EGT0
ENGINEERING TRIPOS PART IA

Thursday 9 June 2022 9 to 12.10

Paper 2

STRUCTURES AND MATERIALS

*Answer **all** questions.*

*The **approximate** number of marks allocated to each part of a question is indicated in the right margin.*

Answers to questions in each section should be tied together and handed in separately.

*Write your candidate number **not** your name on the cover sheet.*

STATIONERY REQUIREMENTS
Single-sided script paper and graph paper

SPECIAL REQUIREMENTS TO BE SUPPLIED FOR THIS EXAM
CUED approved calculator allowed
Engineering Data Book

10 minutes reading time is allowed for this paper at the start of the exam.

You may not start to read the questions printed on the subsequent pages of this question paper until instructed to do so.

You may not remove any stationery from the Examination Room.

SECTION A

1 (**short**)

A steel frame subject to a single point load is shown in Fig. 1. Draw the bending moment diagram for this structure noting all salient values. Use the convention that moment diagrams are drawn on the tension side of the structure. [10]

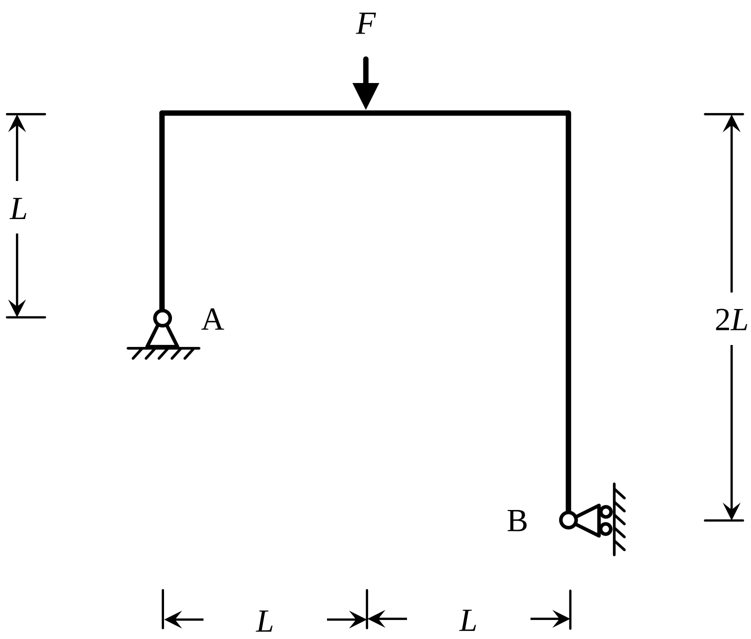


Fig. 1

2 (**short**)

A long-span timber roof structure is constructed as a three-pinned arch as shown in Fig. 2. The arch has a parabolic geometry defined by:

$$y = d\frac{x^2}{L^2}$$

A uniformly distributed load w per unit horizontal length acts over one half of the structure. The self-weight of the structure is negligible. Determine the magnitude of the bending moment at $x = \frac{L}{2}$. [10]

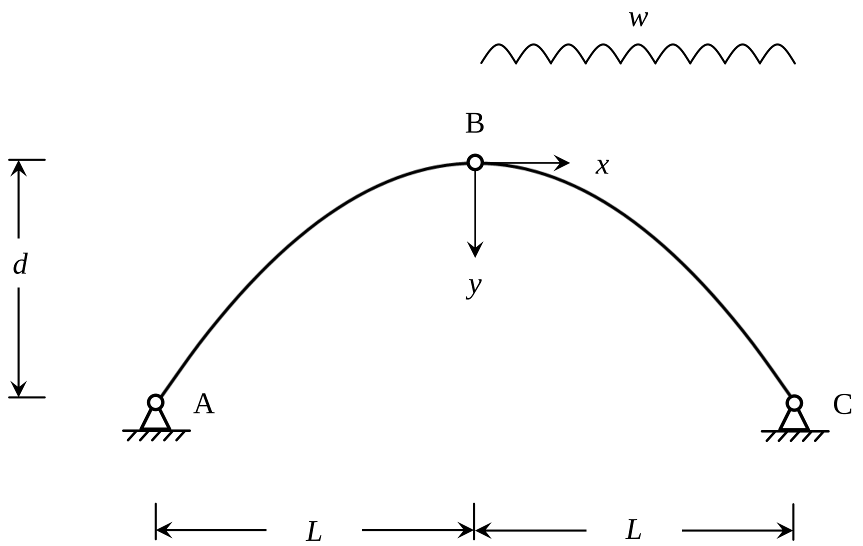


Fig. 2

3 (**short**)

