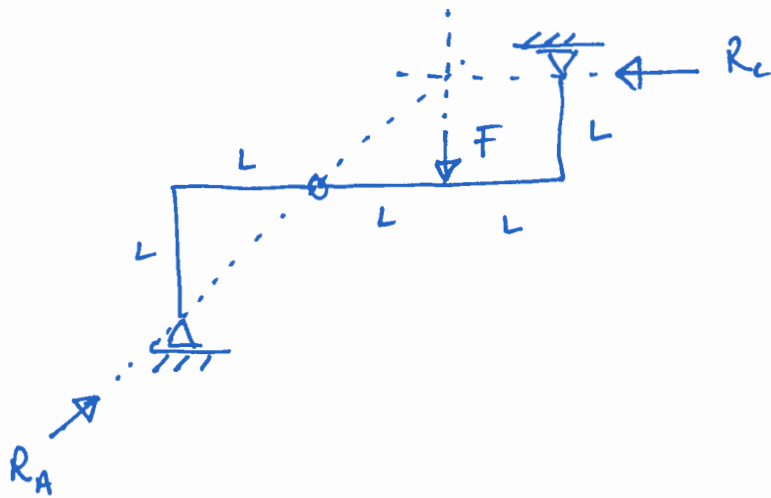
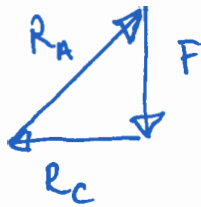


1)



External forces must be in equilibrium, hence must meet at a point.

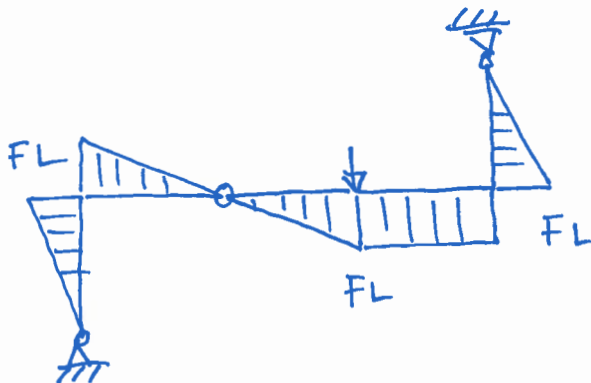


So  $R_c = F$  and  $R_A = \sqrt{2}F$

$\therefore H_c = F, V_c = 0$

$H_A = F, V_A = F$

Hence bending moment diagram:



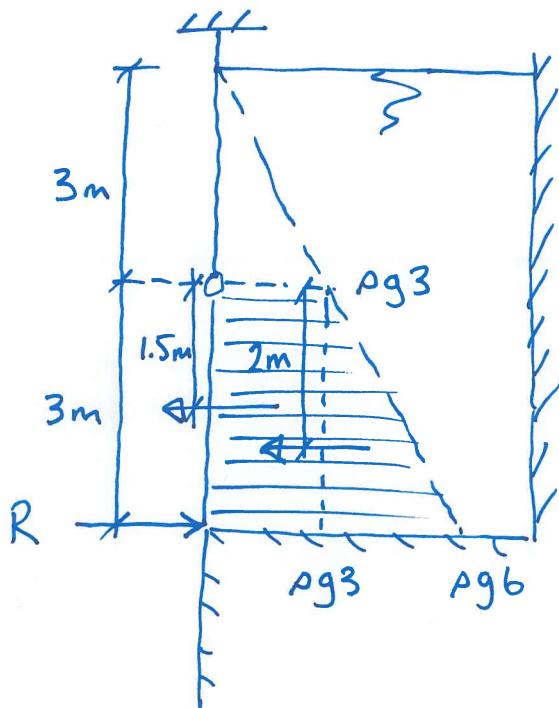
2)

$$\rho = 1000 \text{ kg m}^{-3}$$

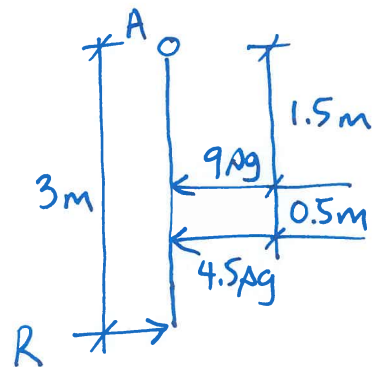
$$g = 10 \text{ m s}^{-2}$$

$h$  = depth below free surface

Since flap is 1m wide,  $\rho gh$  is the force per unit depth.



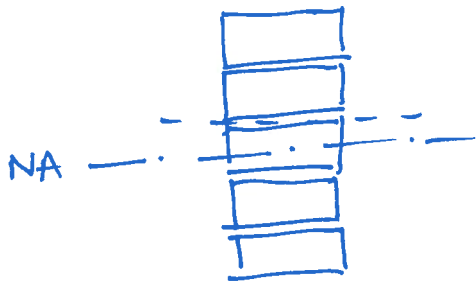
Replacing the stress distributions with equivalent resultant forces acting at the centroid, moment equilibrium about A finds:



$$\text{Hence } R = \frac{1.5 \times 9\rho g + 2 \times 4.5\rho g}{3}$$

$$= \underline{\underline{75 \text{ kN}}}$$

3)



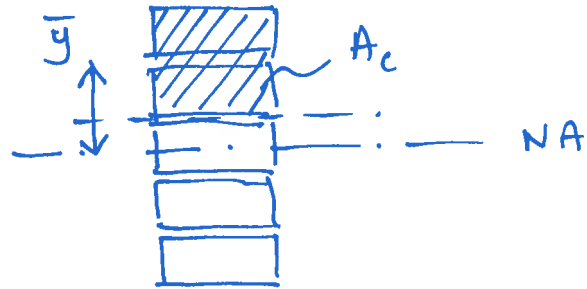
Fully composite action

So shear stress is parabolic with max at NA.

