L/2$. [5]

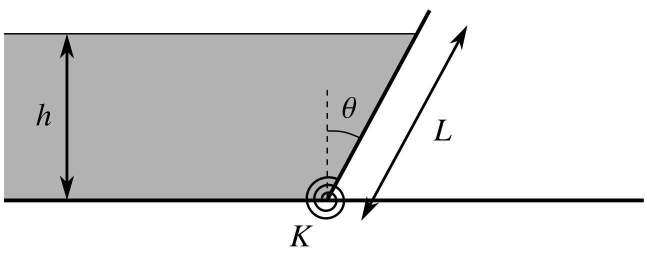

Fig. 2

5 (**short**) An incompressible flow of air with density ρ travelling at velocity V_1 through a circular, straight pipe of diameter D_1 with pressure p_1 undergoes a smooth contraction to diameter D_2. Two Pitot tubes connected by a manometer with closed valves are installed upstream and downstream of the contraction, as indicated in Fig. 3. Friction forces may be neglected throughout.

(a) Find the velocity downstream of the contraction, V_2, in terms of V_1, D_1 and D_2. [2]

(b) Find the pressure downstream of the contraction, p_2, in terms of p_1, ρ, V_1, D_1 and D_2. [4]

(c) If the valves shown in the figure are opened, describe what happens to the height difference across the manometer, Δh, and justify your answer. [4]

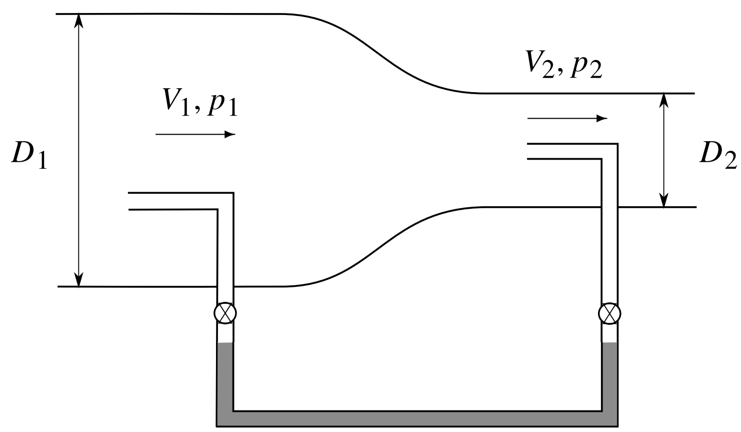

Fig. 3

6 (**long**) A series of two-dimensional aerofoils are laid out in a vertical cascade, such that they deflect the upstream flow from an angle α_1 to an angle α_2 with the horizontal, as shown in Fig. 4. The figure portrays the flow around one of the aerofoils, with streamlines A and B parallel and separated by a vertical distance L. The flow upstream has uniform velocity U_∞, while the flow downstream is non-uniform and has magnitude $V - \delta V \cos(2\pi y/L)$, where $y = \pm L/2$ are the vertical coordinates of streamlines A and B. The pressure upstream and downstream of the cascade is the ambient pressure.

(a) Find the value of V in terms of U_∞, α_1 and α_2. [5]

(b) The streamlines A and B can be considered inviscid. Find the value of δV in terms of U_∞, α_1 and α_2. [5]

(c) Find the value and direction of the horizontal force per unit depth into the page, F_x, exerted by the flow on the aerofoil, as a function of ρ, L, U_∞, α_1 and α_2. Find the ratio of F_x to $1/2\rho U_\infty^2 L$ when $\alpha_1 = \arccos(3/5)$ and $\alpha_2 = \arccos(4/5)$. [10]

(d) Find the value and direction of the vertical force per unit depth into the page, F_y, exerted by the flow on the aerofoil, as a function of ρ, L, U_∞, α_1 and α_2. Find the ratio of F_y to $1/2\rho U_\infty^2 L$ when $\alpha_1 = \arccos(3/5)$ and $\alpha_2 = \arccos(4/5)$. [10]