A symmetrical aluminium T-section is shown in Fig. 3. Calculate the second moment of area of the section about its horizontal neutral axis. [10]

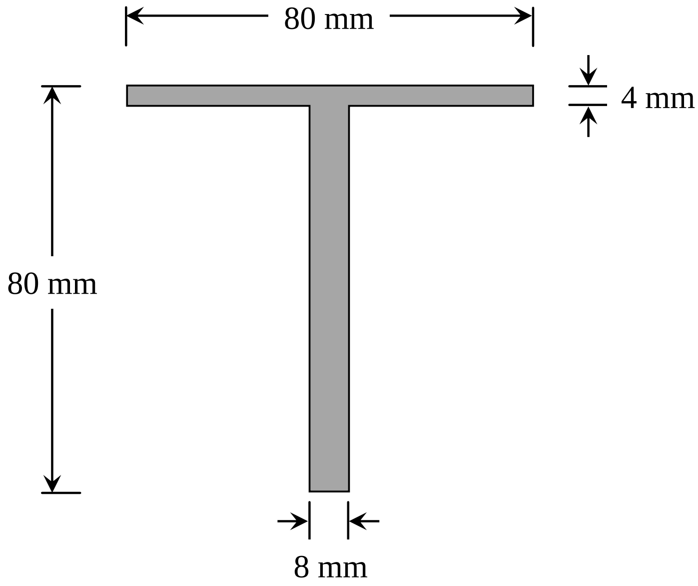


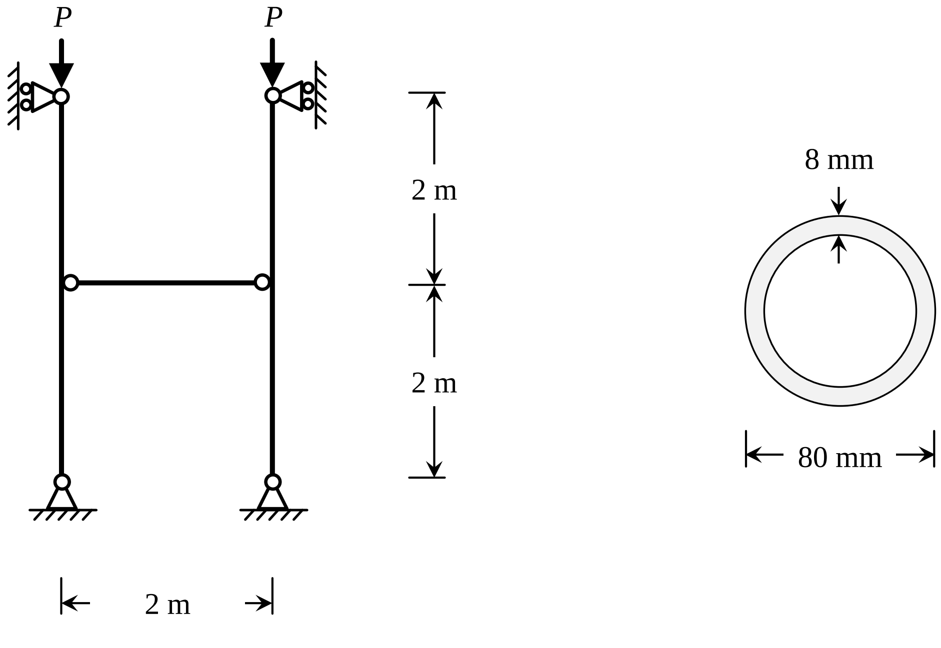
Fig. 3

4 (**short**)

A portion of a bamboo scaffolding structure is idealized in Fig. 4(a). The bamboo members can be assumed to be prismatic and have the cross-section dimensions shown in Fig. 4(b). The bamboo has a Youngs modulus of 15 GPa. The vertical bamboo struts are each subject to a vertical load P applied concentric to the strut.