Glue line closest to NA

thus sees highest shear stress.

$$\tau = \frac{S A_c \bar{y}}{I b}$$



$$A_c \bar{y} = 120 \times 120 \times (60 + 30) = \overset{1296 \times 10^3}{\cancel{12960}} \text{ mm}^2$$

$$I = \frac{120 \times 300^3}{12} = 270 \times 10^6 \text{ mm}^4$$

$$S = 50 \times 10^3 \text{ N}$$

$$b = 120 \text{ mm}$$

$$\tau = \underline{\underline{2 \text{ N/mm}^2}}$$

4)

Vessel is thin-walled. Neglecting end effects.

$$\sigma_{L,F} = \frac{F}{2\pi r t}$$

longitudinal stress due to F.

$$\sigma_{L,P} = \frac{Pr}{2t}$$

longitudinal stress due to P.

$$\sigma_{\theta} = \frac{Pr}{t}$$

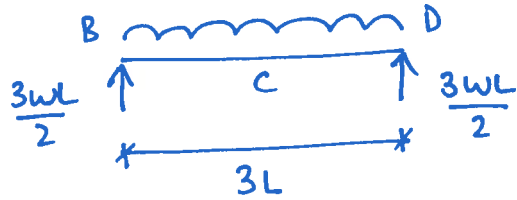
circumferential stress due to P.

$$\text{So, } \frac{-F}{2\pi r t} + \frac{\left(\frac{\sigma_{\theta} t}{r}\right) r}{2t} = 0$$

$$\Rightarrow \sigma_{\theta} = \frac{F}{\pi r t} = \frac{123 \times 10^3}{\pi \times 98 \times 4} = \underline{\underline{100 \text{ MPa}}}$$

5. a)

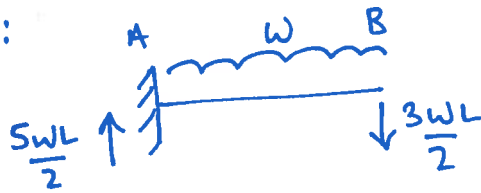
Span B-D is simply supported so:



$$M_B = M_D = 0$$

$$M_C = \frac{w(3L)^2}{8} = \frac{9}{8}wL^2$$

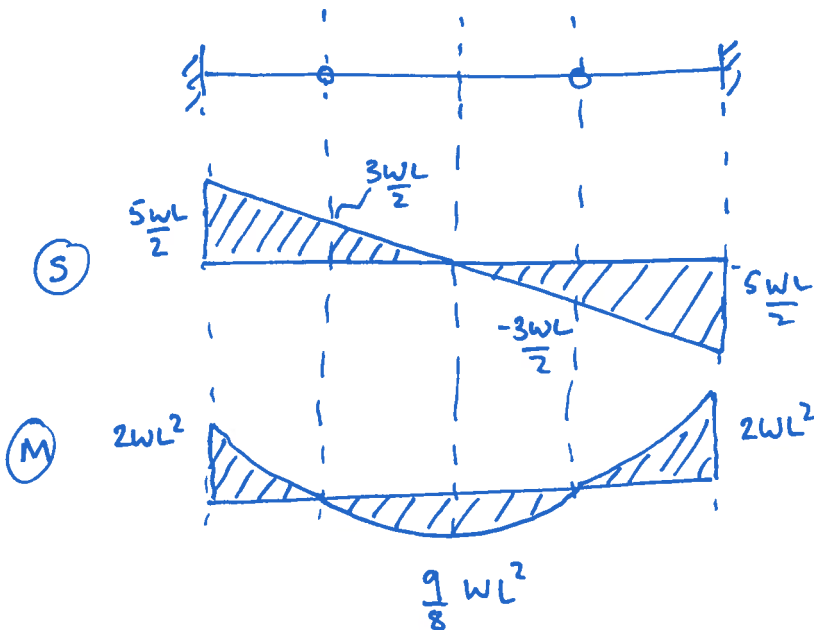
Hence:



$$M_A = wL^2 + \frac{3wL^2}{2} = 2wL^2$$

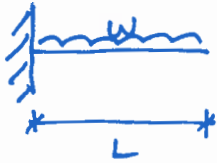
By symmetry:  $M_E = M_A$

∴

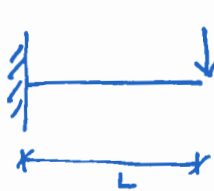


5. b)

By superposition of data book cases.

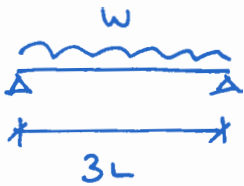


$$\frac{wL^4}{8EI} = \delta_1$$



$$F = \frac{3wL}{2}$$

$$\frac{FL^3}{3EI} = \frac{3wL^4}{6EI} = \delta_2$$



$$\frac{5w(3L)^4}{384EI} = \delta_3$$

$$\delta_c = \delta_1 + \delta_2 + \delta_3 = \frac{215 wL^4}{\cancel{384} EI \cdot 128}$$

5. c)

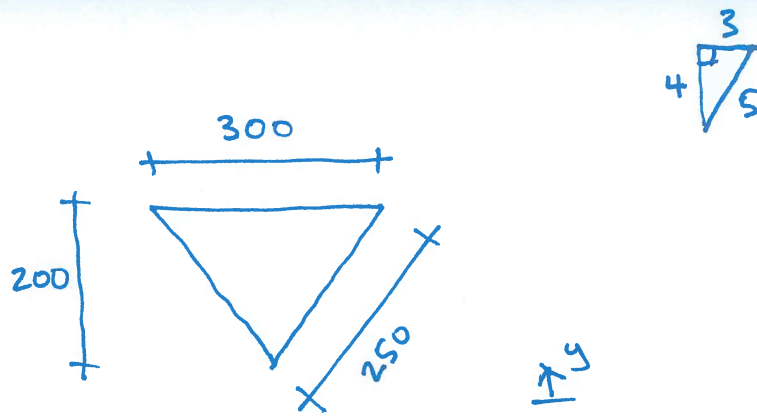


Plate 10 mm thick.

