Version RGM/5

EGT0
ENGINEERING TRIPOS PART IA

Wednesday 5 June 2024   9 to 12.10

Paper 1

MECHANICAL ENGINEERING

*Answer **all** questions.*

*The **approximate** number of marks allocated to each part of a question is indicated in the right margin.*

Answers to questions in each section should be tied together and handed in separately.

*Write your candidate number **not** your name on the top sheet.*

STATIONERY REQUIREMENTS
Single-sided script paper

SPECIAL REQUIREMENTS TO BE SUPPLIED FOR THIS EXAM
CUED approved calculator allowed
Engineering Data Book

10 minutes reading time is allowed for this paper at the start of the exam.

You may not start to read the questions printed on the subsequent pages of this question paper until instructed to do so.

You may not remove any stationery from the Examination Room.

SECTION A

1 (**short**) A thermally-insulated frictionless vertical piston with cross-sectional area $A = 0.01$ m^2 contains a volume $V_1 = 0.01$ m^3 of argon (modelled as a perfect gas with $\gamma = 5/3$) initially at temperature $T_1 = 300$ K and pressure $p_1 = 10^5$ N m^{-2}.

(a) A weight $F = 1000$ N is gently lowered onto the piston such that equilibrium is maintained at all times during the process. What is the pressure p_2 and volume V_2 of the argon at the end of this process? [2]

(b) Calculate the change in internal energy, ΔU, of the argon during this process. [3]

(c) The thermal insulation is removed and the argon returns to $T_3 = T_1 = 300$ K. What is the final volume V_3 of the argon? [1]

(d) How much heat is lost to the surroundings after the thermal insulation is removed? [3]

(e) Explain why your answers to (b) and (d) are different. [1]

2 (**short**) Air with mass flowrate $\dot{m} = 2.00\ \mathrm{kg\,s^{-1}}$ initially at $T_1 = 1000$ K and $p_1 = 2.87$ bar flows adiabatically through a mesh in a horizontal pipe. The pipe has constant cross-sectional area $A = 0.02\ \mathrm{m^2}$. The air temperature drops by 28.66 K. Treat air as a perfect gas with $R = 287\ \mathrm{J\ kg^{-1}K^{-1}}$, $c_p = 1005\ \mathrm{J\ kg^{-1}K^{-1}}$, and $\gamma = 1.4$.

(a) Calculate the density and speed of the air upstream of the mesh. [2]

(b) Calculate the speed and density of the air downstream of the mesh and show that the downstream pressure, p_2 is approximately 1.07 bar. [4]

(c) Sketch this process on a $T - s$ diagram showing relevant lines of constant pressure. [2]

(d) Calculate the rate of irreversible entropy increase across the mesh. [2]

3 (**long**) The Humphrey cycle, which models pulse detonation engines, consists of the following four thermodynamic processes: (i) a gas is compressed isentropically from states 1 to 2, (ii) heat is added at constant volume from states 2 to 3, (iii) the gas is expanded isentropically from states 3 to 4, (iv) heat is rejected at constant pressure from states 4 to 1. This is not a steady flow process. You may consider the gas to be perfect with fixed c_p and $\gamma = 7/5$ throughout the cycle. T_i, p_i and v_i are the temperature, pressure and specific volume, respectively, at each state i.

(a) The pressure ratio, p_2/p_1, of process (i) is denoted r. Write down expressions for T_2/T_1 and v_2/v_1 in terms of r and sketch the cycle on a $T - s$ diagram. [4]

(b) Use a $T\mathrm{d}S$ equation to show that $\mathrm{d}(\ln T)/\mathrm{d}s = 1/c_v$ during process (ii). Express $T_3 - T_2$ in terms of the specific heat added, q, and the specific heat at constant volume, c_v. Show that the change in specific entropy is $s_3 - s_2 = c_v \ln\left(1 + q/(c_v T_2)\right)$. [6]

(c) Use a $T\mathrm{d}S$ equation to show that $\mathrm{d}(\ln T)/\mathrm{d}s = 1/c_p$ during process (iv). [2]

(d) With reference to your previous answers, carefully re-sketch this Humphrey cycle on a new $T - s$ diagram. [6]

(e) The Brayton cycle is identical to the Humphrey cycle but with heat added at constant pressure. For the same pressure ratio, r, and specific heat added, q, calculate T_3 and $s_3 - s_2$. By considering $\mathrm{d}(s_3 - s_2)/\mathrm{d}q$, or otherwise, show that $s_3 - s_2$ is always larger for the Brayton cycle than for the Humphrey cycle and carefully sketch the Brayton cycle on the same $T - s$ diagram as the Humphrey cycle of part (d). [6]

