Version RMF/BL/6

EGT0
ENGINEERING TRIPOS PART IA

Thursday 8 June 2023   9 to 12.10

Paper 2

STRUCTURES AND MATERIALS

*Answer **all** questions.*

*The **approximate** number of marks allocated to each part of a question is indicated in the right margin.*

Answers to questions in each section should be tied together and handed in separately.

*Write your candidate number **not** your name on the cover sheet.*

STATIONERY REQUIREMENTS
Single-sided script paper and graph paper

SPECIAL REQUIREMENTS TO BE SUPPLIED FOR THIS EXAM
CUED approved calculator allowed
Engineering Data Book

10 minutes reading time is allowed for this paper at the start of the exam.

You may not start to read the questions printed on the subsequent pages of this question paper until instructed to do so.

You may not remove any stationery from the Examination Room.

SECTION A

1 (**short**)

A steel frame subject to a distributed load *w* per unit horizontal length is shown in Fig. 1. Draw the bending moment diagram for this structure noting all salient values. Use the convention that moment diagrams are drawn on the tension side of the structure. [10]

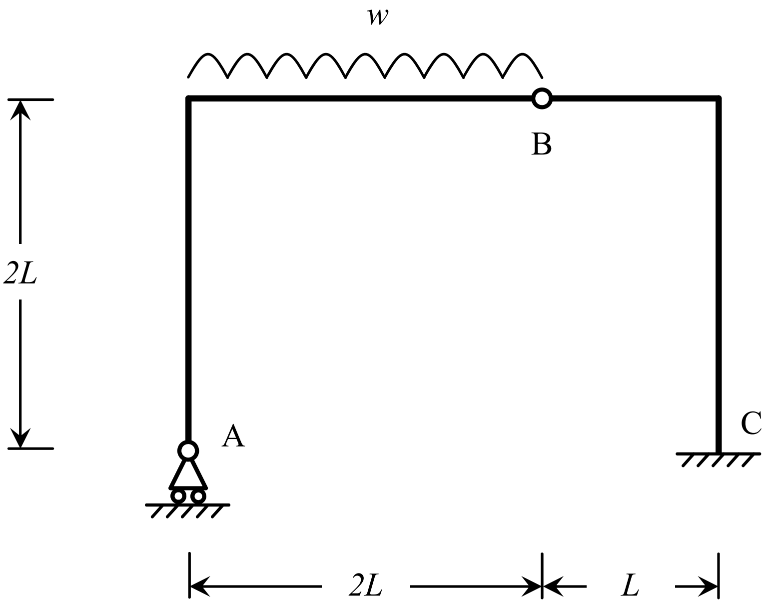

Fig. 1

2 (**short**)

The pin jointed aluminium structure shown in Fig. 2 is composed of two members with cross sectional area A and Young's modulus E such that $L/(AE) = 10^{-6}\,\mathrm{N}^{-1}\,\mathrm{m}$. A vertical point load F is applied at joint A. Using graphical methods or otherwise, determine the horizontal and vertical displacement of joint A when the applied force $F = 1$ kN. [10]

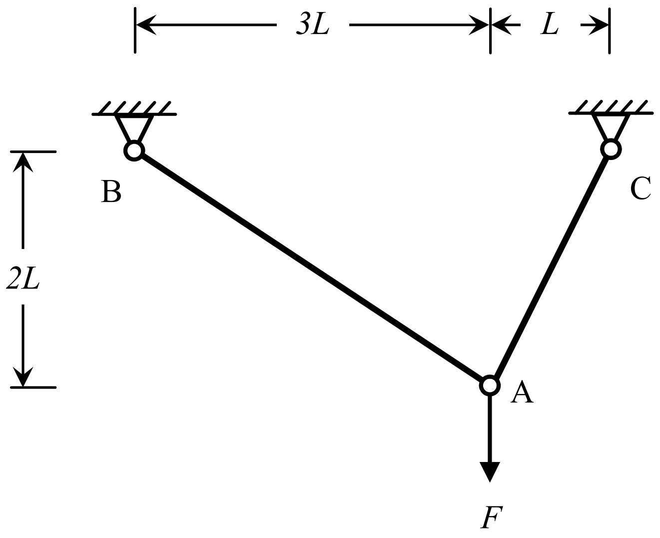

Fig. 2

3 (**short**)