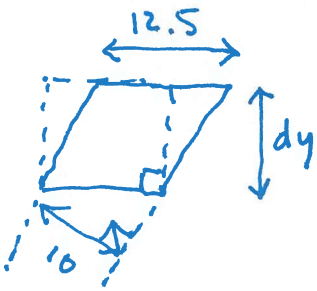
Find N.A. by 1st moments of area:

$$\bar{y} = \frac{\sum Ay}{A} = \frac{(10 \times 300 \times 200) + (2 \times 10 \times 250 \times 100)}{(10 \times 300) + (2 \times 10 \times 250)}$$

$$= \underline{137.5 \text{ mm}}$$

$$I = \int y^2 dA = \int_{57.5}^{62.5} y^2 \times y \times 300 + 2 \int_{-137.5}^{62.5} y^2 \times y \times 12.5$$

$$\approx \underline{35 \times 10^6 \text{ mm}^4}$$



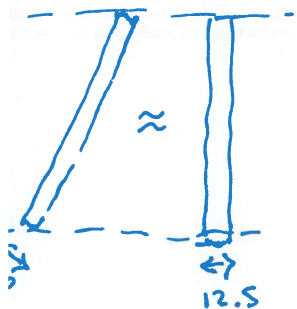
or, by parallel axis theorem:

$$I_{\text{flange}} \approx 10 \times 300 \times 62.5^2 = 11.7 \times 10^6 \text{ mm}^4$$

$$I_{\text{webs}} = 2 \times \left[ \left( \frac{12.5 \times 200^3}{12} \right) + (12.5 \times 200 \times 37.5^2) \right]$$

$$= 11.8 \times 10^6 \text{ mm}^4$$

$$I = I_{\text{flange}} + I_{\text{webs}} \approx \underline{35 \times 10^6 \text{ mm}^4}$$



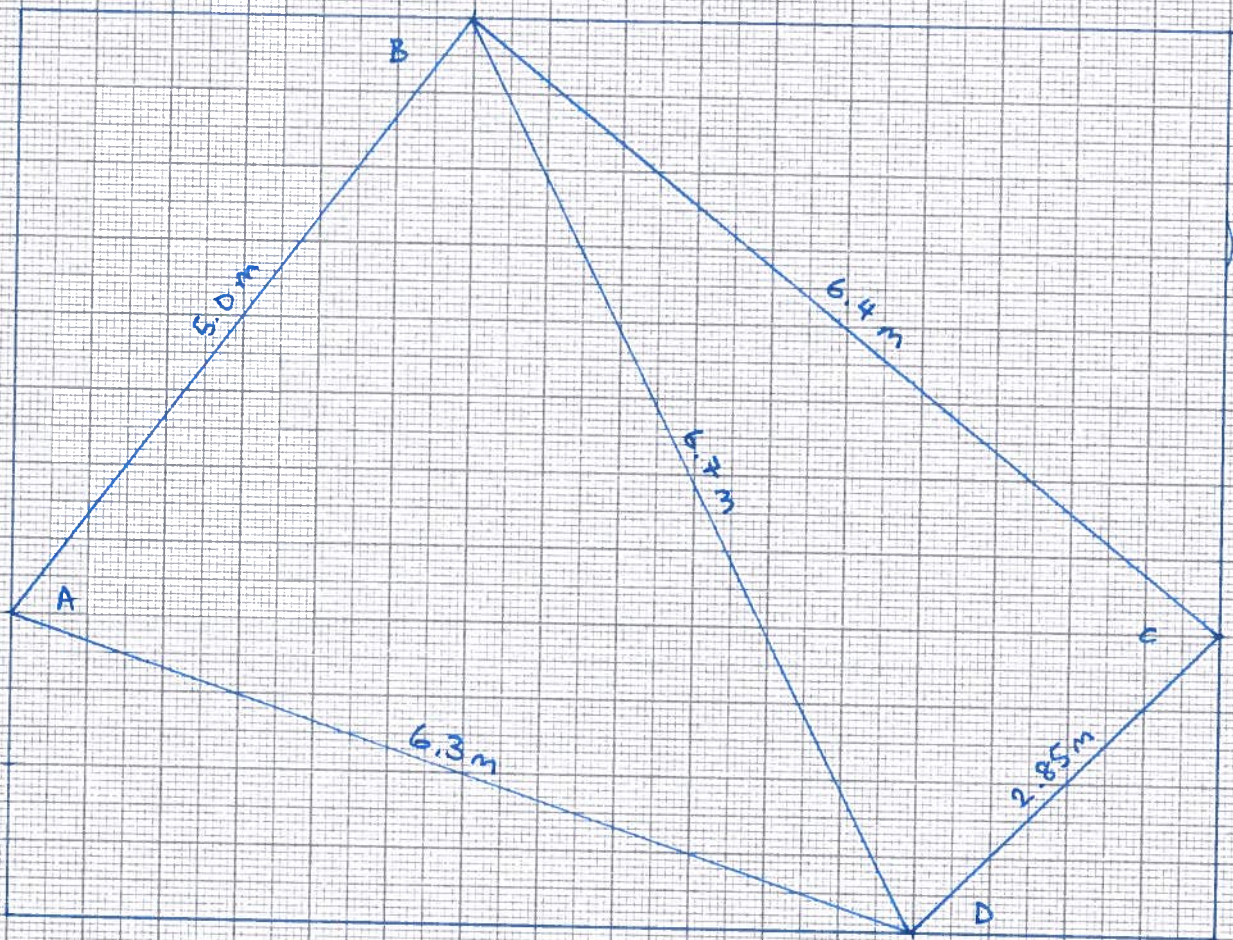
5. c) cont...

$$M = \frac{\delta I}{y}$$

$$\therefore M = \frac{275 \times 35 \times 10^6}{137.5} = \underline{\underline{70 \text{ kNm}}}$$



