Version AJK/3

EGT0
ENGINEERING TRIPOS PART IA

Wednesday 11 June 2025 9 to 12.10

Paper 1

MECHANICAL ENGINEERING

*Answer **all** questions.*

*The **approximate** number of marks allocated to each part of a question is indicated in the right margin.*

Answers to questions in each section should be tied together and handed in separately.

*Write your candidate number **not** your name on the cover sheet.*

STATIONERY REQUIREMENTS
Single-sided script paper

SPECIAL REQUIREMENTS TO BE SUPPLIED FOR THIS EXAM
CUED approved calculator allowed
Engineering Data Book

10 minutes reading time is allowed for this paper at the start of the exam.

You may not start to read the questions printed on the subsequent pages of this question paper until instructed to do so.

You may not remove any stationery from the Examination Room.

SECTION A

1 (**short**) Figure 1 shows a cylindrical tank with diameter d_1. The tank contains an upside-down J-shaped pipe with diameter d_2 that protrudes distance h_2 below the tank and is connected to a cylinder of diameter d_3, whose bottom end is distance h_3 from the bottom of the tank. The cylinder contains a rod and piston. The piston is a one-way valve that permits flow upwards but not downwards through the piston. The tank diameter d_1 is much greater than d_2 and d_3.

(a) The tank, cylinder, and pipe initially contain air. The tank is filled with water until its depth, h, reaches h_1. After filling, what are the gauge pressures at locations 1 to 5? [2]

(b) The rod is pulled up such that the J-shaped pipe completely fills with water, which starts to flow out of the protruding pipe. Stating all assumptions, what are the gauge pressures at locations 1 to 5? [2]

(c) Neglecting friction, derive an expression for the volumetric flowrate out of the tank as a function of the height, h, of water in the tank. [4]

(d) The rod is hollow. How could the rod be altered such that the tank drains to h_3 when the rod is held up but only half-drains when the rod is dropped back down as the tank drains? [2]

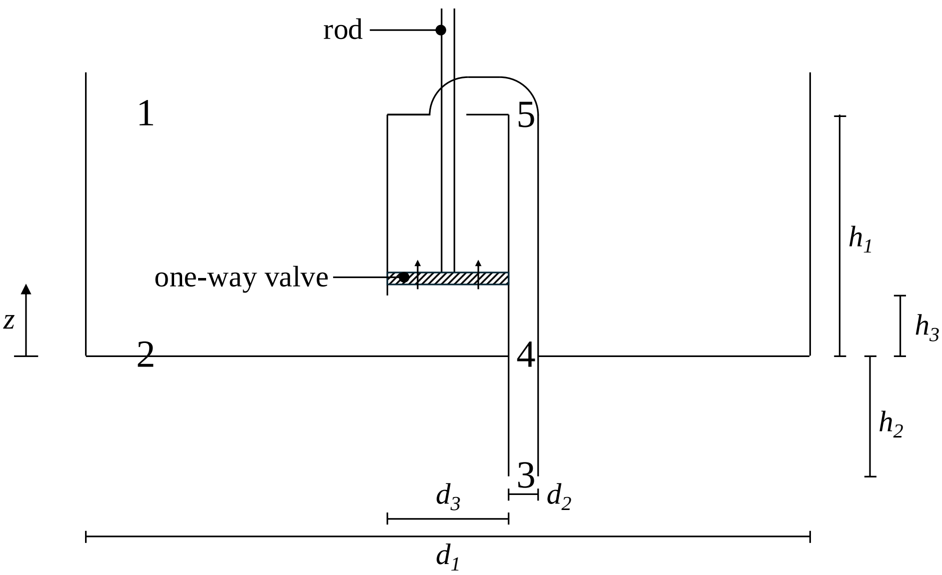

Fig. 1

(TURN OVER

2 (**long**) Figure 2 shows the plan view of a channel formed during a dam break. The bottom of the channel is at the same height as the bottom of the reservoir. The water surface in the reservoir has height h_0, which can be assumed to remain constant. The water in the channel has width $b(x)$, surface height $h(x)$, and speed $v(x)$. The streamline curvature can be neglected, meaning that all flow properties vary only in the x-direction. For this problem, a non-dimensional number is defined as $\mathrm{Fr} = v/\sqrt{gh}$, where g is the acceleration due to gravity.