A flexible inextensible cable spans 40 m between two fixed supports as shown in Fig. 3. The cable supports a uniformly distributed load of 1 kN per unit horizontal length and a vertical point load of 10 kN at B. If the dip at midspan is 1000 mm, calculate the dip at point B. [10]

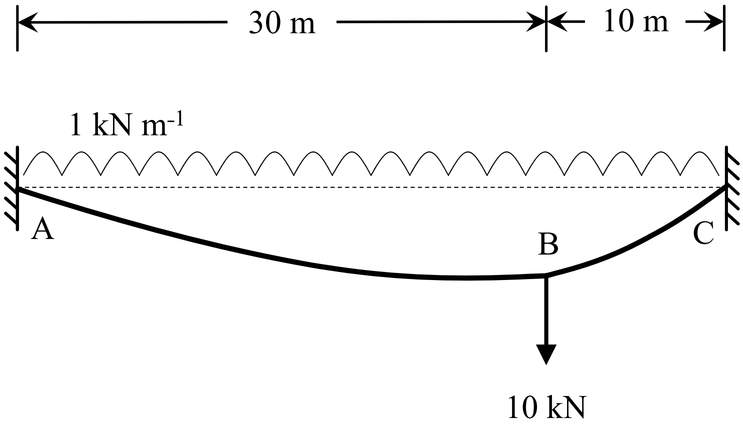

Fig. 3

4 (**short**)

A steel strut is idealised in Fig. 4(a). The column is a prismatic hollow rectangular section fabricated from 5 mm thick steel plate and has the cross-section dimensions shown in Fig. 4(b). The steel has a Young's modulus of 210 GPa and a yield strength of 275 MPa. The strut is subject to a vertical load *P* applied concentric to the strut. If the failure of the strut can be assumed to be governed either by crushing or by Euler buckling, calculate the maximum load *P* that the strut can support. [10]

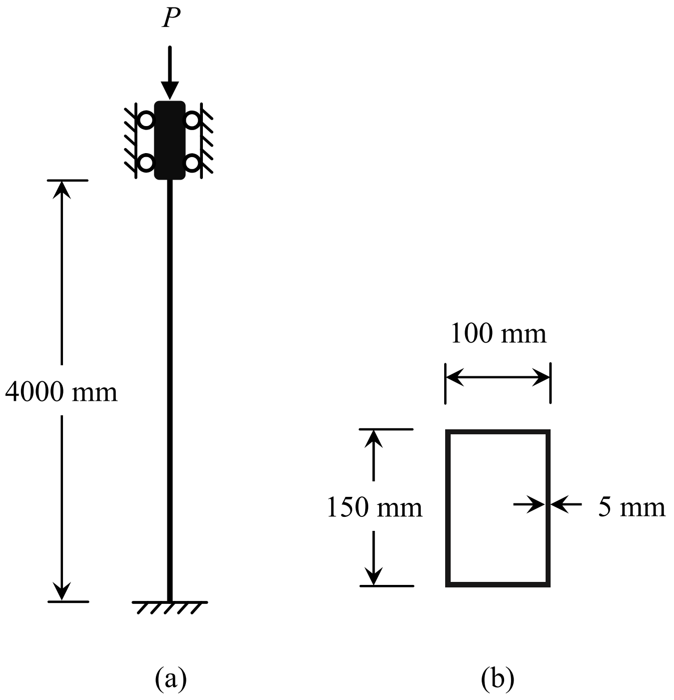

Fig. 4

5 (**long**)

Figure 5 shows an arrangement of beams of uniform cross section having flexural rigidity EI. The beams support a uniformly distributed load w per unit horizontal length.