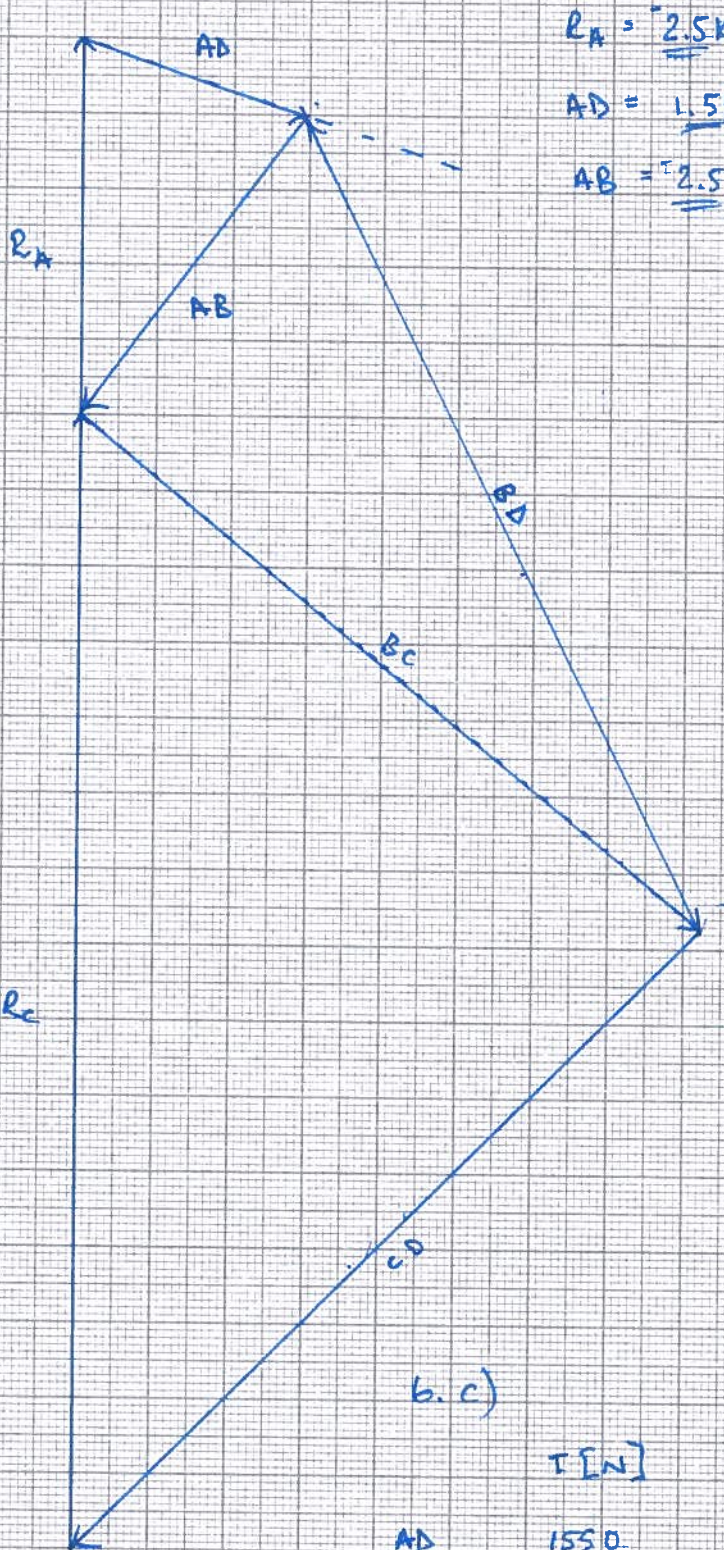
6) Probably quickest to determine lengths and angles from scale drawing of structure.



a) Taking moments about A:  $R_c = \frac{10 \times 6}{8} = \underline{\underline{7.5 \text{ kN}}}$

By vertical equilibrium:  $10 - R_c = R_A = \underline{\underline{2.5 \text{ kN}}}$

6. b) Determine forces graphically



$$R_A = \underline{\underline{2.5 \text{ KN}}}$$

$$AD = \underline{\underline{1.55 \text{ KN}}}$$

$$AB = \underline{\underline{2.5 \text{ KN}}}$$

$$R_c = \underline{\underline{7.5 \text{ KN}}}$$

$$BC = \underline{\underline{5.3 \text{ KN}}}$$

$$CD = \underline{\underline{5.85 \text{ KN}}}$$

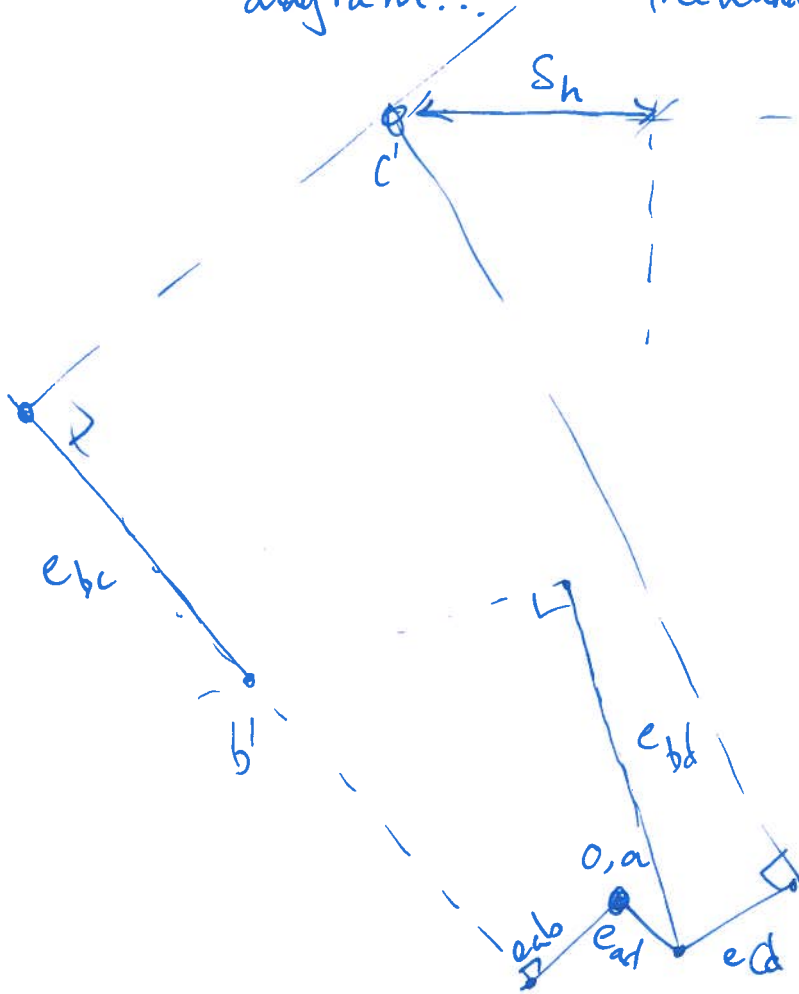
$$BD = \underline{\underline{6.0 \text{ KN}}}$$