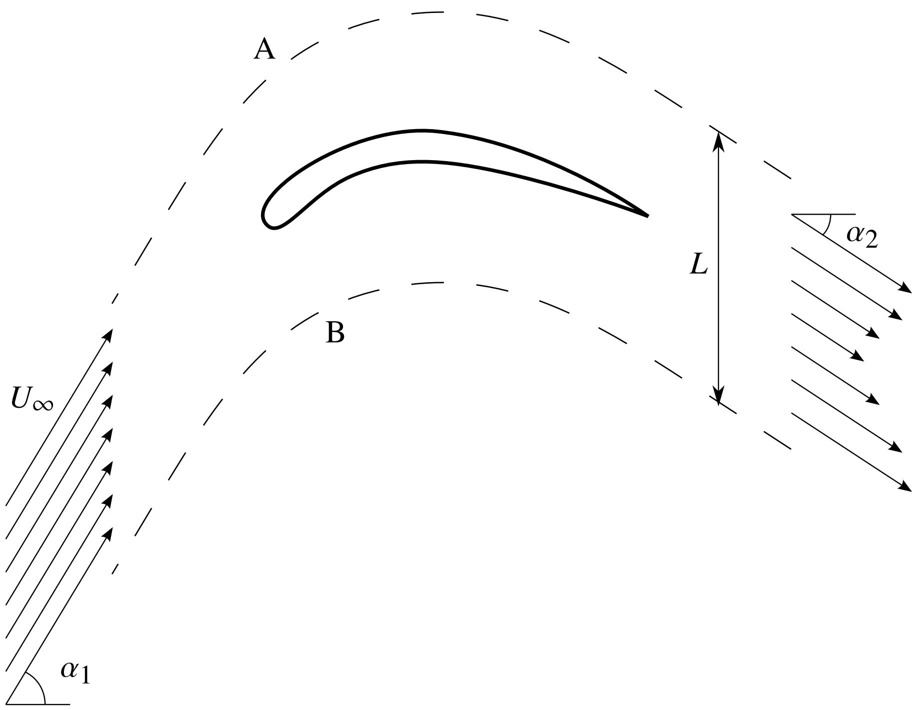

Fig. 4

SECTION B

7 (**short**)

(a) A train is traveling forwards with velocity v. A child throws a tennis ball towards the oncoming train with a horizontal speed u relative to the ground. Assuming the collision is elastic, and the train is far heavier than the ball, at what speed v_B does the ball rebound? [4]

(b) Three balls of masses $m_1 \gg m_2 \gg m_3$ are vertically stacked at a height h, and then dropped to the ground, as shown in Fig. 5. Assuming that the balls start with small initial separations, and that all collisions occur elastically, find the height to which the mass m_3 rebounds. (You may ignore the radius of the balls.) [6]

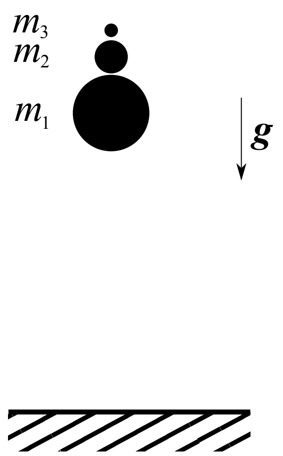

Fig. 5

8 (**short**) An open topped cart is moving at velocity v along a horizontal path, without any friction. It is raining, and the cart is filled by a mass Ω of water each second, as shown in Fig. 6.

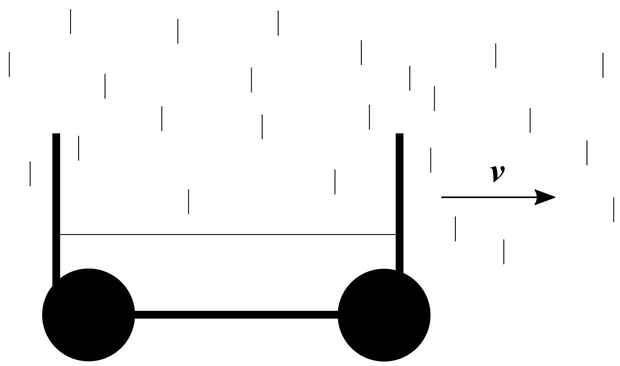

Fig. 6

(a) Find the horizontal force F that must be applied to the cart to maintain its velocity. [4]

(b) Find the power required to maintain the cart's velocity. [3]

(c) Find the efficiency with which this power is converted to kinetic energy. Where does the balance of the energy go? [3]

9 (**long**) A simple seesaw consists of a uniform rod of length $2l$ and mass m with a pivot at its center that allows it to rotate to form an angle θ with the horizontal. Two children, of masses m and $2m$, sit on the ends of the seesaw, with the heavier child touching the ground, and the seesaw making an initial angle θ_0 as shown in Fig. 7. The heavier child then applies an impulse P vertically upwards to launch the seesaw into motion.