(a) Draw the shear and bending moment diagrams for the beam subject to this loading arrangement, noting all salient values. [10]

(b) Determine the displacement of point D. [20]

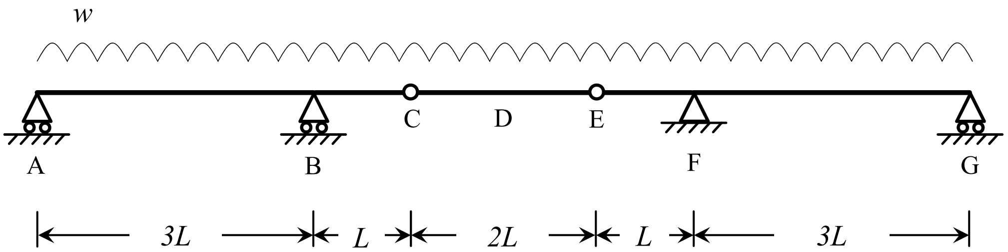

Fig. 5

6 (**long**)

The cross-section of a composite beam built-up from a pair of timber beams and a concrete slab is shown in Fig. 6. The concrete and the timber are connected at the interface by a strong, stiff adhesive that allows fully composite structural behaviour to be developed. The dimensions of the beam are given in Fig. 6.

The Young's moduli of the concrete and the timber are 30 GPa and 10 GPa respectively. The maximum longitudinal stress that the concrete can sustain is 30 MPa in compression and zero MPa in tension. The maximum longitudinal stress that the timber can sustain is 20 MPa in both tension and compression. The composite beam is designed to be simply supported and resist gravity loads only, meaning that the top fibre will be in compression and the bottom fibre will be in tension.

(a) Calculate the depth of the neutral axis, showing that this passes through the timber. Comment on the significance of the neutral axis passing through the timber rather than the concrete. [10]

(b) Determine the flexural rigidity EI of the composite beam. [10]

(c) Determine the maximum bending moment that the cross section can sustain. [10]

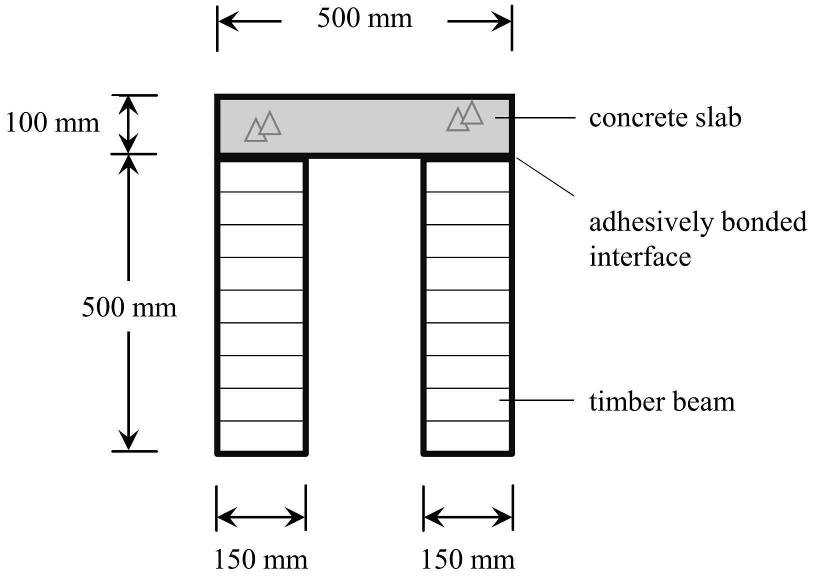

Fig. 6

SECTION B

7 (**short**) A metal is subjected to a Vickers hardness test as sketched in Fig. 7. The test resulted in an indent size $d = 0.48$ mm at a constant load $F = 5$ kg.

(a) Define the Vickers hardness and estimate the metal's yield stress σ_y. [4]

(b) The metal deforms in a standard tensile test according to the following relationship between true stress σ_t and true strain ϵ_t:

$$\sigma_t = K\epsilon_t^{0.5} \quad \text{MPa}$$

Assume the average true strain in a Vickers hardness test is 8%. Use the results from part (a) to calculate the constant K. Find the change in length of a tensile test specimen of initial length 20 cm, loaded with $\sigma_t = 200$ MPa. [6]