b. c)

	$T [N]$	$L [m]$	$e = \frac{TL}{EA} \left[ \frac{mm}{mm} \right]$
AD	1550	6.3	<del>1.0</del> 1.0 mm
AB	-2500	5	-1.25 mm
BC	-5300	6.4	-3.4 mm
CD	5850	2.85	1.7 mm
BD	6000	6.7	4.0 mm

b c)

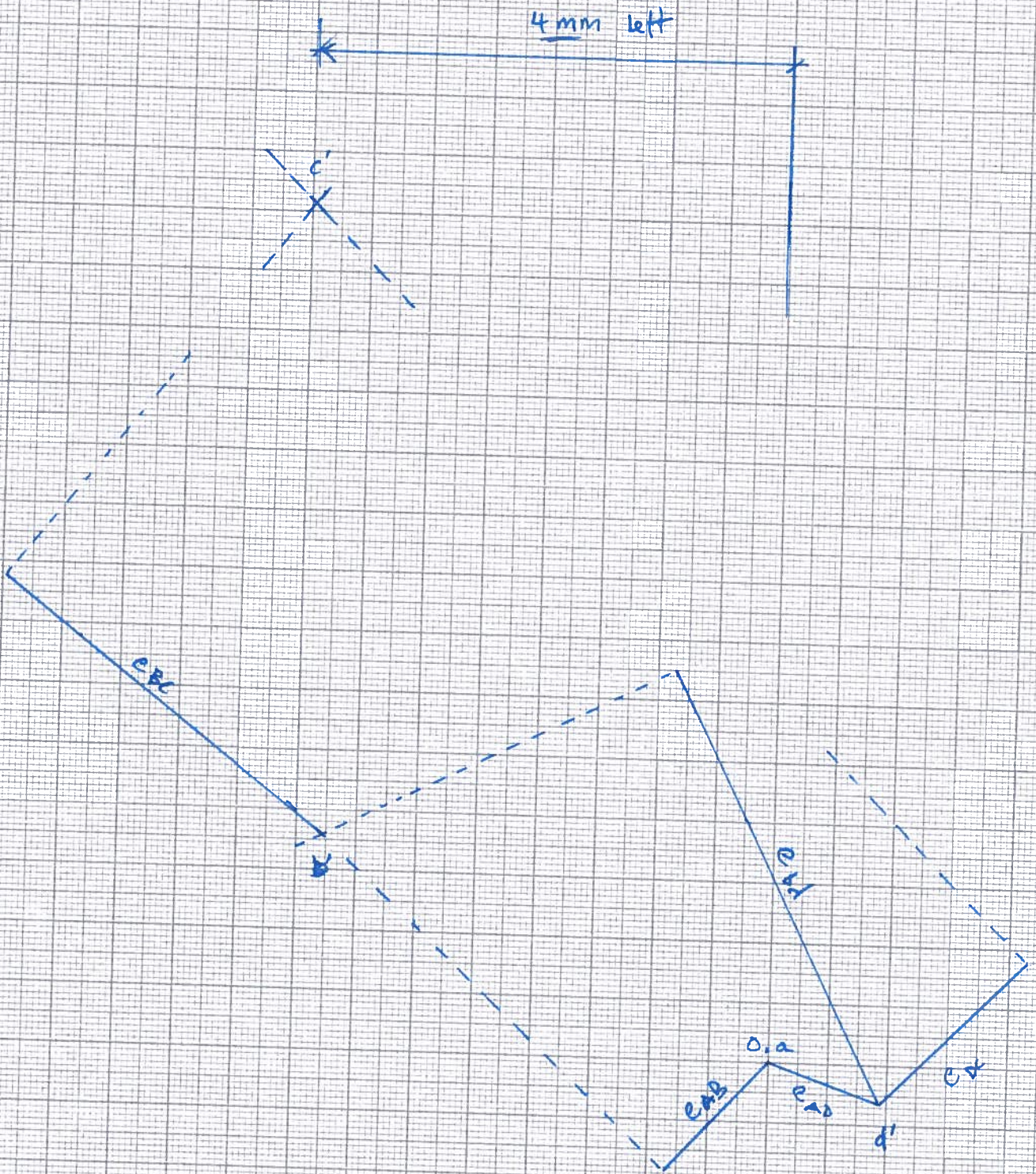
check general shape of displacement diagram... freehand sketch



So start bottom right quadrant of page.

b.c)

Vertical displacement of C known to be 0 due to boundary condition (roller).



Scale  $2\text{ cm} = 1\text{ mm}$

# 1P2 Materials 2024 Crib

## Question 7

(a)

The stress concentration factor is a measure of the degree to which an external stress is amplified at the edge of a defect or a design feature (threads, holes, corners, etc.). It is the ratio of the maximum stress at the defect/design features to the remote stress. The stress concentration factor is a dimensionless number and is valid for blunt features.