(a) Calculate the Euler buckling load of the vertical struts. [7]

(b) Why is the buckling load likely to be less than the load calculated in part a)? [3]



(a) **(b)**

Fig. 4

5 (**long**)

A pin jointed steel truss structure is shown in Fig. 5. The truss is subject to a vertical point load of 10 kN applied at joint D. The members of the truss have cross-sectional area A and Youngs Modulus E such that $EA = 10^7$ N.

(a) Determine the forces in each bar of the truss. [10]

(b) Determine the displacement of joint B. [20]

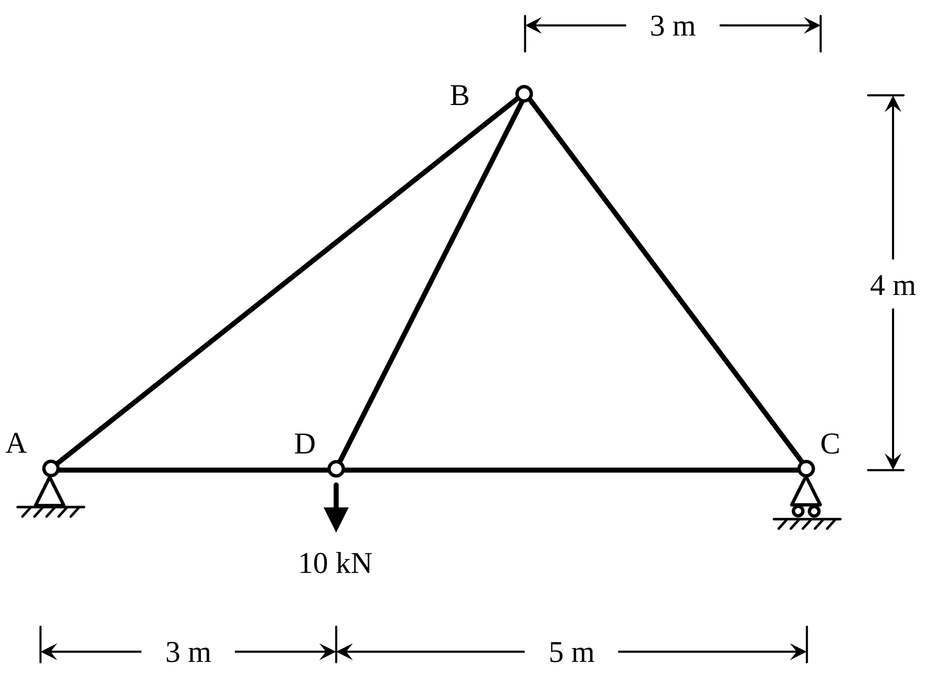


Fig. 5

6 (**long**)

A simply supported timber beam subject to a uniformly distributed load is shown in Fig. 6. Attached to the beam, mid-span, is a vertical cable that passes over a frictionless pulley to support a suspended weight P. The designed weight of P is 300 kN such that the mid-span deflection of the beam is zero when the beam is subject to a uniformly distributed load of 30 kNm^{-1}. The beam has a rectangular cross-section of 600 mm depth and 200 mm breadth. The timber has a Youngs modulus of 11 GPa and sustains a maximum longitudinal stress in bending of 24 MPa.

(a) Draw the shear and bending moment diagrams for the beam subject to this loading arrangement, noting all salient values. [10]

(b) It is found that the as-built weight of P is 20% less than originally designed. What is the resulting maximum longitudinal stress at the critical section? Is the section adequate? [10]

(c) What is the mid-span deflection associated with this reduction in the weight of P? Comment on the magnitude of this deflection and any implications for designers in analogous situations. [10]

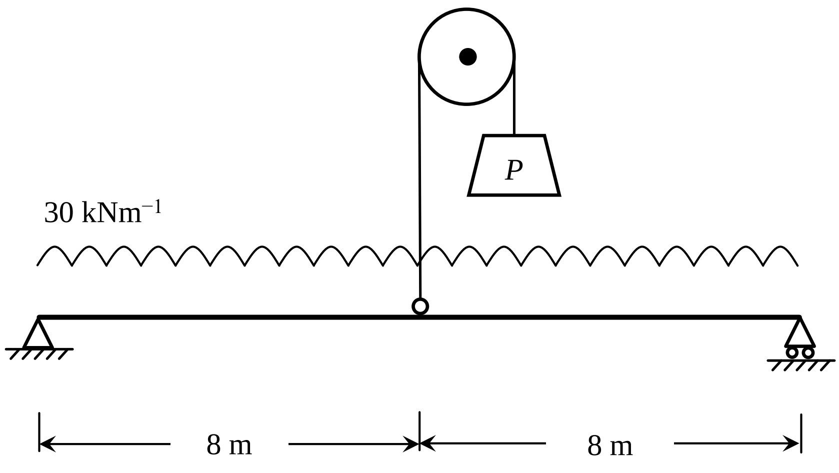


Fig. 6

(TURN OVER

SECTION B

7 (**short**)

