EGT0
ENGINEERING TRIPOS PART IA

Thursday 6 June 2024 9 to 12.10

Paper 2

STRUCTURES AND MATERIALS

*Answer **all** questions.*

*The **approximate** number of marks allocated to each part of a question is indicated in the right margin.*

Answers to questions in each section should be tied together and handed in separately.

*Write your candidate number **not** your name on the cover sheet.*

STATIONERY REQUIREMENTS
Single-sided script paper and graph paper

SPECIAL REQUIREMENTS TO BE SUPPLIED FOR THIS EXAM
CUED approved calculator allowed
Engineering Data Book

10 minutes reading time is allowed for this paper at the start of the exam.

You may not start to read the questions printed on the subsequent pages of this question paper until instructed to do so.

You may not remove any stationery from the Examination Room.

SECTION A

1 (**short**)

A frame subject to a point load F is shown in Fig. 1. Draw the bending moment diagram for this structure noting all salient values. Use the convention that moment diagrams are drawn on the tension side of the structure. [10]

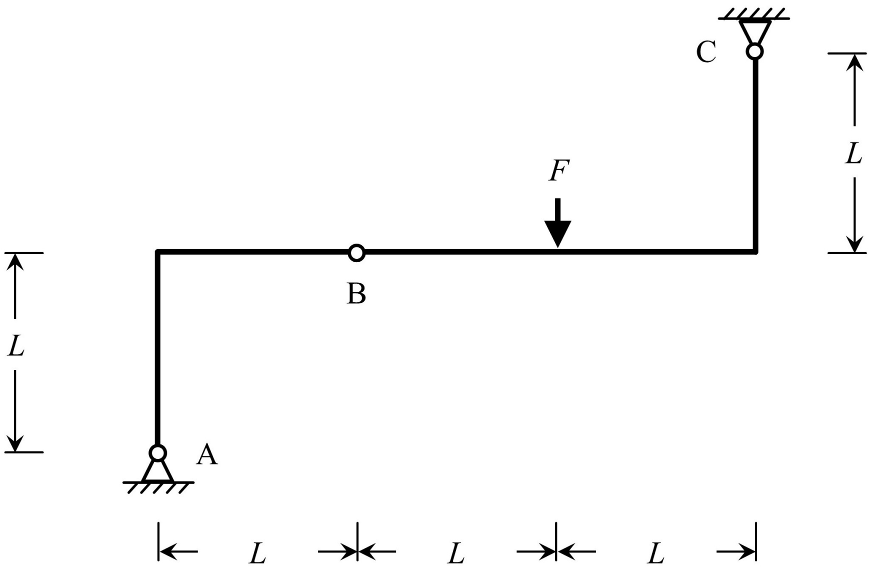

Fig. 1

2 (**short**)

A small open reservoir containing water is shown in Fig. 2. The retaining structure on the left hand side of the reservoir includes a flap sluice gate that can be used to release water from the reservoir. The flap is a plate 1 m wide and 3 m high that is supported from above by a horizontal hinge at A. A horizontal ram is used to control the opening of the flap. If the water is 6 m deep and the flap is closed, what is the magnitude of the force *R* on the ram? [10]

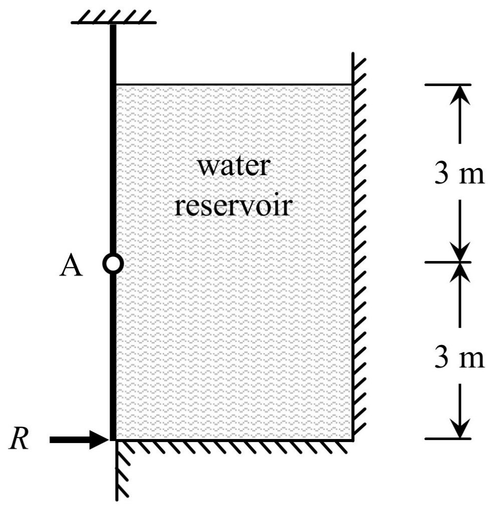

Fig. 2

3 (**short**)

A glued laminated timber beam is shown in Fig. 3. The beam cross section is built up from 60 mm thick layers of timber, adhesively bonded together with a strong, stiff glue. Perfectly composite action of the cross-section may be assumed. If the beam is subjected to a vertical shear force of 50 kN, what is the maximum shear stress on the glue. [10]