The stress intensity factor is a measure of the loading at the tip of a sharp crack. For linear elastic materials, stress intensity factor is given by  $Y\sigma\sqrt{\pi a}$ , where  $Y$  is a dimensionless constant,  $\sigma$  is the remote applied stress, and  $a$  is some measure of the crack length. It has the units MPa  $\sqrt{\text{m}}$ .

The critical stress intensity factor is also known as the fracture toughness. When the stress intensity factor under mode I (crack opening) loading reaches a critical value, fast fracture occurs.

(b)

The crack of length  $a$  is subjected to hoop stress in the vessel wall drives the crack so  $\sigma = \frac{pR}{t}$ . For the fast fracture to not happen, we require (with  $Y = 1$ ):

$$\sigma\sqrt{\pi a} \leq K_{\text{IC}}. \quad (1)$$

Hence,

$$a \leq \frac{K_{\text{IC}}^2 t^2}{\pi p^2 R^2}. \quad (2)$$

The minimum wall thickness can be achieved when the crack length  $a$  matches the thickness of the vessel wall, i.e.,  $a = t$ . Combining this with the the equation above we have:

$$t \geq \frac{\pi p^2 R^2}{K_{\text{IC}}^2} = \frac{\pi \times 10^2 \times 1^2}{110^2} = 0.026 \quad (3)$$

Therefore, the smallest wall thickness is 26 mm.

## Question8

(a)

(i): The nominal strain is  $\frac{2L-L}{L} = 1$ . True strain is  $\ln(\frac{2L}{L}) = 0.69$ .

(ii): The nominal strain is  $\frac{0.5L-L}{L} = -0.5\%$ . True strain is  $\ln(\frac{0.5L}{L}) = -0.69$ .

(b)

From the polymers and foam process chart, thermoplastics can be shaped by:

1. Machining
2. Injection Moulding
3. Blow Moulding
4. Compression Moulding
5. Rotational Moulding.

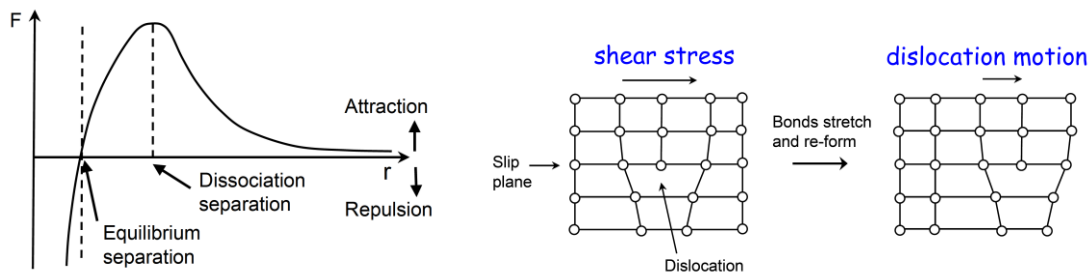
The requirement on mass of 25-35g eliminates Compression Moulding and Rotational Moulding. The requirement on dimensional tolerance of 0.2 mm rules out Blow Moulding. From the economic batch size chart, machining is not appropriate with a target size of 50,000, therefore Injection Moulding is the most suitable shaping process.

**Q9 (short). Microstructural origin and manipulation of plastic properties.**

(a) The *Young's modulus* is determined by the stiffness of interatomic bonds. This is related to the slope of the atomic force vs separation curve (see below) at the equilibrium separation. [2]

The *yield strength* is due to dislocation glide within the material. This occurs by local breaking and reforming of atomic bonds at the dislocation core (see below). [2]

Cold rolling will increase the dislocation density. This increases the interaction between dislocations, impeding their motion. This will increase the yield strength. The Young's modulus will not be affected by cold rolling, as it is not dependent on dislocation motion. [2]



(b) Given that these form a solid solution without precipitates, and are as-cast so there is no work hardening, the only relevant hardening mechanism is solid solution hardening. The hardening effect depends on the obstacle spacing, which in turn depends on the concentration of solute atoms.

In case (i), there will be a large spacing of Ni atoms obstructing dislocation glide in Cu. So the hardening effect vs pure Cu will be small. Similarly in case (iii), but now there is a low spacing of Cu atoms obstructing glide in Ni. So the hardening effect vs pure Ni will be small (which is similar to pure Cu). Case (ii) will provide the smallest obstacle spacing of all these options, and so will have the highest solid solution hardening effect. [4]

**Q10 (short). Weibull statistics of brittle fracture.**

(a) The strength of the ceramic, characterised by the probability of surviving a given load, will depend on the distribution of flaws. The larger the specimen, the greater the probability of flaws above a critical size for the load applied, and so the lower the survival probability. The constants  $m$  and  $\sigma_0$  capture this dependence. [2]

(b) Take these cylinders to be the reference volume  $V_0$ .

Given data:  $P_{s1} = 0.85$  at  $\sigma_1 = 300$  MPa, and  $P_{s2} = 0.20$  at  $\sigma_2 = 400$  MPa.