(f) By inspecting the $T - s$ diagram, and without further calculations, state which cycle is more thermodynamically efficient. Explain your answer. [6]

4 (**short**) Water of density ρ is confined by a solid gate of length $4R$ under the effect of gravity g, as shown in Fig. 1. The problem is homogeneous in the direction into the page, and the gate can rotate about a pivot freely in the clockwise direction, but is prevented from rotating anti-clockwise beyond the vertical position by a backstop as shown in the figure. The section of the gate below the pivot is in a circular cavity of radius R that prevents water leakage. The water level is h.

(a) Find the value of h for which the gate will begin to open. [5]

(b) For h lower than the value found in (a), find the force F produced by the hydrostatic pressure on the gate as a function of ρ, g, R, and h. [5]

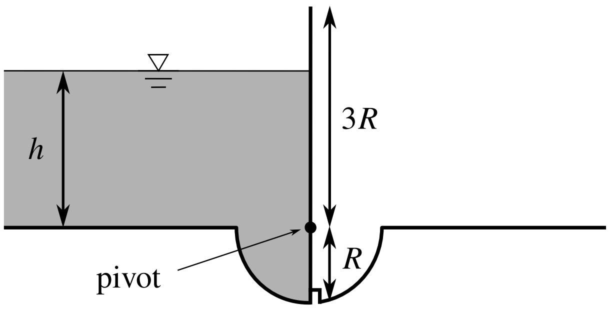

Fig. 1

5 (**short**) A two-dimensional jet of water of density ρ, thickness d_1 and velocity V_1 is deflected by a two-dimensional obstacle forming two diverging jets of uniform velocity V_2, as shown in Fig. 2. The pressure is the atmospheric pressure throughout and friction and gravity may be neglected throughout.

(a) Find the velocity V_2 and thickness d_2 of the diverging jets in terms of V_1 and d_1. [5]

(b) Find the force that the fluid exerts on the obstacle in terms of ρ, V_1 and d_1. [5]

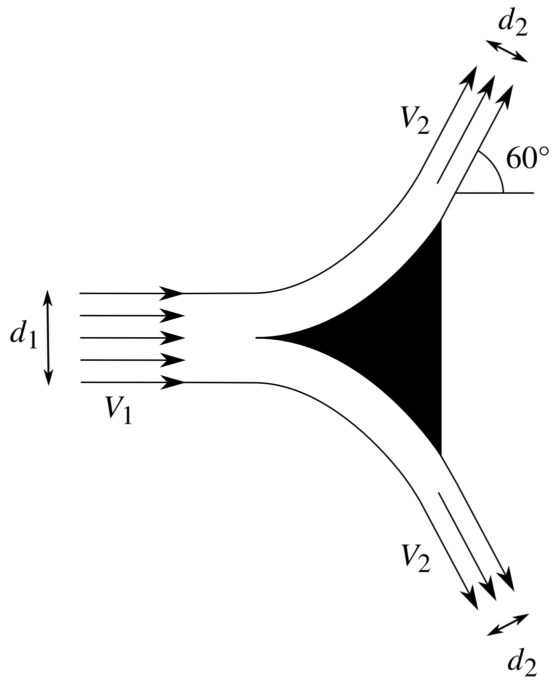

Fig. 2

6 (**long**) A circular, straight pipe enclosing air of density ρ undergoes a smooth contraction from diameter D_1 to diameter D_2. A series of pressure taps are installed along the pipe and connected to manometers containing a column of liquid of density ρ_ℓ, as shown in Fig. 3 when the air is at rest. The air is subsequently made to flow such that upstream of the contraction it has velocity V_1 and pressure p_1. Upstream of the contraction, pressure tap 1 is installed with the tube opening at a right angle to the flow direction. Pressure tap 2 has the opening facing the flow and is connected to a Pitot tube. Downstream of the contraction, pressure taps 3 and 4 are set in a similar arrangement, as shown in the Figure. The manometers are labelled with two digits marking the taps that they connect, e.g. manometer 24 connects taps 2 and 4, and they give as a reading the height difference in their column of water, e.g. h_{24}, with the reading being positive when the right side is higher. The manometers are deep enough not to overflow. Friction forces may be neglected throughout. The effect of gravity g on the air flow may also be neglected.