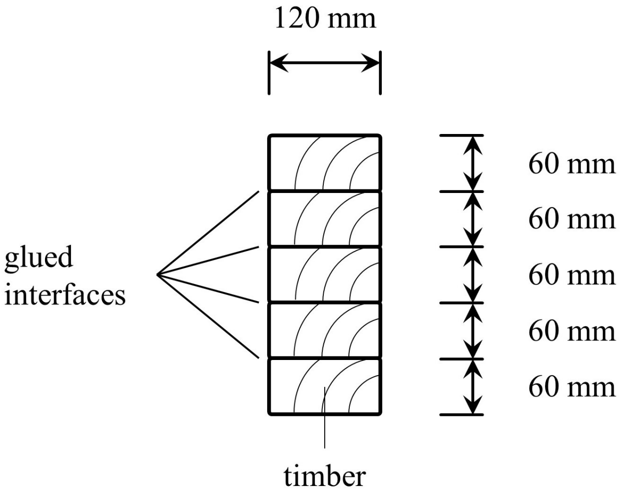

Fig. 3

4 (**short**)

A closed ended cylindrical pressure vessel is shown in Fig. 4. The circumferential walls of the pressure vessel are fabricated from 4 mm thick steel and have an outside diameter of 200 mm. The pressure vessel supports an external vertical load of 123 kN applied concentrically through the closed end plates. If the internal pressure is such that the stress in the longitudinal direction is zero, what is the stress in the circumferential direction? Clearly state any assumptions made. [10]

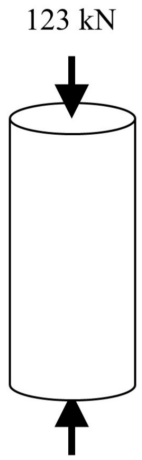

Fig. 4

5 (**long**)

Figure 5(a) shows an arrangement of beams of uniform cross section having flexural rigidity EI. The beams support a uniformly distributed load w per unit horizontal length.

(a) Draw the shear and bending moment diagrams for the beam subject to this loading arrangement, noting all salient values. [10]

(b) Determine the midspan displacement at point C in terms of w, L, E and I. [10]

(c) A possible design for the beam using a welded triangular box section manufactured from 10 mm thick steel plate is shown in Fig. 5(b). If the steel plate has a yield strength of 275 MPa, what is the maximum value of bending moment that the section can sustain before first yield? [10]

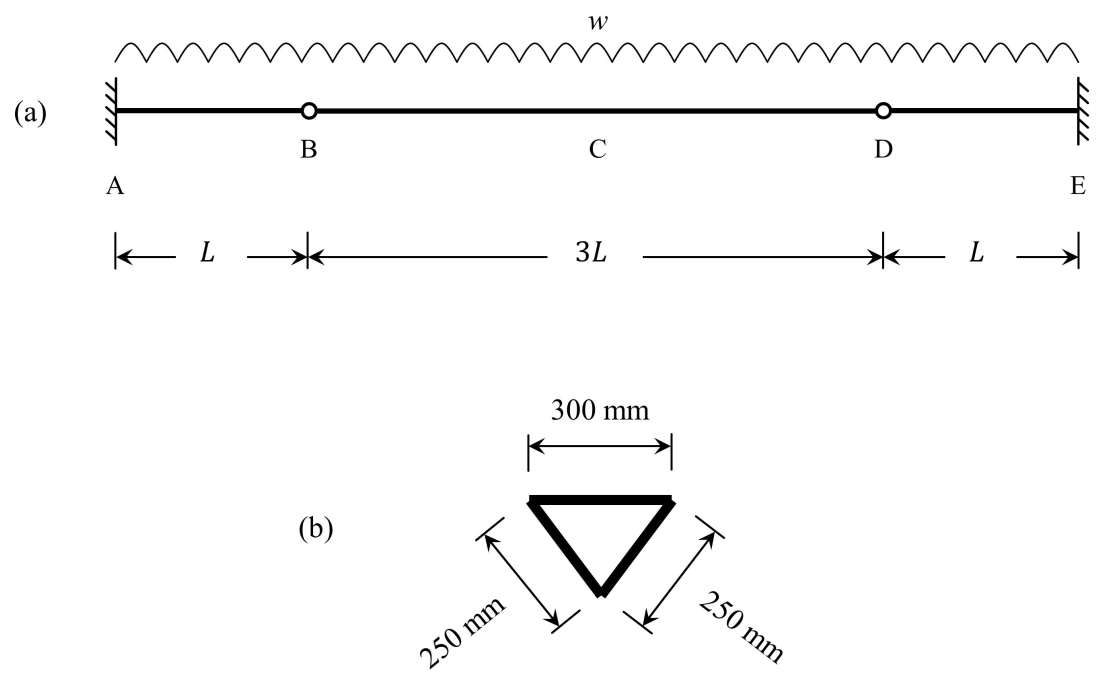

Fig. 5

6 (**long**)

A pin jointed 2D truss structure is shown in Fig. 6. The members of the truss have cross-sectional area A and Young's Modulus E such that $AE = 10^7$ N^{-1} m.