(a) Explain why, at a given x-location, the pressure variation in the vertical z-direction is hydrostatic: i.e. $p(z) + \rho g z = 0$. [2]

(b) If friction with the channel walls can be ignored and there is no sudden increase in h (known as a hydraulic jump), explain why Bernoulli's equation can be used to relate $h(x)$ and $v(x)$ throughout the flow with $v^2 + 2gh = \text{const}$. [4]

(c) Properties at the narrowest section of the channel are labelled with subscript 1. Given knowledge that $Fr_1 = 1$, find the simplest possible expressions for h_1 and v_1 in terms of g and h_0, and find an expression for the volumetric flowrate, Q, through the channel in terms of g, h_0, and b_1. [6]

(d) Downstream of the narrowest section, the depth $h(x)$ continues to drop. What happens to the speed, $v(x)$? What is the maximum speed that the flow can attain? [2]

(e) Further downstream there is a hydraulic jump through which the height $h(x)$ suddenly increases from h_2 to h_3 and Fr changes from Fr_2 to Fr_3. The jump is sufficiently short that you may assume b to be uniform across the jump. Defining $H = h_3/h_2$ and considering a control volume around the hydraulic jump,

(i) use conservation of mass to show that [2]

$$\mathrm{Fr}_2^2 = H^3 \mathrm{Fr}_3^2$$

(ii) use the steady flow momentum equation to show that [8]

$$2\mathrm{Fr}_2^2 + 1 = H^2(2\mathrm{Fr}_3^2 + 1)$$

(f) Check that the above equations are satisfied if there is no jump ($H = 1$). Factorize out this solution and then find the other two solutions for H in terms of Fr_2. State which solution is unphysical. What is H if $\mathrm{Fr}_2 = \sqrt{3}$? [6]

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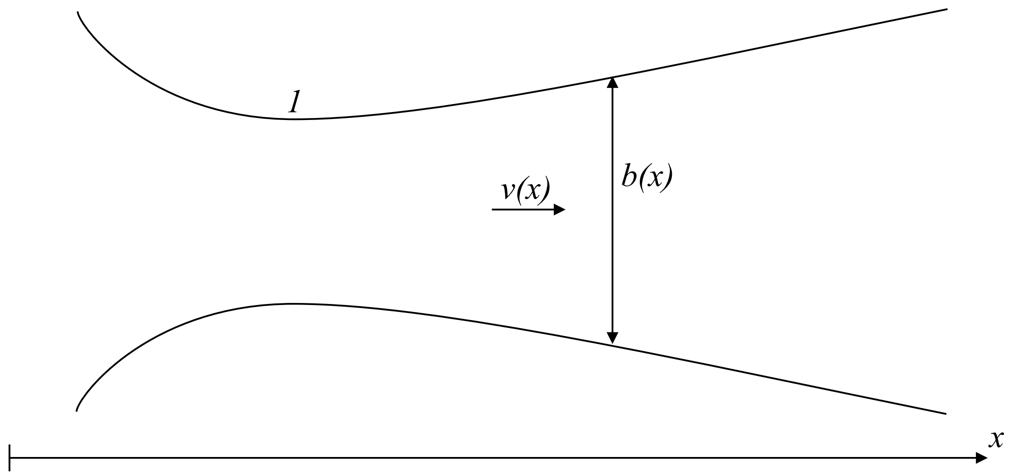

Fig. 2

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3 (**short**) The speed, V, of a cylindrical water jet with cross-sectional area A and density ρ is to be measured by placing a cone with half-angle θ on the centreline of the jet and measuring the force, F, required to keep the cone stationary, as shown in Fig. 3. The jet can be considered to be inviscid.