(a) Fig. 7 shows four tensile nominal stress-strain curves up to failure for materials A, B, C and D. Each of these curves represents either a metal, a polymer, or a ceramic. In each case, identify which it is, giving reasons for your choice. [4]

(b) For the case of the metal, sketch the nominal stress-strain curve in tension, and superimpose a sketch of the corresponding true stress-strain curve in both tension and compression. [2]

(c) For the case of the ceramic, sketch the expected form of the nominal stress-strain response in both tension and compression. By sketching the failure mechanism in each case, explain the relative strength observed in tension and compression. [4]

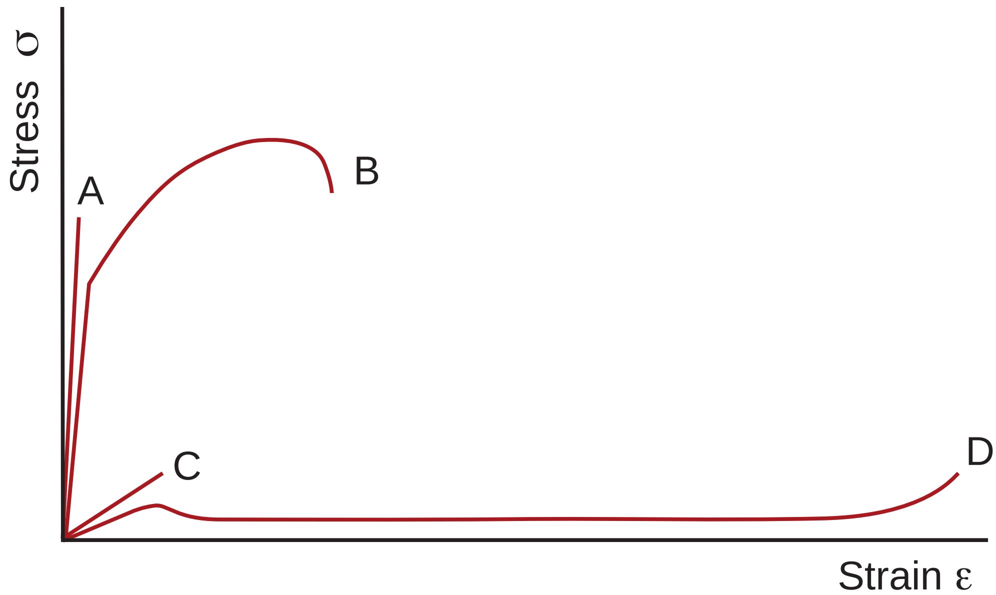


Fig. 7

8 (**short**)

(a) Sketch and explain the response of a dashpot to a step in strain (relaxation response) and a step in stress (creep response). [2]

(b) Indicate which of the curves on Fig. 8 (labelled by the letters a to l) corresponds to:

(i) the relaxation response of model 1; [2]

(ii) the creep response of model 1; [2]

(iii) the relaxation response of model 2; [2]

(iv) the creep response of model 2. [2]

