Version RGM/8

EGT0
ENGINEERING TRIPOS PART IA

Wednesday 8 June 2022   9 to 12.10

Paper 1

MECHANICAL ENGINEERING

*Answer **all** questions.*

*The **approximate** number of marks allocated to each part of a question is indicated in the right margin.*

Answers to questions in each section should be tied together and handed in separately.

*Write your candidate number **not** your name on the top sheet.*

STATIONERY REQUIREMENTS
Single-sided script paper

SPECIAL REQUIREMENTS TO BE SUPPLIED FOR THIS EXAM
CUED approved calculator allowed
Engineering Data Book

10 minutes reading time is allowed for this paper at the start of the exam.

You may not start to read the questions printed on the subsequent pages of this question paper until instructed to do so.

You may not remove any stationery from the Examination Room.

SECTION A

1 (**short**)

(a) An experimentalist passes heat Q to a mass m of a perfect gas in a closed rigid vessel. Write down an expression for the change in temperature, ΔT_v, of the gas. [1]

(b) The experimentalist passes the same heat Q to the same mass m of the same gas in a cylinder bounded by a frictionless piston that is free to expand against atmospheric pressure, p_a. Write down an expression for the change in temperature, ΔT_p, of the gas. [1]

(c) If the perfect gas is air, is $\Delta T_v > \Delta T_p$ or is $\Delta T_p > \Delta T_v$? Why? [2]

(d) Derive an expression for the work done, W, by the gas in part (b), in terms of Q and the ratio of specific heats, γ. Give a physical interpretation of $(\gamma - 1)/\gamma$. [4]

(e) Can you think of a common material (not necessarily a gas) for which ΔT_p would be greater than ΔT_v? Explain your answer. [2]

2 (**short**) The Coefficient of Performance of a heat pump, COP_P, is defined as

$$COP_P = \frac{\text{heat to hot space}}{\text{work in}}$$

(a) A domestic heat pump takes heat from the atmosphere at 5 Celsius. Calculate the maximum COP_P when:

(i) maintaining water at 40 Celsius for central heating;

(ii) maintaining water at 50 Celsius for washing;

(iii) maintaining air at 250 Celsius for cooking in the oven.

Comment on your answers. [4]

(b) Calculate the minimum work required to heat 5 kg of water from 20 Celsius to 90 Celsius taking heat from the atmosphere at 10 Celsius. [6]

(TURN OVER

3 (**long**) The air-standard Atkinson cycle models a reciprocating engine where the isentropic compression (1→2) and constant-volume heat addition (2→3) processes are identical to the air-standard Otto cycle. Work is produced during isentropic expansion (3→4), where the expansion ratio $r_e = V_4/V_3$ from volumes V_3 to V_4 is larger than the compression ratio $r_c = V_1/V_2$ from volumes V_1 to V_2. The expansion ratio results in the final pressure $p_4 = p_1$ such that the heat release is isobaric. Assume that the cycle fluid is air which can be treated as a perfect gas with constant specific heat capacity c_v and ratio of heat capacities γ.

(a) Sketch representative $p - V$ and $T - s$ diagrams for the air-standard Atkinson cycle. [5]

(b) Show that the temperature after heat addition is $T_3 = T_1 r_c^{\gamma-1} + q_{2-3}/c_v$, where T_1 is the initial temperature and q_{2-3} is the heat input per unit mass of air flow. [6]

(c) Show that the ratio $r = r_e/r_c$ for expansion such that $p_4 = p_1$ is

$$r = \left(1 + \frac{q_{2-3}}{T_1 c_v r_c^{\gamma-1}}\right)^{1/\gamma}.$$

[8]

(d) A given air-standard Atkinson engine has $V_1 = 0.3$ m^3 and $r_c = 8$. The conditions prior to compression are temperature $T_1 = 300$ K and pressure $p_1 = 1$ bar. The heat input per unit mass of air flow is $q_{2-3} = 1$ MJ kg^{-1}. The air properties are $\gamma = 1.34$, gas constant $R = 287$ J kg^{-1} K^{-1} and specific heat capacity at constant volume $c_v = 834$ J kg^{-1} K^{-1}.

(i) Calculate the temperatures after heat addition T_3 and after expansion T_4. [5]

(ii) Calculate the efficiency of the engine. [6]

4 (**short**) A tank of water of density ρ feeds two taps, at heights h_1 and h_2 below the tank's free surface, under the effect of gravity g, as shown in Fig. 1. The tank and the taps have uniform cross-sectional areas A_0 and A_1, respectively. Friction may be neglected throughout and the flow can be considered quasi-steady.

