EGT0
ENGINEERING TRIPOS PART IA

Thursday 12 June 2025 9 to 12.10

Paper 2

STRUCTURES AND MATERIALS

*Answer **all** questions.*

*The **approximate** number of marks allocated to each part of a question is indicated in the right margin.*

Answers to questions in each section should be tied together and handed in separately.

*Write your candidate number **not** your name on the cover sheet.*

STATIONERY REQUIREMENTS
Single-sided script paper and graph paper

SPECIAL REQUIREMENTS TO BE SUPPLIED FOR THIS EXAM
CUED approved calculator allowed
Engineering Data Book

10 minutes reading time is allowed for this paper at the start of the exam.

You may not start to read the questions printed on the subsequent pages of this question paper until instructed to do so.

You may not remove any stationery from the Examination Room.

SECTION A

1 (**short**)

A frame subject to a point load F is shown in Fig. 1. Draw the bending moment diagram for this structure noting all salient values. Use the convention that moment diagrams are drawn on the tension side of the structure. [10]

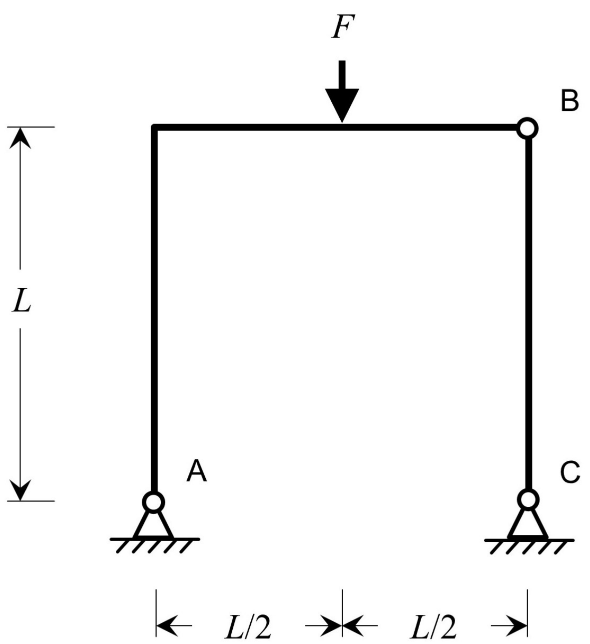


Fig. 1

2 (**short**)

Three blocks of weight W_A, W_B and W_C are connected by cables passing over frictionless pulleys as shown in Fig. 2. Blocks B and C hang freely, while the base of block A is in contact with a slope inclined at 30° to the horizontal. The angle of static friction between the base of block A and the slope is 30°. In terms of W_B, what values of W_A and W_C allow static equilibrium to be maintained in this arrangement? [10]

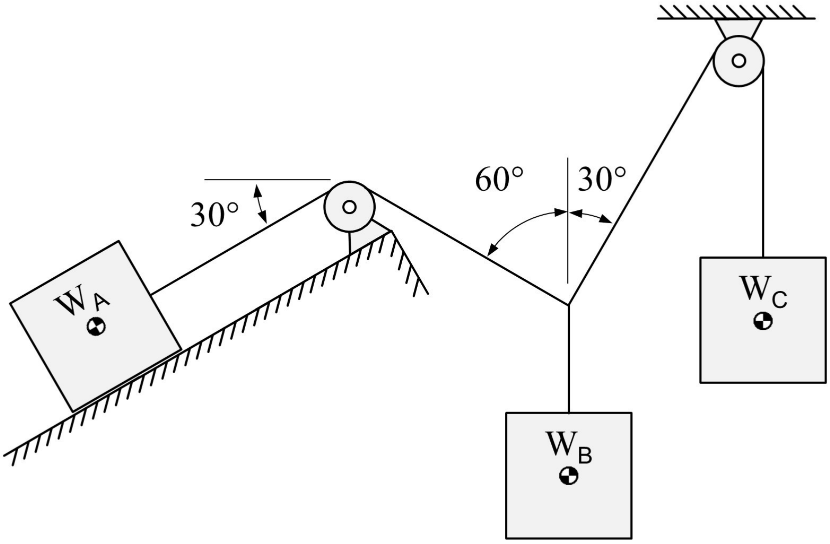
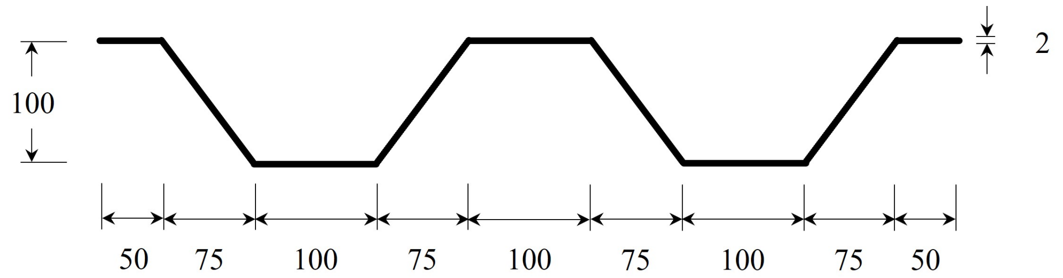