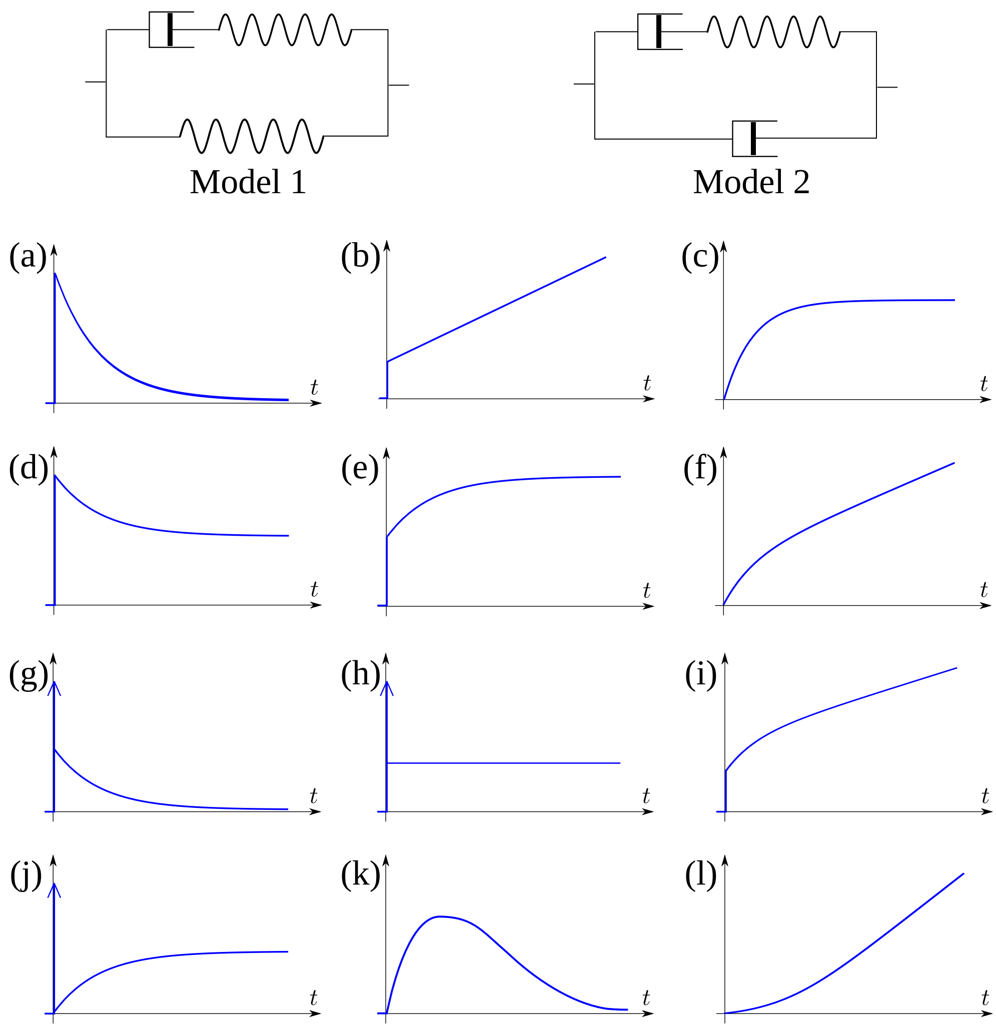


Fig. 8

9 (**short**)

In salt (sodium chloride NaCl), Cl ions form a FCC lattice, and Na ions occupy all the available octahedral holes. The crystal structure of NaCl is shown in Fig. 9.

(a) NaCl has a lattice constant $a = 0.564$ nm. Determine the density of NaCl given that the atomic masses of sodium and chlorine are 22.989 and 35.453 kg/kmol, respectively. [5]

(b) The ratio of the diameters of sodium and chlorine ions is 0.69. Show that the chlorine and sodium ions are touching along the edges of the unit cell. Find the size (as a fraction of the diameter of the Cl ions) of the gaps between the chlorine ions on the diagonal of the face of the unit cell. [5]

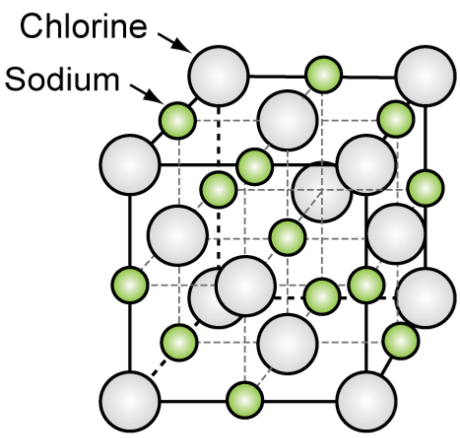


Fig. 9

10 (**short**)

An isotropic elastic material with Young's modulus E and Poisson's ratio ν is stressed in its principal axes with plane strain condition (i.e., $\epsilon_3 = 0$).

