Q1 At balance, $R_2/R_1 = \dfrac{\left(R_3 + \frac{1}{j\omega C}\right)}{\dfrac{1}{1/R_3 + j\omega C}}$

$$R_2/R_1 = \left(R_3 + \frac{1}{j\omega C}\right)\left(1/R_3 + j\omega C\right)$$

$$R_2/R_1 = \left(2 + R_3 j\omega C + \frac{1}{R_3 j\omega C}\right)$$

Equating imaginary parts:

$$R_3 \omega C = \frac{1}{R_3 \omega C}.$$

$$1 = R_3^2 \omega^2 C^2$$

$$\omega = \frac{1}{R_3 C} \Rightarrow f = \frac{1}{2\pi R_3 C}$$

Q2 $470\mu F = \frac{1}{j\omega C} = -j6.77$. $10mH = j\omega L = j3.14$.

Total impedance $= 1 + [1 \parallel (2 + j3.14 - j6.77)]$

$= 1.864 - j0.16$.

$= 1.87 \angle -4.906°$.

Supply $= \frac{120V}{1.87 \angle -4.906°}$

$= 64.17 \angle 4.906°$ Amp.

Resonance happens at $X_L = X_C$.

$2\pi f L = \frac{1}{2\pi f C}$

$f = \frac{1}{2\pi\sqrt{LC}} = 73.4$ Hz.

Q3 For wideband gain region, f will be low enough to keep the 100pF cap. open.

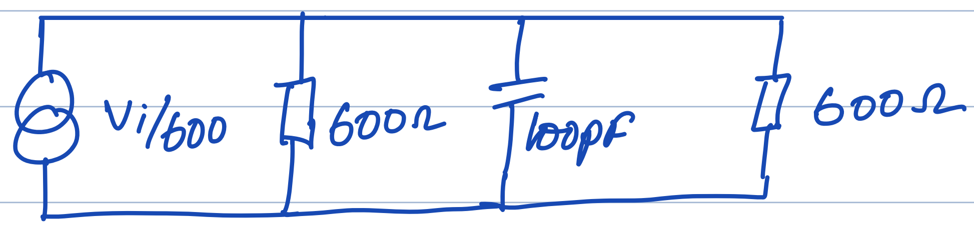
Therefore, combining the resistances, & considering virtual earth at the ideal op-amp input, we get:

$$\frac{V_{in}}{1200} = -\frac{V_{out}}{15000}$$

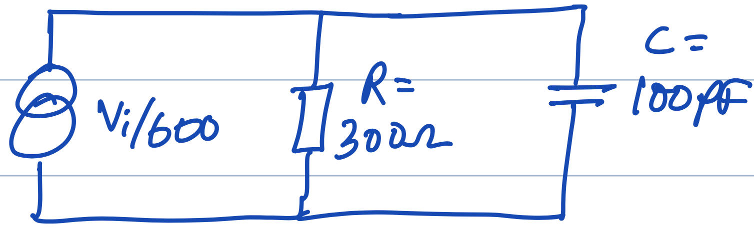
$$V_{out}/V_{in} = -\frac{15000}{1200} = -12.5.$$

At -3dB frequency, Vout drops to $\frac{1}{\sqrt{2}}$ of midband value. The same is true for the voltage across the cap.

Convert to NORTON,



This becomes:



$$V_c = \left(\frac{1}{\frac{1}{R} + j\omega C} \right) \frac{V_i}{2R}$$

$$V_c = \frac{V_i}{2\,(1 + j\omega CR)}$$

equate real & imaginary parts:

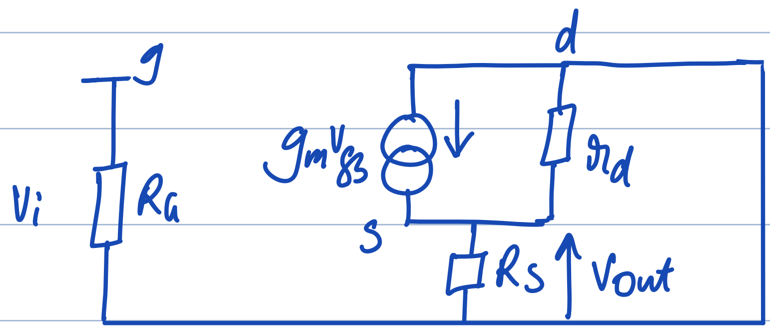
$$\omega CR = 1.$$

$$f = \frac{1}{2\pi\, CR} = \frac{1}{2\pi \times 100 \times 10^{-12} \times 300}\ \mathrm{Hz}$$

$$= 5{\cdot}30\ \mathrm{MHz}.$$

Q4 (long)