Fig. 2

3 (**short**)

A section of trapezoidal steel profiled decking is shown in Fig. 3. The section is formed from a sheet of steel 2 mm thick. What is the second moment of area I of the section about its horizontal neutral axis? [10]



Dimensions in mm.

Fig. 3

4 (**short**)

A flexible, inextensible cable is suspended between points A and B as shown in Fig. 4. The cable is subject to a load per unit horizontal length that varies linearly from 2 kN m^{-1} to 0 kN m^{-1}. At support B, the inclination of the cable is 14 degrees to the horizontal. What is the sag of the cable at midspan? [10]

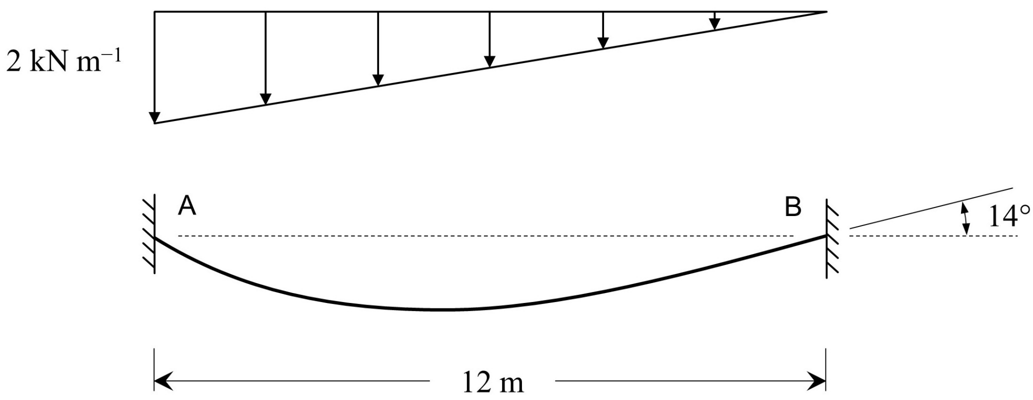


Fig. 4

5 (**long**)

Figure 5 shows a pin jointed planar truss. The nodes of the truss are effectively restrained against out-of-plane movement. The members of the truss have cross-sectional area A and Young's modulus E. A vertical point load P is applied at joint H.