A vertical point load of 10 kN is applied at joint D.

(a) Calculate the support reactions at A and C. [4]

(b) Using graphical methods or otherwise, determine the member forces. [6]

(c) Using graphical methods or otherwise, determine the horizontal displacement of joint C. [20]

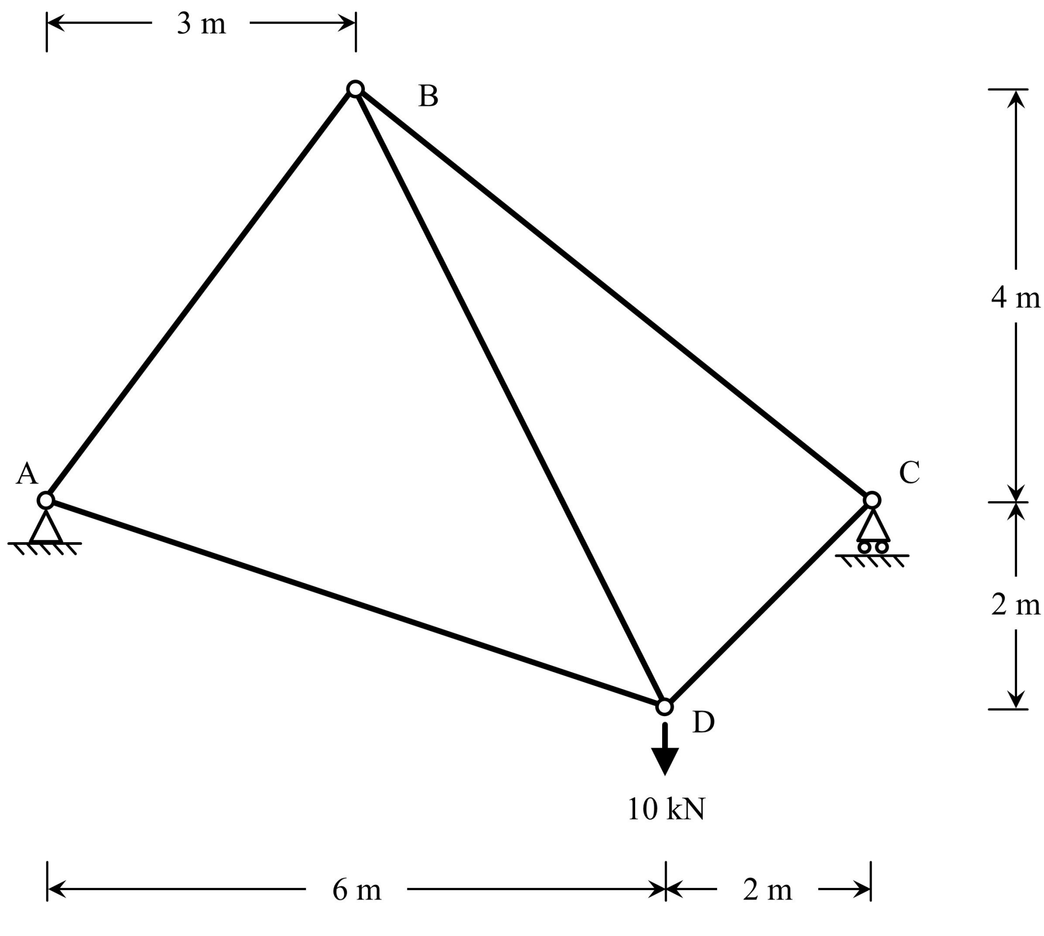

Fig. 6

SECTION B

7 (**short**)

(a) Explain the difference between the stress concentration factor and the stress intensity factor. Define the critical stress intensity factor. [4]