(a) Small signal model:



$$V_{out} = g_m v_{gs} (R_s \parallel r_d)$$

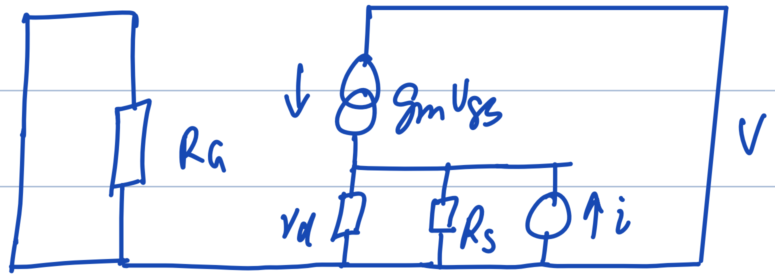
$$V_i = V_{gs} + V_{out}.$$

$$V_{out} = g_m (V_i - V_{out}) (R_s \parallel r_d)$$

$$V_{out} (1 + g_m (R_s \parallel r_d)) = g_m V_i (R_s \parallel r_d)$$

$$V_{out}/V_i = \frac{g_m (R_s \parallel r_d)}{1 + g_m (R_s \parallel r_d)} = 0.955$$

for o/p impedance,



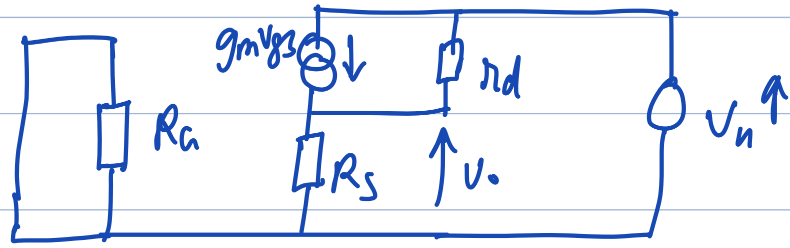
$$(i + g_m V_{gs})(R_s \,//\, r_d) = V = -V_{gs}$$

$$(i - g_m V)(R_s \,//\, r_d) = V.$$

$$V(1 + g_m (R_s \,//\, r_d)) = i\,(R_s \,//\, r_d)$$

$$V/i = \frac{(R_s \,//\, r_d)}{1 + g_m (R_s \,//\, r_d)} = 191\,\Omega.$$

(b) Component of o/p arising from noise:



$$V_o = -V_{gs}.$$

$$V_o/R_s = g_m v_{gs} + \frac{(V_n - V_o)}{r_d}$$

$$V_o = \frac{V_n}{r_d\left(g_m + \frac{1}{R_s} + \frac{1}{r_d}\right)}$$

(c) This needs to be less than 30μV.

Therefore $$\frac{V_n}{r_d\left(1/r_d + 1/R_s + g_m\right)} < 30\times10^{-6}$$

$$V_n < 2{\cdot}355\,mV.$$

Q5 (long)

(a) 4 kg lifted at 1 m/sec.

$$\text{Power} = mgv \qquad g = 9{\cdot}81 \text{ m/sec}^2.$$

$$= 4 \times 9{\cdot}81 \times 1$$

$$= 39{\cdot}24 \text{ Watts.}$$

$$VAR = I^2 \omega L = 2^2 \times 2\pi \times 50 \times 50 \times 10^{-3} = 62{\cdot}83$$

$$VA = \sqrt{W^2 + (VAR)^2} = \sqrt{(39{\cdot}24)^2 + (62{\cdot}83)^2}$$

$$= 74{\cdot}08.$$

$$p.f = \frac{39{\cdot}24}{74{\cdot}08} = 0{\cdot}53$$

(b) $\text{Voltage needed} = \dfrac{74{\cdot}08}{2} = 37{\cdot}04$ volts.

With a 20:1 turns ratio, the i/p voltage is

$$= 37{\cdot}04 \times 20 = 74{\cdot}08 \text{ volts.}$$

A capacitor parallel across the high voltage terminals will need to have input as:

$$VARS = \frac{V^2}{1/\omega C} = (74{\cdot}08)^2 \times 2\pi \times 50 \times C.$$

For a p.f. of unity, this equates to 62·83 VAR.

Then, $C = \dfrac{62 \cdot 83}{(74 \cdot 08)^2 \times 2\pi \times 50}$

$= 0 \cdot 364 \, \mu F$

(c) The winding losses are: 3W & 4 VAR.

Input power $= 39 \cdot 24 + 3 = 42 \cdot 24$ Watts.

Input VAR $= 62 \cdot 83 + 4 = 66 \cdot 83$ VARS.

Therefore, input VA $= \sqrt{(42 \cdot 24)^2 + (66 \cdot 83)^2}$

$= 79 \cdot 05$ VA.