Taking ratios, using given equation: 
$$\frac{\ln(P_{s1})}{\ln(P_{s2})} = \left(\frac{\sigma_1}{\sigma_2}\right)^m \quad \frac{\ln(0.85)}{\ln(0.2)} = 0.101 = \left(\frac{300}{400}\right)^m \quad \therefore m = 7.97$$

Rearranging given equation: 
$$\sigma_0 = \frac{\sigma_1}{(-\ln P_{s1})^{1/m}} = \frac{300}{(-\ln 0.85)^{1/7.97}} = 377 \text{ MPa} \quad [4]$$

(c) Survival probability for a number of identical rods under uniaxial stress, from the data book:

$$P_s = \exp \left[ -\frac{V}{V_0} \left( \frac{\sigma}{\sigma_0} \right)^m \right]$$

If there are  $n$  identical rods, then  $V/V_0 = n$ . Set the tensile stress equal to the failure stress of the adhesive,  $\sigma = 290$  MPa, and calculate the value of  $n$  that gives  $P_s = 0.5$ , i.e. when failure in the ceramic becomes more likely.

$$0.5 = \exp\left[-n\left(\frac{290}{377}\right)^{7.97}\right] \therefore n = 5.59$$

As there must be an integer number of rods, failure is more likely in the ceramic when  $n \geq 6$ . [4]

**Q11 (long). Material selection and environmental impact.**

(a) (i) Objective: time constant  $\tau = \frac{d^2 \rho c}{\lambda}$

Constraint:  $F = \frac{4\pi^2 E d^3}{3L} \geq 700$  N

Obtain the free variable from the constraint:  $d = \left(\frac{3FL}{4\pi^2 E}\right)^{1/3}$

Substitute into the objective:  $\tau = \left(\frac{3FL}{4\pi^2}\right)^{2/3} \frac{\rho c}{\lambda E^{2/3}}$

Therefore the material index to maximise is:  $M = \frac{\rho c}{\lambda E^{2/3}}$  [4]

(ii) Evaluating the index for the materials in the table:

Cork:  $M = 72.3 \text{ m}^{-2} \text{ s Pa}^{-2/3}$

Polystyrene foam:  $M = 18.2 \text{ m}^{-2} \text{ s Pa}^{-2/3}$

Polyethylene:  $M = 4.45 \text{ m}^{-2} \text{ s Pa}^{-2/3}$

Therefore, **cork** is the best option. Evaluating  $d$  from the constraint equation and the material properties for cork:

$$d \geq \left(\frac{3FL}{4\pi^2 E}\right)^{1/3} = 7.93 \text{ mm}$$

Note the inequality: these design parameters only set a minimum value of  $d$ . The objective  $\tau$  would be improved if  $d$  was larger than this. [4]

(b) (i) Given two constraints, evaluate the objective  $d$  for each constraint.

Constraint 1: :  $F \geq 700$  N and so  $d \geq \left(\frac{3FL}{4\pi^2 E}\right)^{1/3}$

Cork:  $d \geq 7.93$  mm (as in part a)

Polystyrene foam:  $d \geq 3.85$  mm

Polyethylene:  $d \geq 2.61$  mm

Constraint 2: :  $\tau \geq 1800$  s and so  $d \geq \left(\frac{\lambda \tau}{\rho c}\right)^{1/2}$

Cork:  $d \geq 15.7$  mm

Polystyrene foam:  $d \geq 15.2$  mm

Polyethylene:  $d \geq 20.8$  mm

Check which constraint is active: for all materials it is constraint 2.

Then pick the material that minimises  $d$ : **polystyrene foam**, with  $d = 15.2$  mm. [7]



- (ii) The revised selection criteria means that a single value of the wall thickness  $d$  can now be specified. The constraint on thermal properties is active, and dominates the design. All solutions satisfying this constraint easily meet the mechanical constraint. [1]

Additional constraints (any two)?

Mass: The constraint on  $\tau$  would lead to a very heavy design in the case of polyethylene. This is important for handling and transport of the crate.

Cost: This would affect the competitiveness of the design. The high mass of the PE solution would likely make it by far the most expensive, in terms of material cost and manufacturing cost. As a natural material, cork will have higher material and processing costs vs polystyrene foam.

Manufacturability: Natural materials like cork are more difficult to shape. The high wall thickness of the polyethylene solution would impact processability and manufacturing cost.

Strength: The buckling constraint is not active in this design, but the compressive strength of the walls (yielding or crushing strengths) should be checked.

Toughness: Resistance to cracking, if the crate were damaged in service. [2]

- (c) (i) The constraint on buckling load is now an equality  $F = \frac{4\pi^2 E d^3}{3L} = 700 \text{ N}$ , which fixes the wall thickness for each material. These have already been calculated in part (b):

Cork:  $d = 7.93 \text{ mm}$   
 Polyethylene:  $d = 2.61 \text{ mm}$

To evaluate the refrigeration power we need the volume of the crates  $V = (L + 2d)^3$ :

Cork:  $V = 0.0315 \text{ m}^3$   
 Polyethylene:  $V = 0.0284 \text{ m}^3$

and the time constant  $\tau = \frac{d^2 \rho c}{\lambda}$ :

Cork:  $\tau = 458 \text{ s}$   
 Polyethylene:  $\tau = 28.2 \text{ s}$

The refrigeration power per crate  $P_r = 500V \left(1 - \frac{\tau}{1800}\right)$ :

Cork:  $P_r = 11.7 \text{ W}$   
 Polyethylene:  $P_r = 14.0 \text{ W}$

The refrigeration energy is the power  $P_r \times \text{time} = P_r \times 1000 \text{ km} / 40 \text{ km/hr}$ :

Cork:  $1057 \text{ kJ}$   
 Polyethylene:  $1259 \text{ kJ}$

To evaluate the energy to move the crate we need the mass  $m = \rho(V - L^3)$

Cork:  $m = 0.812 \text{ kg}$   
 Polyethylene:  $m = 1.38 \text{ kg}$

The energy in kJ to move the crate is  $0.46 \times \text{distance} (1000 \text{ km}) \times \text{mass } m$  :

Cork:	374 kJ
Polyethylene:	633 kJ

The total transport energy is therefore:

Cork:	1431 kJ
Polyethylene:	1892 kJ

So **cork** minimises the total transport energy. [9]

(ii) Environmental impact during the life cycle: [4]

- Material production: Differences in energy costs for natural materials vs polymers.
- Product manufacture: Processing of natural materials is more difficult, and differences vs polymer processing would need to be considered.
- Product disposal: Cork will be biodegradable. Polyethylene is recyclable,

## Question12

(a)