(b) A thin-walled, long cylindrical vessel of 2 m diameter is to hold an internal pressure of 10 MPa. Non-destructive testing revealed the presence of small microscopic cracks in the inner wall. The cracks are expected to gradually extend through the pressurised wall by fatigue. The fracture toughness of the vessel material is 110 MPa$\sqrt{\text{m}}$. Find the minimum wall thickness of the vessel that can be used while ensuring that the vessel will not fail by fast fracture. Assume the dimensionless constant $Y = 1$. [6]

8 (**short**)

(a) A uniform solid cylinder of length L is subjected to axial loading. Determine the nominal and true strains in the cylinder in the following two scenarios:

(i) the cylinder is stretched to twice its original length; [3]

(ii) the cylinder is compressed to half its original length. [3]

(b) Thermoplastic CD cases with weight between 25-35 g and a minimum section thickness of 1-2 mm are required. These cases should have surface roughness of 1 μm, and dimensional tolerance of 0.2 mm. The target batch size is $50,000$. Using the relevant process attribute charts in the databook, identify the most suitable shaping process for the CD cases. [4]

9 (**short**)

(a) Explain briefly, using sketches to support your answer, the microstructural origins of the Young's modulus and the yield strength of a pure metal. Would you expect both to change after cold rolling? [6]

(b) Pure Cu and pure Ni have yield strengths of 60 MPa and 80 MPa, respectively. Alloys of Cu and Ni form a solid solution at room temperature, without precipitation, for any composition. Explain, with reference to the relevant hardening mechanisms, which of the following alloys (expressed in number fraction of atoms) will have the highest yield strength, measured after casting: (i) Cu - 10% Ni, (ii) Cu - 50% Ni, (iii) Cu - 90% Ni. [4]

10 (**short**) A reference volume V_0 of ceramic subjected to a uniform tensile stress σ has a survival probability

$$P_s\left(V_0\right) = \exp\left[-\left(\frac{\sigma}{\sigma_0}\right)^m\right]$$

where σ_0 and m are constants.

(a) Explain briefly why the values of σ_0 and m depend on the specimen volume V_0. [2]

(b) A batch of identical ceramic rods are tested to failure in tension. The rods are solid circular cylinders, loaded parallel to the axis. 85% of the specimens survive a tensile stress σ = 300 MPa, and 20% survive σ = 400 MPa. Calculate the constants σ_0 and m for the rods. [4]

(c) A number of the rods described in (b) are joined together end-to-end using an adhesive, applied uniformly across the circular faces, to create a longer rod of the same diameter. This is loaded in tension parallel to the cylinder axis. If the adhesive fails at a tensile stress σ = 290 MPa, how many rods can be joined together before failure is more likely to occur in the ceramic than the adhesive? [4]

11 (**long**)

A packing crate is to be designed to transport frozen produce within a refrigerated truck. The storage volume is a cube of side length L = 0.3 m, which is a fixed dimension. This is enclosed on all sides by walls of uniform thickness d. The crate wall material and thickness d are free variables. A cross-section through the crate is shown in Fig. 7.

The thermal insulation is characterised by the time scale τ for heat transfer through the walls

$$\tau = \frac{d^2 \rho c}{\lambda}$$

where a larger τ indicates better insulation. When crates are stacked, the walls will buckle when the force F on a crate is

$$F = \frac{4\pi^2 E d^3}{3L} \ .$$

Candidate materials and their property values are given in Table 1.

(a) A material is to be selected for the crate that maximises the time constant τ, subject to the constraint $F \geq 700$ N.

(i) Derive a material performance index that should be maximised. [4]

(ii) Identify the best material in Table 1, and give the value of d. [4]

(b) To refine the selection, a material is to be chosen that minimises the wall thickness d subject to the constraints $F \geq 700$ N and $\tau \geq 1800$ s.

(i) Identify the best material in Table 1, and give the value of d. [7]

(ii) Comment on the effect of the revised selection criteria, and suggest two additional constraints that should be considered. [3]

	cork	polystyrene foam	polyethylene
Young's modulus, E (GPa)	0.032	0.28	0.90
density, ρ (kg m^{-3})	180	270	960
thermal conductivity, λ (W m^{-1} K^{-1})	0.042	0.052	0.44
specific heat capacity, c (J kg^{-1} K^{-1})	1700	1500	1900

Table 1

 (cont.

(c) The crate design influences two contributions to the transport energy consumption. First, the power (in Watts) required to refrigerate the truck per crate is

$$P_r = \begin{cases} 500V\left(1 - \frac{\tau}{1800}\right) & \text{if } 0 \le \tau \le 1800 \text{ s} \\ 0 & \text{if } \tau > 1800 \text{ s} \end{cases}$$

where $V = (L + 2d)^3$ is the volume occupied by a crate in m^3 and τ is in seconds. Second, the energy required to move the truck per unit mass transported, per unit distance travelled, is 0.46 kJ kg^{-1} km^{-1}.