(a) Using 3D Hooke's law, find an expression for σ_3 in terms of σ_1 and σ_2, and express the matrix C such that

$$\begin{bmatrix} \epsilon_1 \\ \epsilon_2 \end{bmatrix} = C \begin{bmatrix} \sigma_1 \\ \sigma_2 \end{bmatrix}$$

in terms of E and ν. [5]

(b) Find the relationship between ϵ_1 and ϵ_2 when $\nu = 0.5$. What is the physical interpretation of this equation? [5]

11 (**long**)

(a) Describe the following concepts used in fracture mechanics:

(i) stress intensity factor K; [2]

(ii) fracture toughness K_{IC}; [2]

(iii) fast fracture. [2]

(b) An aircraft component is manufactured from an aluminium alloy with a fracture toughness of 28 MPa m$^{1/2}$. Testing has established that fracture occurs at a remote stress of 260 MPa when the maximum internal crack length is 2.4 mm. For this same component, alloy and crack orientation, determine whether fast fracture will occur at a stress level of 340 MPa when the maximum internal crack is 1.2 mm. Justify your answer. [7]

(c) A steel girder undergoes sinusoidal cyclic loading between 225 MPa (tension) and 60 MPa (compression). Fatigue crack growth follows the Paris law with exponent $n = 2.5$ and constant 1.5×10^{-10} MN^{-n} m$^{1+1.5n}$. The steel used has a fracture toughness $K_{IC} = 95$ MPa m$^{1/2}$. The largest surface crack present, identified by X-ray inspection, is 2.5 mm in length.

(i) Calculate the fatigue life of the girder. In the expression for the stress intensity factor, assume that the dimensionless constant Y is equal to 1. [8]

(ii) Find the decrease in stress range necessary to increase the fatigue life of the girder, estimated in (i), by a factor of 5. [6]

(iii) It is found that in service the loading amplitude fluctuates with a fraction of the runs being at a lower load amplitude. How would you calculate the effect this has on a fatigue life? [3]

12 (**long**)

A beam is designed to be as light as possible for the chassis of a race car. The beam is simply supported, carries a distributed load w (we can neglect self-weight here), and has a length L. The cross-sectional area A is uniform along the beam but its value may be varied. The deflection δ and the bending moment M_0 at mid-span are:

$$\delta = \frac{5wL^4}{384EI} \quad \text{and} \quad M_0 = \frac{wL^2}{8},$$

where E is the Young's modulus and I is the second moment of area. The maximum tensile stress

$$\sigma_0 = \frac{M_0 y_{\max}}{I}$$

is found at the maximum distance $y_{\max}$ of the beam cross-section from the neutral axis.

(a) In a first design iteration, the section shape of the beam may be changed.

(i) Using a solid square section as reference shape, derive the following expressions ϕ_e and ϕ_f of the shape factors for bending stiffness and strength,

$$\phi_e = \frac{12I}{A^2} \quad \text{and} \quad \phi_f = \frac{6(I/y_{\max})}{A^{3/2}},$$

respectively. [3]

(ii) There are two possible design constraints to consider: the beam deflection should not exceed $\delta_{\max}$, and the beam should not fail, $\sigma_0 \leq \sigma_f$ (the failure strength). For each constraint, derive the performance index to be maximised for minimum weight. [7]

(iii) For each constraint, rank the materials given in Table 1 from best to worst. [4]

(b) In a refined model, the beam is a cylindrical tube with length $L = 1$ m, radius $R = 2$ cm and a thickness t that can be chosen. The distributed load $w = 10^3$ N m^{-1} and the maximum deflection $\delta_{\max} = 2.5$ mm. The tube is thin walled, $t \ll R$, so we may use $A = 2\pi Rt$ and $I = \pi R^3 t$. Both stiffness and strength constraints identified in (a.ii) must now be satisfied.

(i) State the objective, geometric constraints, functional constraints, and the free variables for this design. [3]

(ii) Calculate the required value of the thickness t for the materials in Table 1. [7]

(iii) For each material, and using the values of t found in (b.ii), evaluate the relevant shape factor for the tube. Identify the lightest material for the beam, briefly noting other factors to consider. [6]

Material	Density ρ ($\mathrm{Mg\,m^{-3}}$)	Modulus E (GPa)	Strength σ_f (MPa)	Maximum shape factor ϕ_e (ref. solid square)	Maximum shape factor ϕ_f (ref. solid square)
Steel	7.8	210	900	64	13
Al alloy	2.6	70	200	49	10
Ti alloy	4.5	100	700	57	11

Table 1

END OF PAPER