(a) Find the two expressions for the exit velocities at the taps, V_1 and V_2, as a function of ρ, g, h_1, h_2, and the velocity of the water at the free surface in the tank, V_0. Assuming that $V_0 \ll \sqrt{2gh_1}$, simplify the above expressions and give V_1 and V_2 as a function of ρ, g, h_1, h_2. [6]

(b) Use mass conservation to obtain V_0 as a function of A_0 and A_1 and the previously obtained V_1 and V_2. For a system with g = 10 m s^{-2}, h_1 = 3 m, h_2 = 6 m, A_0 = 0.4 m^2 and A_1 = 4 cm^2, check if the above simplified method is justified. [4]

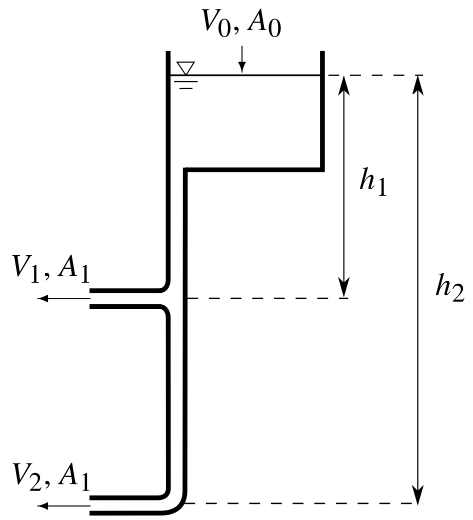


Fig. 1

5 (**short**) An incompressible flow moves in circular, concentric streamlines between radius R_0 and R_1, as illustrated in Fig. 2. The velocity at $r = R_0$ is V_0.

(a) Derive an expression from first principles for the gradient of the pressure normal to the streamlines in terms of ρ, v, and r, where v is the local value of the velocity. [4]

(b) The Bernoulli constant is the same for all streamlines (i.e. it does not depend on r). By differentiating the Bernoulli equation in the direction normal to the streamlines, and using the result from (a), or otherwise, find the distribution of velocity $v(r)$, and give the value V_1 of the velocity at $r = R_1$. [6]

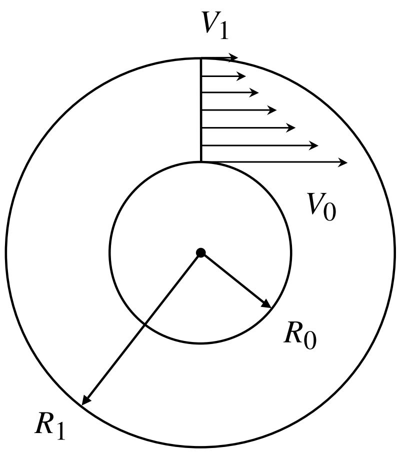


Fig. 2

6 (**long**) Flow in a pipe of cross-sectional area A_1 undergoes a sudden expansion into a pipe of cross-sectional area A_2, as shown in Fig. 3. The flow is incompressible with density ρ. Far upstream of the expansion, the flow has uniform velocity V_1 and pressure p_1. Far downstream of the expansion, the flow has unknown uniform velocity V_2 and pressure p_2. In the expansion, the flow enters section B as a uniform jet with velocity V_1, and subsequently undergoes mixing. The pressure in section B can be considered uniform throughout the section. The outside ambient pressure is p_a. Friction with the pipe walls may be neglected throughout.

(a) Find the velocity downstream of the expansion, V_2, in terms of V_1, A_1 and A_2. [2]

(b) By considering the control volume that encloses the fluid between sections B and C, or otherwise, find the pressure downstream of the expansion, p_2, in terms of ρ, p_1, V_1, A_1 and A_2. [12]

(c) By considering a different control volume, or otherwise, find the magnitude and direction of the force required to restrain the pipe, F. [6]

(d) Two Pitot tubes connected by a manometer with closed valves containing a column of liquid of density ρ_ℓ are installed upstream and downstream of the contraction, as shown in Fig. 3. If the valves are opened, find the resulting height difference across the manometer, Δh, in terms of ρ, ρ_ℓ, g, p_1, V_1, A_1 and A_2. [10]

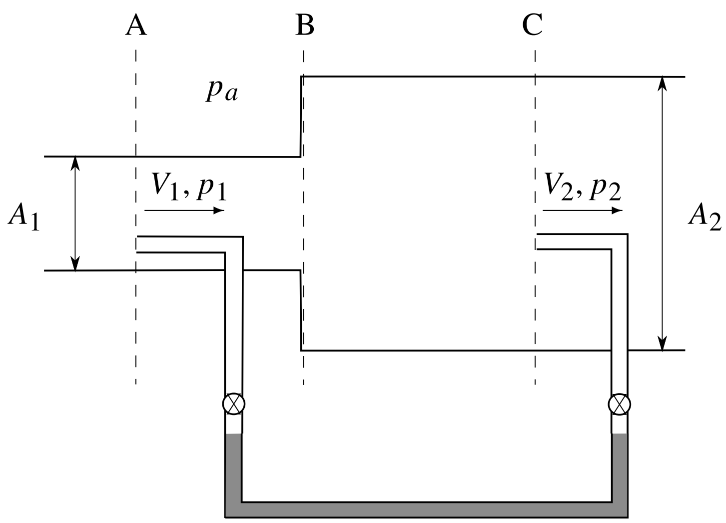


Fig. 3

SECTION B

7 (**short**) A pendulum shock testing machine consists of a mass m on the end of a light rigid rod of length l that swings freely to impact a viscous buffer of damping rate λ located directly below the pendulum pivot, as shown in Fig. 4.