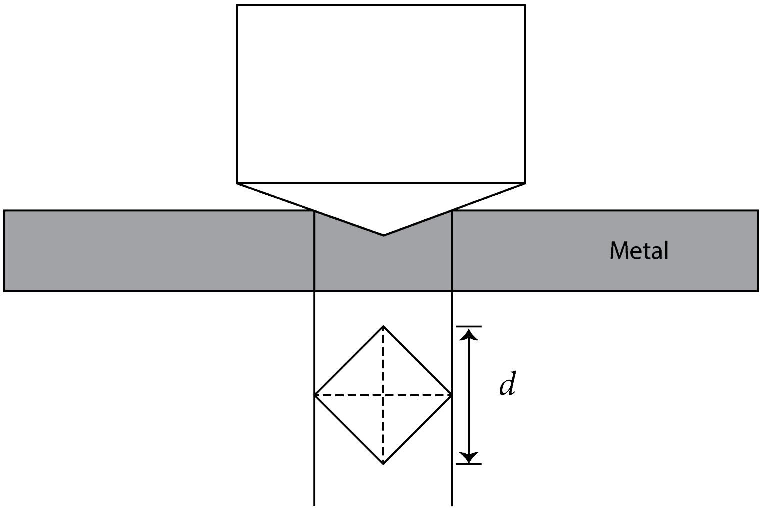

Fig. 7

8 (**short**)

(a) Describe with sketches how substitutional solid solutions provide pinning points to dislocations, increasing the yield stress. [4]

(b) The Cu-Ni system forms solid solutions for all compositions between pure Cu and pure Ni. Figure 8 shows the variation of the yield stress with composition of Ni in weight %. For low concentrations (< 20%), the yield stress σ_y is expected to vary with composition C as

$$\sigma_y = \sigma_0 + \alpha C^n \quad \text{MPa}$$

where α and n are constants.

(i) Define the physical meaning of σ_0. [2]

(ii) Use the graph to find the constants α and n for dilute solutions of Ni in Cu, with stress σ in MPa and composition C in wt%. [2]

(iii) Assuming the value of n is the same for dilute solutions of Cu in Ni, state by inspection whether the constant α is higher or lower in this case. [2]

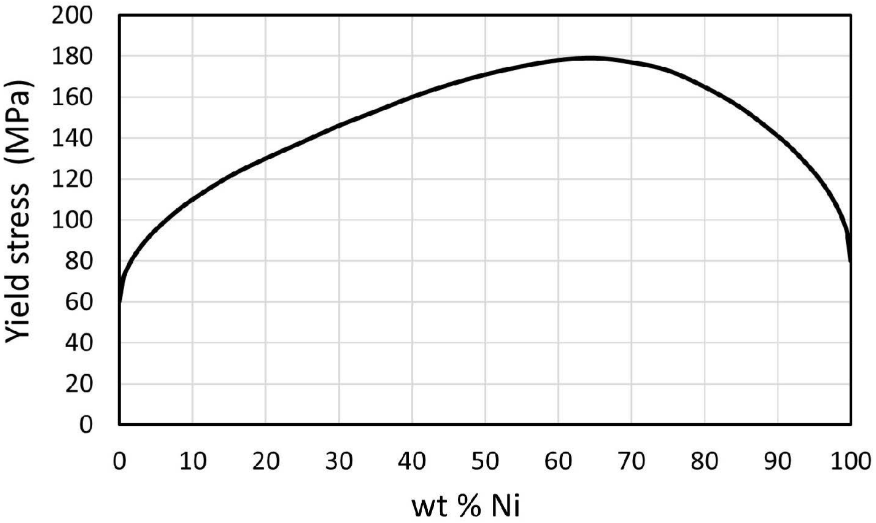

Fig. 8

9 (**short**) Some uncracked bicycle forks are subject to fatigue loading. Approximate $S - N$ data for the material used are given in Fig. 9, for zero mean stress.