(a) Explain why the water can be assumed to leave the rear edge of the cone at speed V. [2]

(b) With a simple model, calculate the horizontal force, F, on the cone. Sketch F as a function of θ for $0 < \theta < \pi$. When and why will this simple model break down? [4]

(c) Downstream of the cone, the conical sheet of water has radius r and thickness t, where $r \gg t$. Neglecting gravitational effects, derive an expression for t as a function of r. [4]

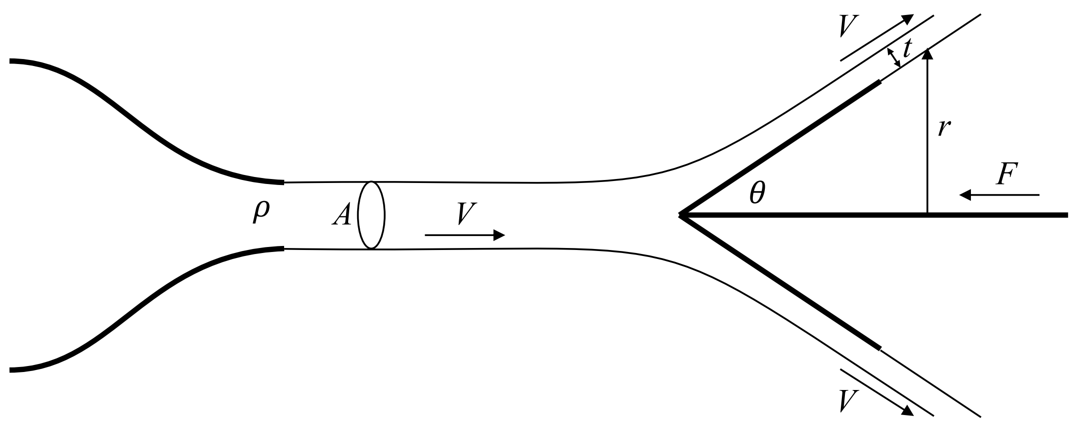

Fig. 3

4 (**short**) In a polytropic process, the pressure, p, and specific volume, v, of a perfect gas are related by $pv^n = \text{const}$.

(a) What types of process are represented by $n = \gamma, 1$, and 0. What is n for a constant volume process? [2]

(b) A gas in a cylinder expands against a fully-resisted piston in a polytropic process. Derive an expression for the work done by the gas in terms of its initial state (p_1, v_1), its final state (p_2, v_2), and n, where $n \neq 1$. [4]

(c) Sketch $p - v$ and $T - s$ diagrams of the process in (b) for

(i) $n = \gamma$,

(ii) $n = 1$,

from the same initial state (p_1, v_1) to the same final volume v_2. [4]

(TURN OVER

5 (**long**) A ramjet is an air-breathing propulsion engine that contains no moving parts, as shown in Fig. 4. Air at thermodynamic state 1 enters at speed v_1, decelerates to thermodynamic state 2, which has negligible speed, is heated at uniform pressure to thermodynamic state 3, which also has negligible speed, and then accelerates to thermodynamic state 4 at speed v_4. You may assume that air is a perfect gas with $\gamma = 7/5$, $c_p = 1005$ J kg^{-1} K^{-1}, and $R = 287.1$ J kg^{-1} K^{-1}.

(a) Show that

$$\frac{T_2}{T_1} = 1 + \frac{v_1^2}{2c_pT_1}$$ [2]

(b) Assuming for now that the flow decelerates from 1 to 2 reversibly and adiabatically, derive an expression for the isentropic compression ratio $p_{2_{isen}}/p_1$in terms of $v_1/\sqrt{2c_pT_1}$ and sketch $p_{2_{isen}}/p_1$ as a function of $v_1/\sqrt{2c_pT_1}$. [6]

(c) Calculate T_2 and $p_{2_{isen}}$ if $T_1 = 200$ K, $p_1 = 0.03$ MPa, and $v_1/\sqrt{2c_pT_1} = 2$. Comment on your results. What value of v_1 does this correspond to? [4]

(d) In reality, the flow between 1 and 2 decelerates irreversibly and adiabatically and the real compression ratio p_2/p_1 is 20% of the isentropic compression ratio calculated above. Show that the specific entropy increase is $s_2 - s_1 = 462.1$ J kg^{-1} K^{-1} and sketch this process on a $T - s$ diagram. [6]

(e) The heat addition per unit mass of air is $q = 1.005 \times 10^6$ J kg^{-1}. Calculate T_3. [4]

(f) If the exit area is carefully chosen such that $p_4 = p_1$ and if the acceleration from state 3 to state 4 is isentropic, sketch the full cycle on a new $T - s$ diagram. [4]