(a) Calculate the velocity of the mass at the moment of impact with the buffer if the pendulum is released from rest at the horizontal position. [1]

(b) Derive the equation of motion of the mass in terms of its velocity after the impact and calculate the peak acceleration. [5]

(c) Stating your assumptions, calculate the distance travelled by the mass. [4]

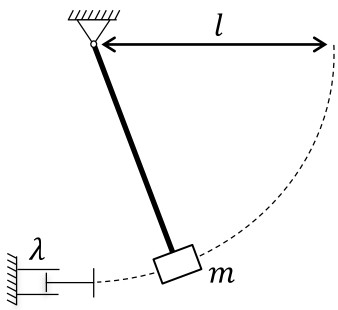


Fig. 4

8 (**short**) A spinning firework is made from a uniform disk of radius a and mass M that is free to rotate about a pin through its central axis, as shown in Fig. 5. Two rockets are mounted at the edge of the disk such that they fire in opposite directions, causing the disk to spin.

(a) By modelling each rocket as a thin rod of length $3a/2$ and mass $M/2$, calculate the mass moment of inertia of the firework. [3]

(b) Each rocket ejects mass at a constant rate $\dot{m}$ with relative velocity v. Ignoring any effect of gravity, calculate the initial angular acceleration of the firework. [4]

(c) Assuming that air resistance is negligible, that the ejected mass is removed uniformly along the rods, and that their whole mass can be ejected, sketch the variation of the angular acceleration with time. [3]

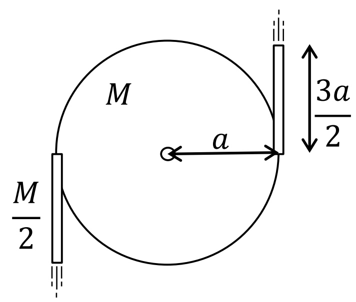


Fig. 5

9 (**long**) A rigid uniform plank of length $\sqrt{3}\ell$ rests on two supports with its midpoint located a distance $\ell/2$ from each support, as shown in Fig. 6(a). Support B fails suddenly and the plank rotates about support A before hitting a fixed horizontal rail C located a distance $\ell/2 + c\ell/2$ from A at angle θ_0, as shown in Fig. 6(b).

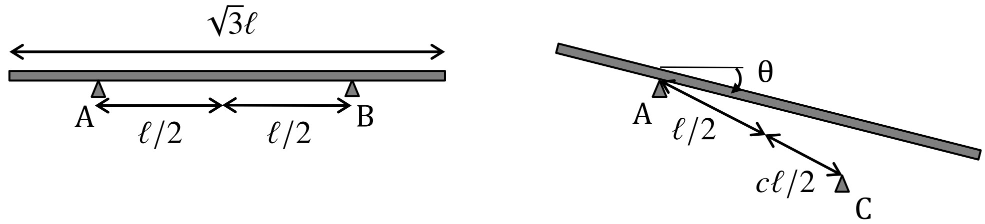
(a) Assuming no slip at A, find an expression for the angular acceleration of the plank $\ddot{\theta}$ as a function of its rotation θ. [6]

(b) Find a similar expression for the angular velocity of the plank $\dot{\theta}$ and its value ω_0 at the moment of impact with the rail. [6]

(c) If the coefficient of restitution between the plank and the rail is e, calculate the subsequent angular velocity of the plank ω_1 assuming there is no impulsive reaction at support A. [8]

(d) By considering the subsequent velocity of point A on the plank, find the limiting value of the distance AC (i.e. the value of constant c) such that there is no impulsive reaction at support A. [6]

(e) Assuming an elastic impact with the rail and c = 1, find the greatest height reached by the midpoint of the plank during the subsequent motion. [4]



(a) (b)

Fig. 6

10 (**short**) A particle of mass m slides on a frictionless horizontal table. The particle is attached by a long light inextensible string, which passes through a small frictionless hole, to a second mass, also of mass m, attached to a spring of stiffness k, as shown in Fig. 7.

(a) The particle is moving in a circular orbit of radius R such that the tension in the spring is zero. Determine the tension in the string. [2]

(b) A tangential impulse is applied to the orbiting particle such that it moves out to a maximum radius of $2R$.