(a) The loading cycle due to road roughness is assumed to have a constant stress range $\Delta\sigma$ of 1200 MPa and a mean stress σ_m of zero. How many loading cycles will the forks withstand before failing? [2]

(b) Due to a constant rider load the mean stress is 100 MPa. Use Goodman's rule to estimate the percentage reduction in lifetime associated with this mean stress. The tensile strength σ_{ts} of the steel is 1100 MPa. [4]

(c) Define the fatigue limit. What practical changes could be made to the forks to bring the stress range below the fatigue limit and avoid fatigue failure? [2]

(d) Closer inspection of a failed fork reveals that the failures occur where the forks increase in section. Explain why failure occurs at this location, and suggest an alternative change in fork design to avoid failure. [2]

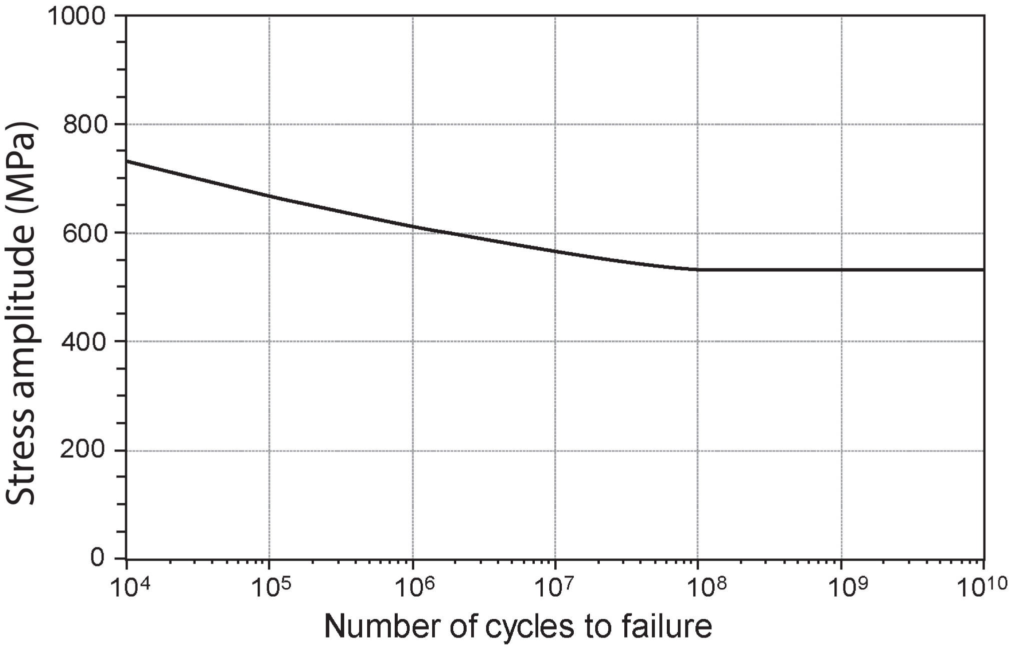

Fig. 9

10 (**short**)

(a) Explain the physical basis of Young's modulus for:

(i) glass,

(ii) polyethylene.

For which of these materials can Young's modulus be manipulated significantly via processing? Explain your reasoning. [4]

(b) Foaming is a popular method for manipulating elastic properties of solids. A simple model predicts Young's modulus of a foam, E_f, scales with that of the solid E_s as:

$$E_f = E_s(\frac{\rho_f}{\rho_s})^2$$

where ρ_f and ρ_s are the density of the foam and the solid respectively.

(i) In the model, which mechanism is assumed to dominate the elastic response of this foam? [2]

(ii) A foam made of Nickel has Young's modulus 1.3 GPa. Bulk Nickel has density ρ_s = 8.9 Mg m^{-3} and Young's modulus E_s = 200 GPa. Find the density of this foam. [2]

(iii) Using the appropriate material property chart in the data book, identify an existing material with which the Nickel foam from part (ii) appears to compete, in terms of modulus and density. Comment on one other property that could determine the suitability of replacing this material with the Nickel foam. [2]

11 (**long**) The owner of a smallholding plans to install a ground-based set of solar panels, and wishes to investigate their sustainability and costs. The owner decides to design and build the frame themselves before having the panels installed professionally.