(g) Calculate v_4 and the engine's net thrust per unit mass of air passing through the engine. You may assume that the mass of fuel is negligible. [4]

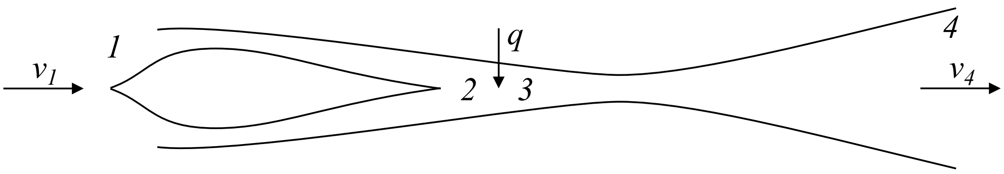

Fig. 4

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6 (**short**) A college is considering whether to heat a building with a gas boiler or an electrically-driven heat pump.

(a) If the heat pump operates between a cold-side temperature T_c and a hot-side temperature T_h, what is its maximum possible coefficient of performance. [2]

(b) Due to irreversibilities in the heat pump, the actual work required is a multiple α of the work that would be required if the heat pump were reversible. What is the actual coefficient of performance of the heat pump? [2]

(c) If the electricity is generated exclusively by a gas-fired power station that converts a proportion, η, of the chemical power into electrical power, at what ratio T_h/T_c would the gas boiler be more energy-efficient than the heat pump? For simplicity, you may assume that the gas boiler converts all of the chemical power into heat. [2]

(d) If $T_c = 273$ K, $\eta = 0.6$, and $\alpha = 2.0$, at what value of T_h would consumption of gas be reduced by using the gas boiler instead of the heat pump? Comment on your answer. What other engineering factors should the college consider? [4]

SECTION B

7 (**short**) A geostationary satellite orbits the Earth directly above the equator, at the same rotational speed as the Earth's rotation. This means that it remains fixed over a specific point on the Earth's surface.

(a) Find the altitude of the satellite above the Earth's surface. [6]

(b) Calculate the work done on the satellite to launch it into orbit from Earth. Assume that the take-off site is equatorial and that the initial kinetic energy due to Earth's rotation may be neglected. [4]

Gravitational constant	G	6.67×10^{-11} N m^2 kg^{-2}
Radius of the Earth	R	6380 km
Mass of the Earth	M	5.97×10^{24} kg
Mass of the satellite	m	1000 kg

8 (**short**) As shown in Fig. 5, a rigid uniform bar of mass m and length l hangs vertically under gravity from a frictionless hinge located at the top end of the bar. The bar is initially at rest. The bar is then impacted by a projectile moving horizontally at speed v and mass m at a distance $2l/3$ below the hinge. The projectile remains attached to the bar after impact.

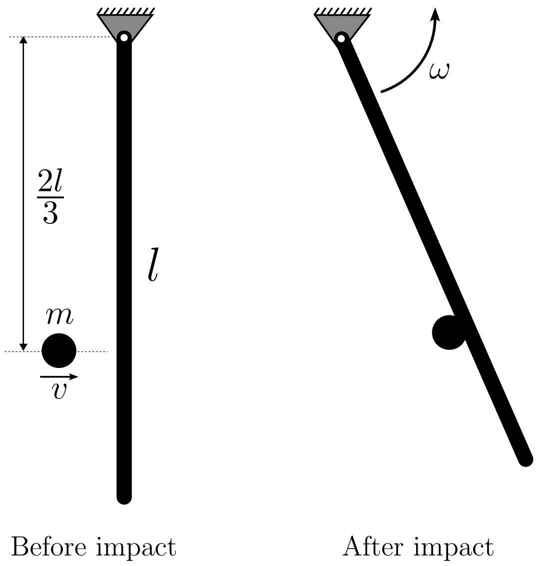

Fig. 5

(a) Find the angular velocity ω of the bar just after impact. [5]

(b) Find an expression for the speed v of the projectile that is required to rotate the bar at least 180 degrees after impact, i.e. vertically above the hinge. [5]

9 (**long**) Figure 6 shows a mechanism in a vertical plane. A pendulum is made of a point mass m attached by a light rod of length l to a pulley of radius $r < l$. An inextensible cable is anchored on the rim of the pulley, and then connected to a second mass m through another pulley. The pulleys are massless and frictionless. The cable and rod are massless. The two masses are subject to gravity. The angle θ is the rotation of the pendulum from vertical.