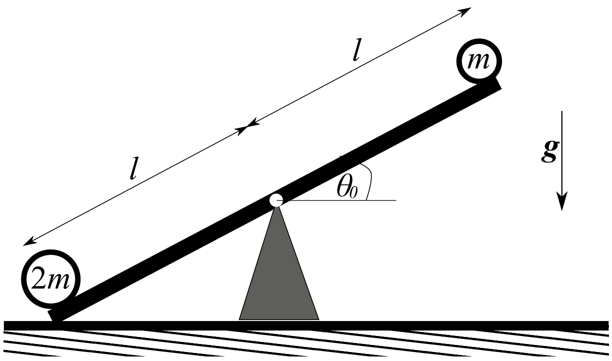

Fig. 7

(a) Show that angular velocity, $\dot{\theta}_0$, of the seesaw directly after the impulse is: [5]

$$\dot{\theta}_0 = -\frac{3P\cos\theta_0}{10ml}.$$

(b) Find the reaction impulse applied to the rod by the pivot, giving your answer in horizontal and vertical components. [5]

During the motion of the seesaw, sliding within the pivot joint causes a frictional torque τ of constant magnitude. When either child reaches the ground, they apply a vertical impulse that reverses their velocity.

(c) Find the equation of motion for θ, and describe a forward Euler numerical integration scheme to predict the seesaw's subsequent motion over multiple oscillations. [8]

(d) Sketch a graph of $\theta(t)$ for the case where the lighter child touches the ground twice. [6]

(e) The initial impulse is chosen such that the lighter child just makes contact with the ground. Find an expression for the initial impulse. [6]

10 (**short**) A particle moves in a spiral trajectory defined by constant angular velocity $\dot{\theta} = \Omega$ and radial position given by $r = r_0\,\mathrm{e}^{\beta t}$, where r_0 and β are positive constants and t is time.

(a) Calculate the velocity and acceleration of the particle in polar coordinates. [6]

(b) For what combination of parameters does the radial component of the acceleration equal 0? [4]

 (TURN OVER

11 (**short**) A micromachined accelerometer embedded in a smartphone is represented by a mass-spring-dashpot system as shown schematically in Fig. 8. The natural frequency for this system is 1000 Hz.

(a) The maximum overshoot in the accelerometer response when the smartphone is subject to unit step input in acceleration is observed to be 25% higher than the final nominal value. Estimate the damping factor and logarithmic decrement of the device. [5]

(b) The phone is left on a machine vibrating at 1000 Hz subjecting the device to a base acceleration, a, of amplitude 1 m s^{-2} at this frequency. Estimate the magnitude of the steady-state displacement of the device. [5]

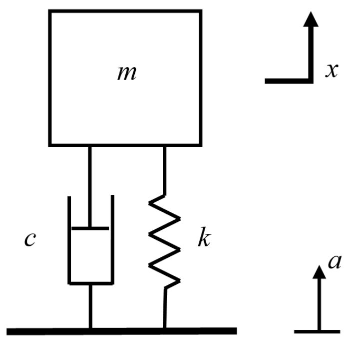

Fig. 8

12 (**long**) A building is being modelled for its response to an earthquake. A schematic of the building is shown in Fig. 9 consisting of a system of two spring coupled masses coupled to the ground by another spring as shown. The value of each mass m is 10^5 kg and the value of the spring constant k is 2×10^5 N m^{-1}. In response to an earthquake the ground displacement is $y = y_0 \cos \omega t$ where ω is the frequency of ground oscillations and y_0 is the ground displacement amplitude.

(a) Show that the equation of motion for this system can be written in the form: [6]

$$\begin{bmatrix} m & 0 \\ 0 & m \end{bmatrix} \begin{bmatrix} \ddot{x}_1 \\ \ddot{x}_2 \end{bmatrix} + \begin{bmatrix} 3k & -k \\ -k & k \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 2ky \\ 0 \end{bmatrix}$$

(b) Determine the natural frequencies and mode shapes for this system. [8]

(c) Determine expressions for the frequency response of the displacements of the two masses and sketch the frequency response for both masses separately. Show on your sketch the impact of added damping. [10]

(d) During an earthquake the ground oscillates at an amplitude $y_0 = 1$ cm at an angular frequency $\omega = 1$ rad s^{-1}. Determine the displacement amplitudes of the masses for this case. [6]

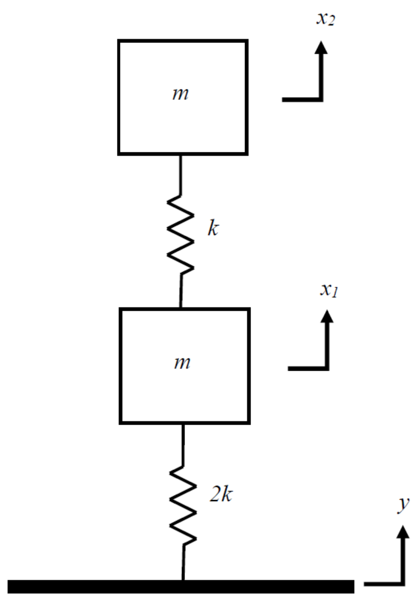

Fig. 9

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