(a) Considering first the support frame, they assess a range of structural materials without constraining the shape of the members. These will be of fixed length L, with a specified bending stiffness S, to withstand the panel weight and wind loading without excessive deflection. The bending stiffness is given by

$$S = C_1 EI/L^3$$

where C_1 is a constant depending on the support constraints, E is Young's modulus, and I is the second moment of area of the members. For a beam of cross-sectional area A, the shape factor ϕ_e is defined as

$$\phi_e = 12I/A^2.$$

Derive a performance index for the minimum embodied energy H for the frame members, for a specified bending stiffness, when both A and ϕ_e may vary. Rank the materials in Table 1 using this performance index, assuming that each material is available with the maximum shape factor indicated. [8]

(b) The owner contacts the panel installer and learns that their standard mounting system requires the frame to be built using circular sections with a fixed diameter D = 60 mm. They recommend using steel tubing of wall thickness t = 2.5 mm.

(i) Find the bending stiffness EI of the recommended steel tubing. Subsequently, identify which one of the other three materials can match the steel tubing for stiffness in the same diameter, assuming in the first instance that a solid cross-section will be required. [5]

(ii) Calculate the mass per unit length of the steel tubing, and of the other material options that meet the stiffness requirement in (i), again assuming a solid cross-section is selected. Find the embodied energy per unit length of each viable material. [4]

(iii) All the suitable frame materials are manufactured in the UK, but need to be transported 500 km by truck, for which the transport energy is 1.5 MJ tonne^{-1} km^{-1}. For a total length of frame material of 40 m, identify which material from (ii) offers the lowest combination of embodied energy plus transport energy. [3]

(c) The panel array consists of 8 panels, each weighing 18 kg, of which 80% by weight is Si and the rest is equal fractions of Al alloy and glass. The embodied energies of Si and glass are 120 and 39 MJ kg^{-1}, respectively. The panels are shipped 11,500 km from Asia to the UK by sea freight, for which the transport energy is 0.18 MJ tonne^{-1} km^{-1}, followed by a further distance of 700 km by truck within the UK. Find the total embodied energy and transport energy for the solar panels. [5]

(d) The power output of the panel array averaged over a year is 300 W.

(i) Find the total energy output of the panel array per year, and hence estimate the energy payback period, accounting for the embodied and transport energies of both frame and panels. [3]

(ii) The total installed cost is estimated to be £7000. For an electricity unit cost of £0.40 kWh^{-1}, estimate the cost payback period. [2]

Material	Density ρ (Mg m^{-3})	Young's Modulus E (GPa)	Embodied energy H_m (MJ kg^{-1})	Maximum shape factor ϕ_e
Steel	7.8	210	32	20
Al alloy	2.8	72	190	17
GFRP	1.9	20	120	14
Softwood	0.5	9.3	11	3

Table 1

12 (**long**)

(a) A thin-walled cylindrical vessel of radius r, thickness t, and fracture toughness K_{IC} is pressurised with gas of internal pressure P. Derive an expression for the critical flaw size a_{crit}. What will happen if $a_{\mathrm{crit}} > t$? [8]

(b) Silicon nitride rods with constant cross-sectional area $A = 0.25$ m^2 are subjected to constant tensile stress. A rod of length 1 m fails at 0.9 GPa with a failure probability of 0.01. A rod of length 0.4 m fails at 1 GPa with the same failure probability. Using the Weibull statistics of fracture from the Materials data book, find the stress that will cause a rod of length 0.2 m to fail with a probability of 0.01. [10]

(c) A turbine blade made of the silicon nitride from part (b) is attached to a rotor of radius $a = 0.2$ m, as shown in Figure 10. The axial stress distribution in the turbine blade can be approximated as:

$$\sigma(x) = -6x + 2.4 \text{ GPa}$$

where x is the distance from the centre of the rotor. The blade has a tip radius $b = 0.4$ m with constant cross-sectional area $A_b = 0.1$ m^2. Use the results from part (b) to find the failure probability of the blade. [12]

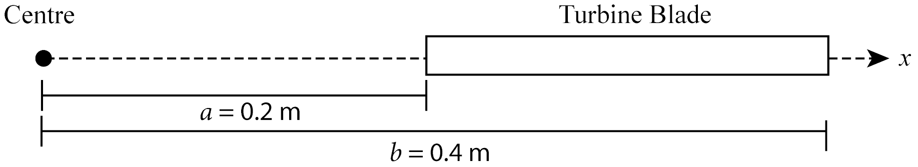

Fig. 10

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