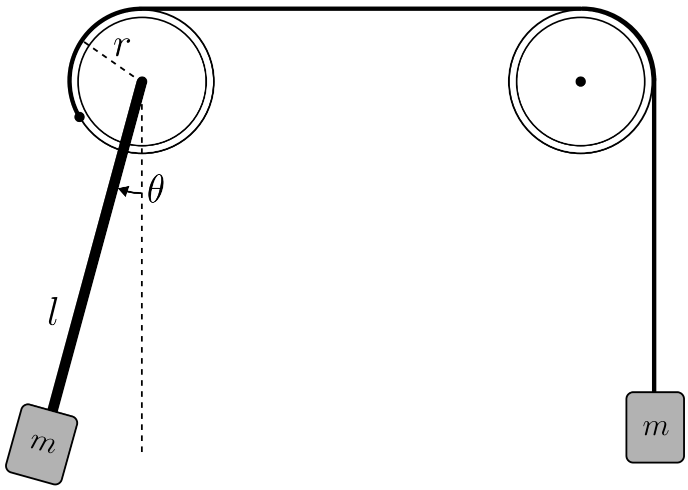

Fig. 6

(a) Find the equilibrium angle θ_{eq} of the pendulum. [7]

(b) The pendulum is displaced by a small angle $\delta\theta$ from its equilibrium position θ_{eq} and released from rest.

(i) Derive the equation of motion of the system by taking moments about the pivot point of the left pulley, and find an expression for the angular frequency of the oscillations. [8]

(ii) Write an expression for the total energy E of the system (to within an arbitrary additive constant). [5]

(iii) Writing the angle θ as $\theta_{eq} + \delta\theta$ and using the fact that $\delta\theta \ll \theta_{eq}$, show that this expression can be approximated as:

$$E = \frac{1}{2}m(r^2 + l^2)\delta\dot{\theta}^2 + \frac{1}{2}mgl\cos(\theta_{eq})\delta\theta^2 + \text{constant}$$

[5]

(iv) By differentiating this expression with respect to time or otherwise, verify your calculation of part (b)(i) for the angular frequency of the oscillations. [5]

10 (**short**) Figure 7 shows the geometry of an oscillating engine, where the cylinder AB rocks as the link BC rotates continuously. Suppose that BC rotates at constant $\omega = \pi$ rad s^{-1}. At $t = 0$, $\theta = 0$ and x is at its minimum value.

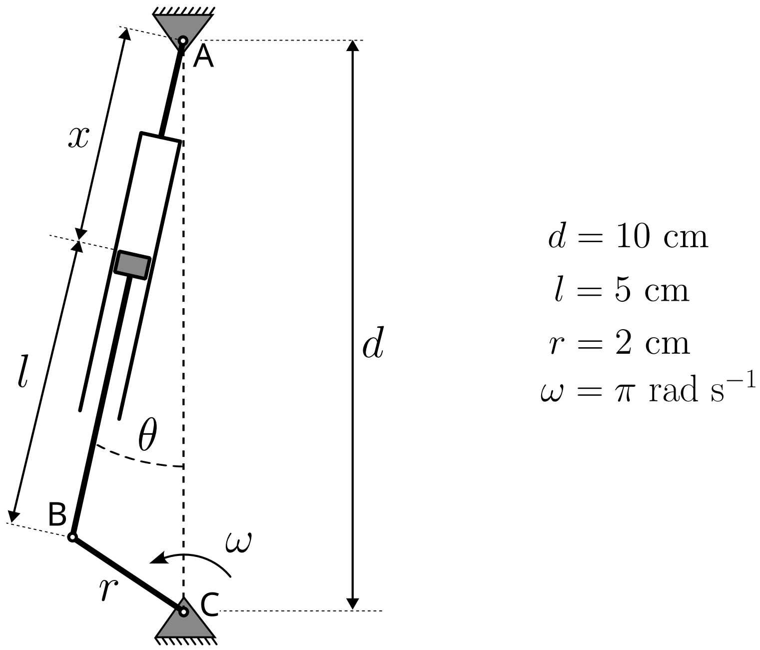

Fig. 7

(a) Sketch the functions $x(t)$ and $\theta(t)$ for $0 \leq t \leq 2$ s. Calculate the minimum and maximum values of x and θ and the times at which they are reached. Show these positions on your sketches. [7]

(b) Calculate $\dot{x}$ and $\dot{\theta}$ at $t = 0$. [3]

11 (**short**) Figure 8 shows a rigid flywheel with radius R and mass M mounted at one end of a vertical, elastic, and massless shaft with torsional stiffness k. The other end of the shaft is fixed. The angle of rotation of the flywheel from its equilibrium position is α.