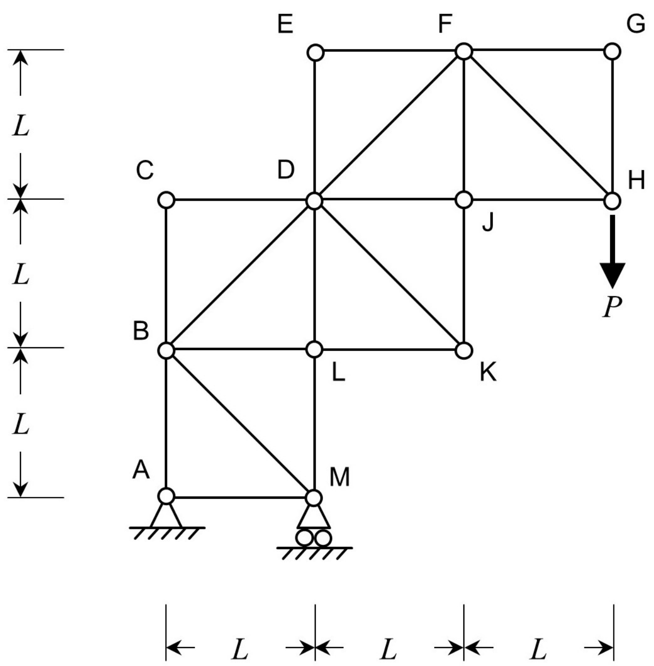


Fig. 5

(a) Determine the member forces in the truss in terms of P. [10]

(b) Determine the horizontal displacement of joint K in terms of P, L, A and E. [10]

(c) If L = 4 m and a steel circular hollow section of outside diameter 101.6 mm, wall thickness 4.0 mm, yield strength σ_y = 275 MPa and Young's modulus E = 200 GPa is selected for the design of all members, what is the maximum value of P that the truss can sustain elastically? [10]

6 (**long**)

A simply supported L-shaped elastic beam is shown in Fig. 6(a). The beam has second moment of area I and Young's modulus E. The beam is subjected to a horizontal point load of magnitude W in the middle of span AB and a vertical load of magnitude W distributed uniformly over span BC.

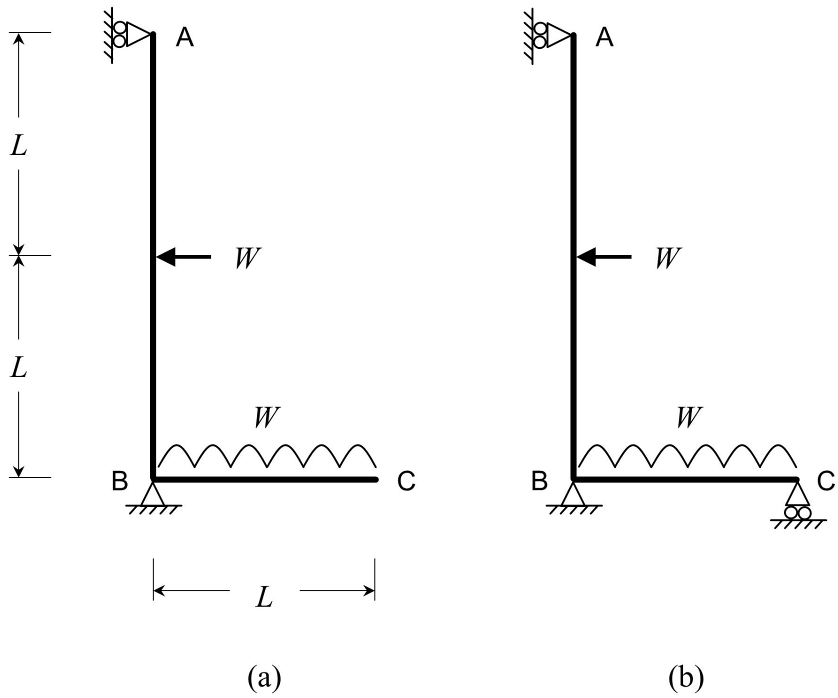


Fig. 6

(a) Draw the shear and bending moment diagrams for the beam subject to this loading arrangement, noting all salient values. [10]

(b) Determine the vertical deflection of point C. [10]

(c) A roller support is added at point C as shown in Fig. 6(b), rendering the structure indeterminate. Determine the magnitude of the vertical reaction provided by the roller support at C. [10]

SECTION B

7 (**short**)

(a) A composite material has a matrix of polypropylene and contains 10% glass reinforcement by volume. Polypropylene has a Young's modulus of 1 GPa, while glass has a Young's modulus of 70 GPa.