(a) Find the velocity downstream of the contraction, V_2, in terms of V_1, D_1 and D_2. [2]

(b) Find the pressure downstream of the contraction, p_2, in terms of p_1, ρ, V_1, D_1 and D_2. [4]

(c) For all the manometers, explain which readings you expect to be zero, and which ones equal and which ones opposite to each other. [14]

(d) Give in terms of ρ, ρ_ℓ, g, V_1, D_1 and D_2:

(i) the reading in manometer 12, h_{12}. [2]

(ii) the reading in manometer 13, h_{13}. [4]

(iii) the reading in manometer 23, h_{23}. [4]

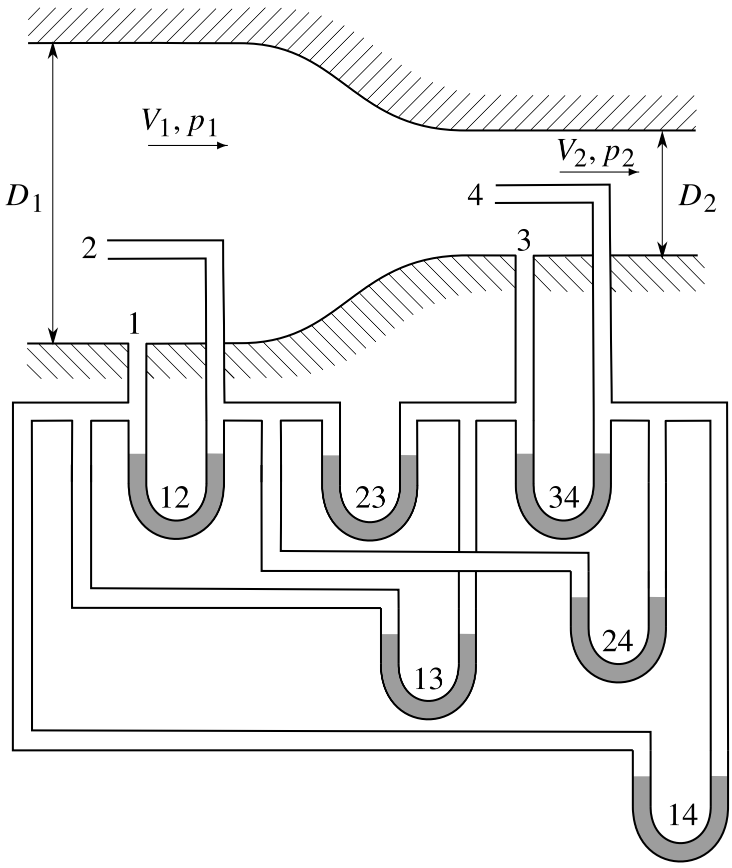

Fig. 3

SECTION B

7 (**long**) A solid sphere of mass m and radius r is rolling down a slope whose angle to the horizontal is α as shown in Fig. 4. The velocity of the sphere is denoted v and its angular velocity is ω.

(a) Find an expression for the angular acceleration $\dot{\omega}$ in terms of the acceleration $\dot{v}$ in the case where the sphere rolls without slip. [4]

(b) Show that the normal reaction N between the sphere and the table is $N = mg\cos\alpha$. [4]

(c) Using d'Alembert's principle or otherwise find an expression in terms of α for the minimum coefficient of friction μ required for the sphere to roll without slip. For the case $\alpha = \pi/4$ show that $\mu = \frac{2}{7}$. [15]

(d) For the case $\alpha = \pi/4$ and the coefficient of friction is 1/7, calculate $\dot{\omega}$ and $\dot{v}$. [7]

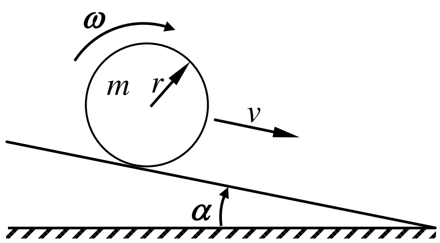

Fig. 4

8 (**short**) A puck of mass m is sliding on an air table as shown in Fig. 5. It is connected by a light inextensible string to a mass $2m$ hanging below the table. The string passes through a frictionless hole in the table. The motion of the puck is described by its distance r from the hole and its angular velocity ω.