(a) Derive an expression for the natural frequency of the system for torsional oscillations. Determine the natural frequency for M = 40 kg, R = 0.3 m, and $k = 1.2\times10^6$ Nm rad^{-1}. [3]

(b) If $\alpha = 0$ and $\dot{\alpha} = 200$ rad s^{-1} at t = 0, find the angular amplitude of the subsequent motion. [3]

(c) In a real system, it is observed that after 3 cycles, the amplitude of the motion has decreased by 95%. Compare the damped and undamped natural frequencies. [4]

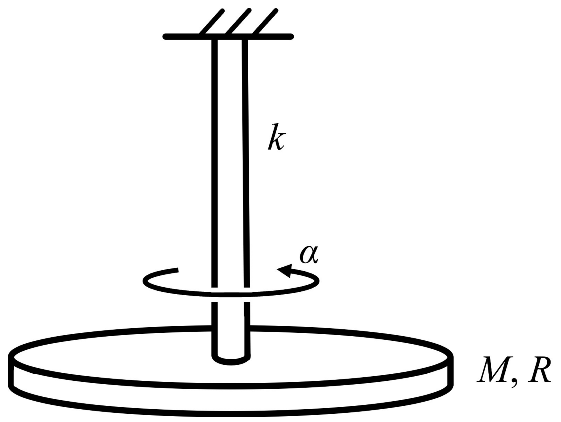

Fig. 8

12 (**long**)

Figure 9 shows a simplified model which may be used to analyse the vibrations of a vehicle. The mass of the wheel is m and the mass of the vehicle it supports is $3m$. The masses are connected by a spring of stiffness k, accounting for the coil spring of the suspension. The mass m is contacting the road surface via a spring of stiffness $2k$ modelling the elasticity of the tyre. The vertical displacements of the masses $3m$ and m are y_1 and y_2, respectively, measured upwards from the equilibrium position. The input excitation from road irregularities is x.

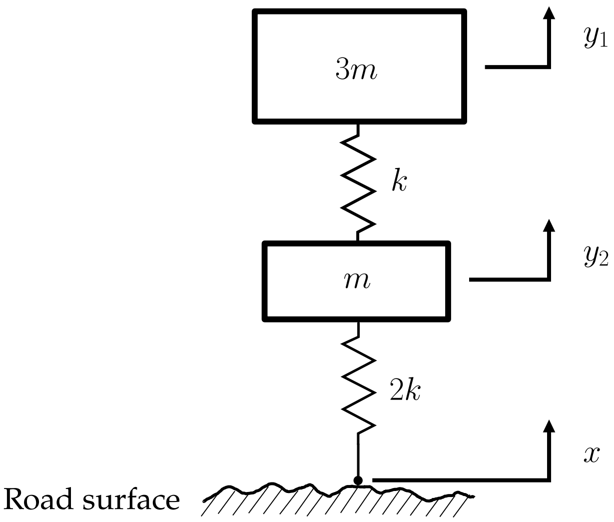

Fig. 9

(a) Find mass and stiffness matrices for the system. Use these to find the natural frequencies and mode shapes of the system and sketch the mode shapes. [12]

(b) For $x = X\cos(\omega t)$, the vehicle and wheel displacements will have the form $y_1 = Y_1\cos(\omega t)$ and $y_2 = Y_2\cos(\omega t)$, respectively. Sketch Y_1/X and Y_2/X as a function of ω. Label all salient values on both axes. [12]

(c) Sketch $|Y_1/X|$ and $|Y_2/X|$ on a new figure, and show qualitatively the effect of damping in the tyre and suspension on these curves. [6]

END OF PAPER