(i) Estimate the Young's modulus of the composite along the fibre direction, assuming the glass reinforcement is in the form of long, parallel fibres. [3]

(ii) Estimate the Young's modulus of the composite, assuming the glass reinforcement is in the form of small dispersed particles. [3]

(b) A cubic element of a linear elastic material with Young's modulus E and Poisson's ratio ν is subjected to uniaxial tensile stresses $\sigma_1 = \sigma$, $\sigma_2 = 0$ and $\sigma_3 = 0$. Find an expression of volumetric strain (or dilation) Δ. For what value of Poisson's ratio is the volume of the element conserved? [4]

8 (**short**)

(a) List the four main microstructural mechanisms by which metals are hardened. [4]

(b) Two cubes of aluminium of side length 1 cm are produced in annealed (soft) and cold-worked (hard) conditions. The dislocation densities in the two samples are 10^5 mm^{-2} and 10^9 mm^{-2}, respectively. Assuming a square array of parallel dislocations, estimate:

(i) the total length of dislocation in each sample; [2]

(ii) the distance between dislocations in the cold-worked sample; [2]

(iii) the ratio of the contribution made by dislocation pinning to the yield strength of the cold-worked sample, to that of the annealed sample. [2]

9 (**short**)

(a) The molecular structure of crystalline polyethylene is shown in Fig. 7. Calculate the density of this polyethylene structure, and explain whether this value is an upper or lower bound for practical polyethylenes. [4]

(b) Would the following polymers be used mostly above or below their glass transition temperature? Justify your answer with reference to the molecular structure, and its influence on their properties.

(i) Semi-crystalline thermoplastic.

(ii) Amorphous thermoplastic.

(iii) Natural rubber. [6]

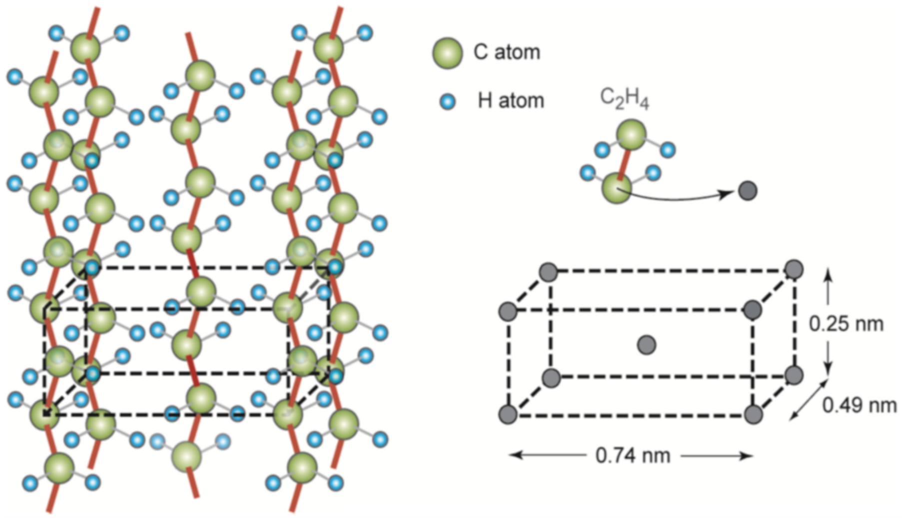


Fig. 7

10 (**short**)

A company supplying mineral water is considering switching from PET polymer bottles to aluminium (Al) alloy cans. A life cycle analysis shows that the embodied energy of the container material is the most significant component of the total energy usage, per unit volume of water contained. Data for each container are given in Table 1.

(a) Calculate the embodied energy of each container, per unit volume of water contained, if they are manufactured from 100% virgin material. [2]

(b) Calculate the fraction of recycled Al alloy that must be used to manufacture each can for it to have the same embodied energy as a PET bottle, per unit volume of water contained, if the PET bottles use 30% recycled material, a typical maximum. [4]

(c) Outline two strategies to reduce the embodied energy of the Al alloy cans, per unit volume of water contained. In your answer, explain briefly the technical challenges the strategy presents. [4]

	PET bottle	Al alloy can
Container capacity ($m^3 \times 10^{-6}$)	500	330
Container mass (kg $\times 10^{-3}$)	30	12
Embodied energy, virgin material (MJ kg^{-1})	84	200
Embodied energy, recycled material (MJ kg^{-1})	39	25

Table 1

11 (**long**)

A pipe in a power station heat exchanger carries gas at high pressure and temperature. The pipe is a thin-walled circular cylinder of diameter $D = 0.025$ m and wall thickness t (which is a design variable). The gauge pressure inside the pipe is p. The temperature of the gas inside the pipe is T. Outside the pipe, the gauge pressure is zero and the temperature is T_0.