(a) The puck is moving in steady circular motion at radius r. Find an expression for the speed of the puck. [4]

(b) An impulse tangent to its motion is now delivered to the puck. The effect of this impulse is to send the puck to a maximum distance from the hole of $2r$. During this motion find the minimum speed of the puck. [6]

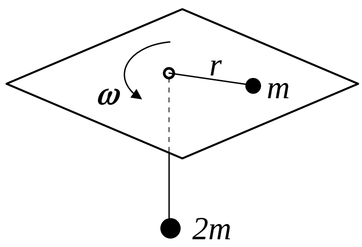

Fig. 5

9 (**short**) A rigid circular disc of mass m and radius r is shown in Fig. 6. A point A is located on the edge of the disc at $\{x, y\} = r\{1, 1\}/\sqrt{2}$

(a) Find the moment of inertia I_{zz} at A. [5]

(b) A shaft parallel to the z axis is fixed to the disc at A. Find the angular acceleration of the disc when a torque Q is applied along the axis of this shaft. [5]

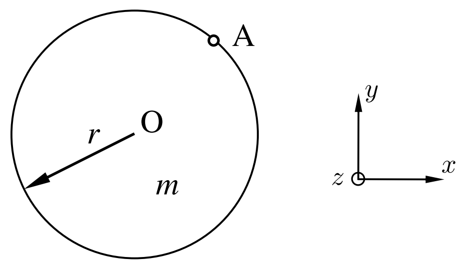

Fig. 6

10 (**short**) A flexible chain of length L and mass m is resting on a set of scales. At time $t = 0$ the chain is lifted from one end at a steady speed v as shown in Fig. 7. The time-varying force required to lift the chain is $f(t)$.

(a) Find an expression for the force $f(t)$ at $t = 0$, the time at which the chain first lifts off the scales. [4]

(b) Sketch a graph of $f(t)$ vs time for the entire duration of the motion. Show also the reading on the scales for this same period. [6]

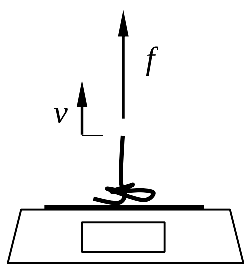

Fig. 7

11 (**short**) An electron microscope is situated in the basement of a building. It is sensitive to horizontal vibration and so is vibration isolated as shown in Fig. 8. The microscope is modelled as a rigid mass m whose displacement is y. The basement motion is x. The total isolation stiffness and damping are $2k$ and 2λ respectively.

(a) Find a differential equation for the motion y of the microscope given the basement motion x. [4]

(b) The basement is moving sinusoidally according to $x(t) = X \cos \omega t$ and motion of the microscope likewise is $y(t) = Y \cos \omega t$. Use the Mechanics Data Book or otherwise to sketch a graph of the ratio Y/X versus frequency ω. [6]

For part (b) use m = 1000 kg, k = 8 N / m and λ = 400 N s / m.

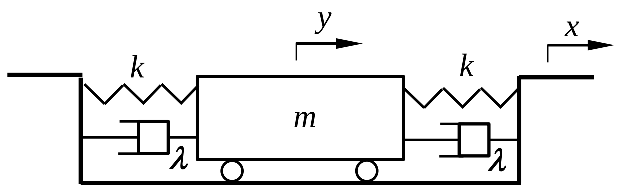

Fig. 8

12 (**long**) A two-degree-of-freedom system is shown in Fig. 9 comprising two masses m and $2m$ described by displacements y_1 and y_2 respectively. The masses are linked by springs k and $2k$ as shown. A force f is applied to mass m.

(a) Find a pair of differential equations describing the motion of the system and express this in matrix form. Identify the mass and stiffness matrices. [8]

(b) For the case $f(t) = F \cos \omega t$ find expressions for Y_1 and Y_2 as functions of frequency where $y_1(t) = Y_1 \cos \omega t$ and $y_2(t) = Y_2 \cos \omega t$. [8]

(c) Show that the two resonance frequencies for the system are at $\omega = \sqrt{2 \pm \sqrt{3}}\sqrt{\frac{k}{m}}$. [6]

(d) Sketch a graph of Y_1 and Y_2 as functions of frequency, noting carefully the two resonance frequencies and the behaviour at $\omega = \sqrt{\frac{k}{m}}$. [8]

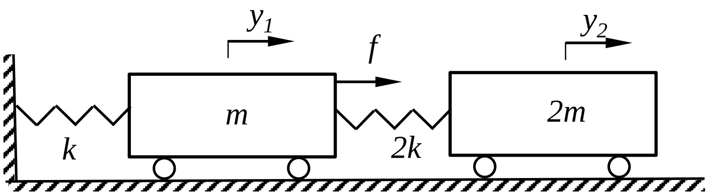

Fig. 9

END OF PAPER