(i) Considering only cork and polyethylene, identify the material that minimises the transport energy consumption per crate, subject to the constraint $F = 700$ N. The lifespan of a crate is 1000 km of travel at an average speed of 40 km hr^{-1}. [9]

(ii) Comment on how the material selection may affect the environmental impact during other stages of the crate's life-cycle. [3]

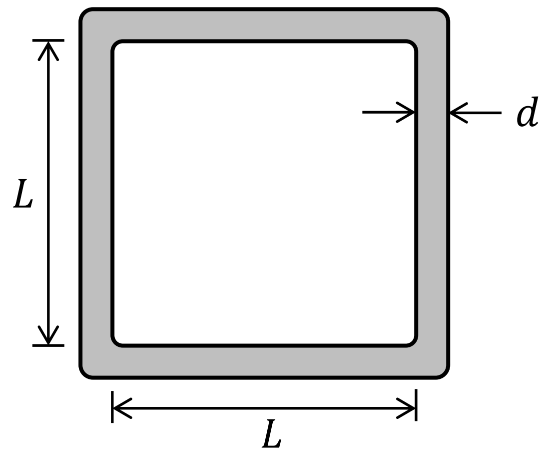

Fig. 7

12 (**long**)

(a) A thin film characterised by Young's modulus E, Poisson's ratio ν, thermal expansion coefficient α_1 and thickness t is perfectly bonded to a component with a significantly larger thickness and a different thermal expansion coefficient α_2, as shown in Fig. 8(a). The material system undergoes a temperature increase of ΔT.

(i) Derive an expression for the strains ε_x and ε_y in the film following this temperature change. [6]

(ii) Using your answer from part (i) and assuming $\sigma_z = 0$, derive an expression for the stresses σ_x and σ_y in the film. [9]

(b) An aluminium bar and a steel bar are firmly attached to the opposing sides of a rigid plate, which is thin compared to the length of the bars. The assembly is fixed between two rigid supports, as shown in Fig. 8(b). The aluminium bar is 15 m long with a cross-sectional area of 2 m^2, and the steel bar is 10 m long with a cross-sectional area of 3 m^2. Aluminium has a Young's modulus of 68 GPa and a thermal expansion coefficient of 22×10^{-6} K^{-1}, while steel has a Young's modulus of 210 GPa and a thermal expansion coefficient of 12.5×10^{-6} K^{-1}. Assume the bars behave elastically without buckling and ignore the self-weight of the system. Initially at 20 °C, the assembly is stress free.

(i) An external force of $F = 5000$ kN is applied to the rigid plate at 20 °C. Find the stresses in the Aluminium bar and the Steel bar. [6]

(ii) The assembly is subsequently heated from 20 °C to 80 °C, with the external force F still applied. Find the stresses in the aluminium and the steel bars after the temperature change. [9]

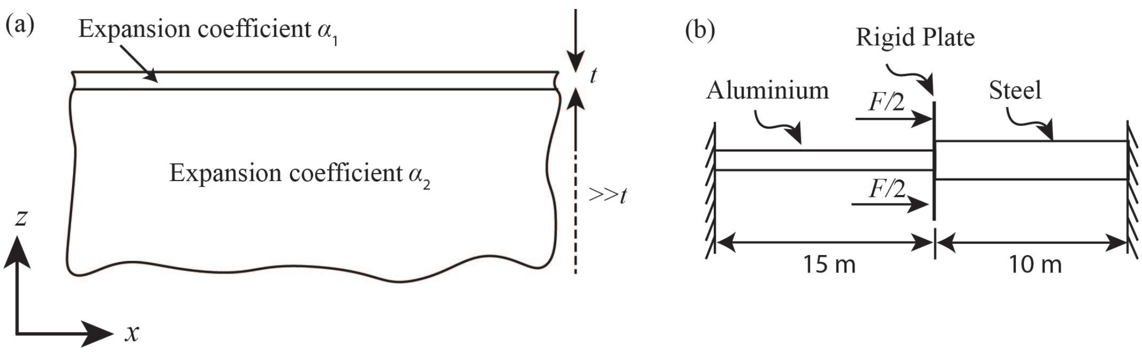

Fig. 8

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