(a) A material is to be selected for the pipe from the options in Table 2. The rate of heat flow through the pipe wall per unit length ($\dot{Q}$) is to be maximised, where

$$\dot{Q} = (T - T_0)\, 2\pi\lambda \frac{D}{t}$$

and λ is the thermal conductivity of the pipe.

(i) If the hoop stress in the pipe σ_h must be less than the yield strength σ_y when $p = 3$ MPa, derive a material performance index that should be maximised, and hence identify the best material and the required wall thickness. [6]

(ii) A second constraint is added. A length $L = 2$ m of empty pipe is simply supported at both ends. It must deflect less than $\delta = 1.5$ mm mid-span due to self weight. Identify whether this constraint alters the material choice and the rate of heat transfer. [8]

(iii) Describe two other constraints to consider in the material selection, and comment on how they may influence the design and performance of the pipe. [4]

(b) A thin protective coating (thickness much less than t) is perfectly bonded to the inside of the pipe. The coating and pipe are strain free when $p = 0$ and $T = T_0$. When $p > 0$ and $T > T_0$, the pipe is supported such that the longitudinal stresses in the pipe wall remain zero, and there is no constraint against longitudinal expansion. The pipe has Young's modulus E_p and Poisson's ratio ν_p. For the coating, these are E_c and ν_c. The coefficients of thermal expansion of the pipe and coating are α_p and α_c, respectively.

(i) Derive an expression for the hoop strain ε_{hc} and longitudinal strain ε_{lc} in the coating when $p > 0$ and the coating and pipe wall are at a uniform temperature $T = T_0$. [4]

(ii) Derive an expression for the hoop stress σ_{hc} in the coating when $p > 0$ and the coating and pipe wall are at a uniform temperature $T > T_0$. [8]

	Cu alloy	stainless steel	Al alloy
Young's modulus, E (GPa)	130	200	75
Density, ρ (kg m^{-3})	8200	7850	2700
Yield strength, σ_y (MPa)	230	550	300
Thermal conductivity, λ (W m^{-1} K^{-1})	280	20	160

Table 2

 (TURN OVER

12 (**long**)

(a) For a pre-cracked specimen subjected to fatigue loading, sketch the logarithm of crack growth per cycle $\mathrm{d}a/\mathrm{d}N$ as a function of the logarithm of the stress intensity factor range ΔK. On your diagram, label and discuss the different regions. [6]

(b) A thin-walled, cylindrical steel pressure vessel of 8 m diameter and 40 mm wall thickness is to operate at a working pressure of 4.1 MPa. Non-destructive testing revealed the presence of small semi-circular cracks in the inside wall which will gradually extend through the wall by fatigue. Assume for this crack geometry that $K = \sigma\sqrt{\pi c}$, where c is the radius of a semi-circular crack and σ is the hoop stress in the vessel. The fracture toughness K_{IC} of the steel is 200 MPa $\mathrm{m}^{1/2}$.

(i) Will the vessel fail in service by leaking or by fast fracture? [6]

(ii) During service, the vessel is subjected to repeated cycles from zero to working pressure. In fatigue conditions, crack growth is given by Paris' law (page 7 of the Materials Databook), with $n = 4$ and $A = 2.44 \times 10^{-14}$ MPa^{-4} m^{-1}. Find the initial crack size c_0 if the vessel is to survive 2000 pressurisation cycles. [10]

(iii) The vessel is subjected to a proof test so that any initial cracks larger than c_0 will fracture the vessel catastrophically. Find the pressure to which the vessel must be subjected to survive the proof test. [8]

END OF PAPER