(i) Imagine the layer is detached and consider a block of the film and the thick component of original length  $L_0$ . A temperature increase of  $\Delta T$  causes the film to change in length by  $\delta L_1 = \alpha_1 L_0 \Delta T$  in both the 1 and 2 directions. Meanwhile, the substrate to which it was previously bonded stretches by  $\delta L_2 = \alpha_2 L_0 \Delta T$  in the same directions. If we want to stick the film back onto the thick component, covering the same surface as before, we must stretch it by applying strain (for  $\alpha_1 \ll 1$ )

$$\epsilon_1 = \epsilon_2 \approx \frac{\delta L_1 - \delta L_2}{L_0 + \delta L_1} = \frac{\Delta T(\alpha_1 - \alpha_2)}{1 + \alpha_1 \Delta T} \approx \Delta T(\alpha_1 - \alpha_2).$$

(ii) From 3D Hooke's law and  $\sigma_3 = 0$ :

$$\begin{aligned}\epsilon_1 &= \frac{1}{E}(\sigma_1 - \nu\sigma_2) \\ \epsilon_2 &= \frac{1}{E}(\sigma_2 - \nu\sigma_1)\end{aligned}$$

Rearranging, we get

$$\begin{aligned}\sigma_1 &= \frac{E}{1 - \nu^2}(\epsilon_1 + \nu\epsilon_2) \\ \sigma_2 &= \frac{E}{1 - \nu^2}(\epsilon_2 + \nu\epsilon_1)\end{aligned}$$

Substituting  $\epsilon_1 = \epsilon_2 = \Delta T(\alpha_1 - \alpha_2)$  we get

$$\sigma_1 = \sigma_2 = \frac{E}{1 - \nu} \Delta T(\alpha_1 - \alpha_2)$$

(b)

(i) Let  $E_1, \alpha_1, L_1, A_1$  be the Young's modulus, thermal expansion coefficient, original length, and cross-sectional area of the Aluminum bar, and  $E_2, \alpha_2, L_2, A_2$  be those of the Steel bar. Further, let  $\epsilon_{e1}$  be the elastic strain of the Aluminum bar and  $\epsilon_{e2}$  be that of the Steel bar. Let  $l_1, l_2$  be the lengths of the Aluminum and Steel sections, respectively, after the external force is applied. Then, by definition,

$$\begin{aligned}\epsilon_{e1} &= \frac{l_1 - L_1}{L_1} \\ \epsilon_{e2} &= \frac{l_2 - L_2}{L_2}\end{aligned}$$

Further note

$$l_1 + l_2 = L_1 + L_2$$

Therefore,

$$L_1\epsilon_{e1} + L_2\epsilon_{e2} = 0 \quad (4)$$

Now consider stress equilibrium, let  $\sigma_1$  be the stress in Aluminium and  $\sigma_2$  be the stress in Steel. The force balance at the rigid plate:

$$-\sigma_1 A_1 + \sigma_2 A_2 + F = 0 \quad (5)$$

Further

$$\sigma_1 = E_1\epsilon_{e1}, \quad \sigma_2 = E_2\epsilon_{e2}$$

Combine the above equation with (1):

$$L_1 \frac{\sigma_1}{E_1} + L_2 \frac{\sigma_2}{E_2} = 0 \quad (6)$$

Solving the system of equations (2) and (3) with unknown stresses, we obtain  $\sigma_1 = 0.31$  MPa and  $\sigma_2 = -1.45$  MPa. Therefore the stress in Aluminium is 0.31 MPa in tension, and the stress in Steel is 1.45 MPa in compression.

(ii) Let  $\epsilon_1, \epsilon_{t1}, \epsilon_{e1}$  be the total, thermal and elastic strains of Aluminium bar and  $\epsilon_2, \epsilon_{t2}, \epsilon_{e2}$  be those of the Steel bar at 80 °C.

First from kinematics, the total strain for both Aluminum and Steel is given by:

$$\epsilon_1 = \epsilon_{e1} + \epsilon_{t1}$$

$$\epsilon_2 = \epsilon_{e2} + \epsilon_{t2}$$

Let  $l_1, l_2$  be the length of the Aluminum and Steel bars at 80 °C. Then from the definition of total strain

$$\epsilon_1 = \frac{l_1 - L_1}{L_1}$$

$$\epsilon_2 = \frac{l_2 - L_2}{L_2}$$

Further note

$$l_1 + l_2 = L_1 + L_2$$

Therefore,

$$(\epsilon_{e1} + \epsilon_{t1})L_1 + (\epsilon_{e2} + \epsilon_{t2})L_2 = 0$$

Note  $\epsilon_{t1} = \alpha_1 \Delta T$  and  $\epsilon_{t2} = \alpha_2 \Delta T$ . The equation above can be written as

$$L_1 \epsilon_{e1} + L_2 \epsilon_{e2} = -(L_1 \alpha_1 \Delta T + L_2 \alpha_2 \Delta T)$$

Now consider stress equilibrium, let  $\sigma_1$  be the stress in Aluminium and  $\sigma_2$  be the stress in Steel. The force balance at the rigid plate:

$$-\sigma_1 A_1 + \sigma_2 A_2 + F = 0$$

Further

$$\sigma_1 = E_1 \epsilon_{e1}, \quad \sigma_2 = E_2 \epsilon_{e2}$$

We therefore have the following system of equations with unknown elastic stresses in both bars

$$\begin{aligned} \sigma_1 A_1 - \sigma_2 A_2 &= F \\ L_1 \frac{\sigma_1}{E_1} + L_2 \frac{\sigma_2}{E_2} &= -(L_1 \alpha_1 \Delta T + L_2 \alpha_2 \Delta T) \end{aligned}$$

Solving the above we have  $\sigma_1 = -107.9$  MPa and  $\sigma_2 = -73.6$  MPa.

Therefore the stress in the Aluminum bar is 107.9 MPa in compression, and the stress in the Steel bar is 73.6 MPa in compression.