(i) Calculate the velocity of the particle at this radius. [5]

(ii) Calculate the new tension in the string. [3]

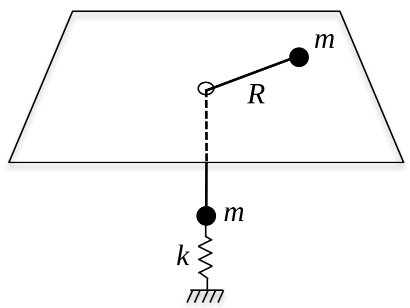


Fig. 7

11 (**short**) A rubber ball of mass m bounces off a hard horizontal floor, as shown in Fig. 8. The ball makes contact with the floor at time $t = 0$ with a downward speed U and it leaves the floor at time $t = t_1$ with an upward speed V. This contact period is shown shaded in the graph of the motion. During the bounce the contact with the floor is modelled as a spring k in parallel with a dashpot λ.

For this question you should refer to the impulse response information on page 7 of the Mechanics Data Book. You may assume that the damping ratio $\zeta \ll 1$.

(a) Show that $t_1 \approx \frac{\pi}{\omega_n}$ where $\omega_n = \sqrt{\frac{k}{m}}$. [3]

(b) The coefficient of restitution for the bounce is defined as $e = \frac{V}{U}$, as shown in the graph of the motion.

Show that $e \approx 1 - a\zeta$ and find the value of the constant a. [7]

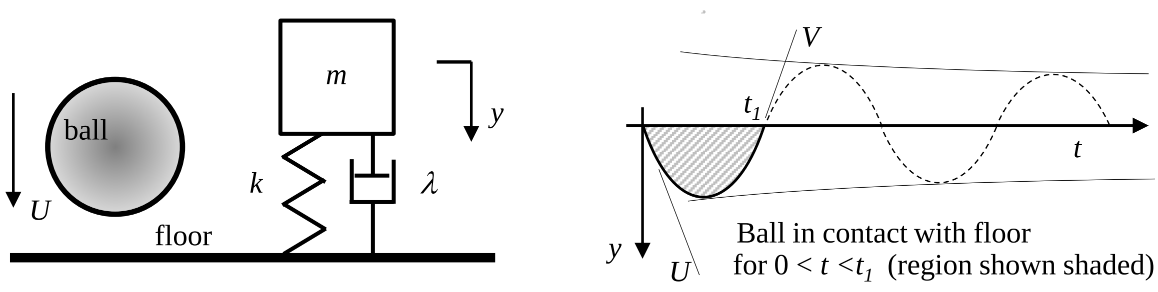


Fig. 8

12 (**long**) Two carts of mass m, connected by a spring of stiffness k, are rolling without friction on a horizontal track, as shown in Fig. 9(a). Motion of cart 1 is described by its displacement x and that of cart 2 by its displacement y.

(a) (i) Write down (or derive) a matrix equation of motion for the system of carts. [5]

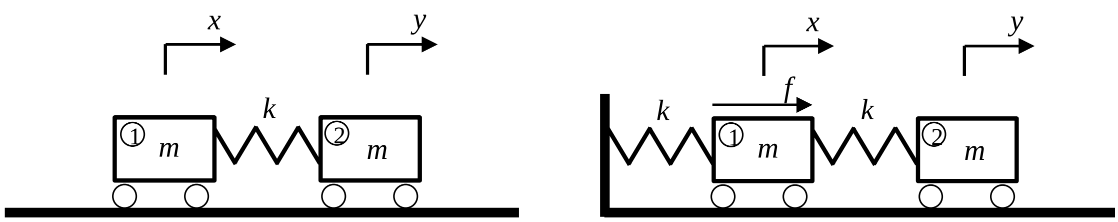
(ii) By inspection or otherwise find the natural frequencies and mode shapes. [5]

(b) A second spring, also of stiffness k, is used to anchor cart 1 to the ground, as shown in Fig. 9(b). A sinusoidal force f is applied to cart 1 at frequency ω such that $f(t) = F\cos(\omega t)$ and the responses are $x(t) = X\cos(\omega t)$ and $y(t) = Y\cos(\omega t)$. The amplitude of motion of cart 1 is given by the expression

$$X(\omega) = \frac{A(1 - \omega^2/\omega_0^2)}{(1 - \omega^2/\omega_1^2)(1 - \omega^2/\omega_2^2)}$$

(i) By solving the equations of motion or otherwise find expressions for A, ω_0, ω_1 and ω_2 in terms of k, m and F. [15]

(ii) Sketch the variation of $X(\omega)$ with angular frequency ω. Indicate on your sketch the values of ω_0, ω_1 and ω_2. [5]



(a) (b)

Fig. 9

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