Current is $2/20 = 1/10$ Amp.

Input $V = 790 \cdot 5$ volts.

Input p.f. $= \dfrac{42 \cdot 24}{79 \cdot 05} = 0 \cdot 534$.

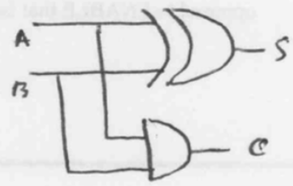
6 (Short)

(a) Half adder. The logic table, logic expression and the implementation of a half adder using a XOR gate are shown below:

A	B	S	C
0	0	0	0
0	1	1	0
1	0	1	0
1	1	0	1

$$S = \bar{A}B + A\bar{B} = A \oplus B$$

$$C = AB$$

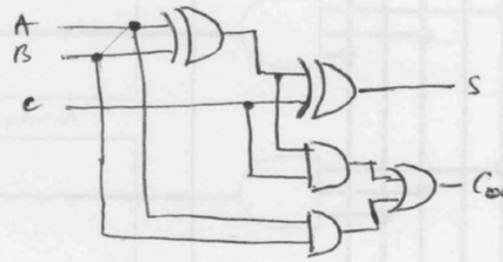


(b) (b) The full adder. The logic table, logic expression and the implementation of a half adder using a XOR gate are shown below:

A	B	Cin	Cout	S
0	0	0	0	0
0	0	1	0	1
0	1	0	0	1
0	1	1	1	0
1	0	0	0	1
1	0	1	1	0
1	1	0	1	0
1	1	1	1	1

$$S = A \oplus B \oplus C_{in}$$

$$C_{out} = AB + (C_{in} \cdot A \oplus B))$$



7 (Short)

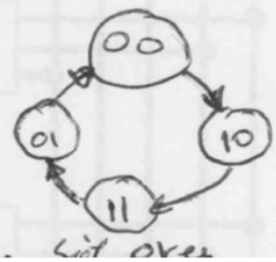
(a)

$Q^+ = D$ for D bistables

$Q_2^+ = \overline{Q_1}$

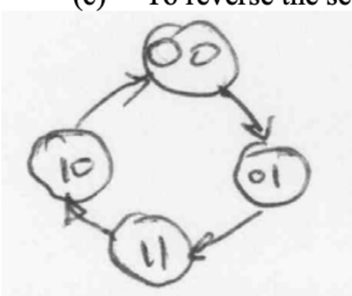
$Q_1^+ = Q_2$

$Q_2 Q_1$	$Q_2^+ Q_1^+$
0 0	1 0
1 0	1 1
1 1	0 1
0 1	0 0



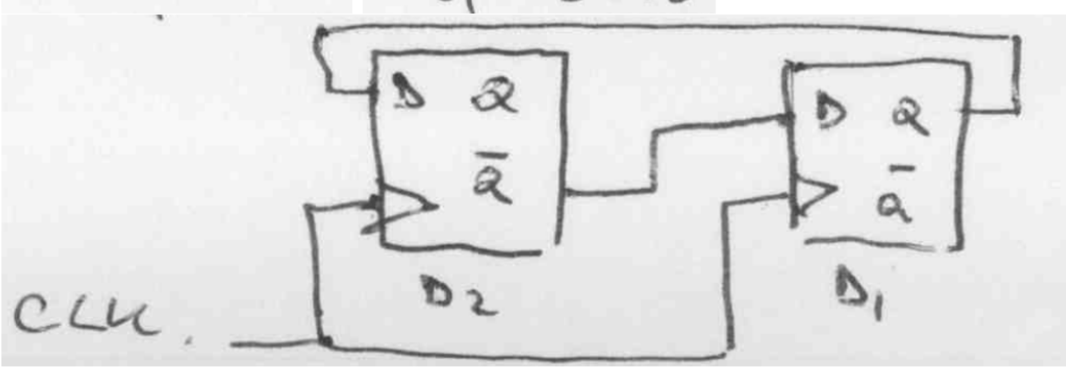
(b) This is a Grey code, in which only one bit changes at a time. It is used in sequential logic applications where dynamic hazards must be avoided

(c) To reverse the sequence



$Q_2^+ = Q_1$

$Q_1^+ = \overline{Q_2}$



8 (**short**)

(a) For the memory chip, 256 kilobyte = number of data lines x $2^{\text{address lines}}$

256 x 1024 x 8 bits = 16 x $2^{\text{address lines}}$

address lines = 17 **[4]**

(b) A microprocessor with 20 address lines can take eight of these memory chips. The most significant 3 lines will be used to address these individual chips. The least significant 17 lines will used for addressing within those individual chips.

The address ranges of these chips in hexadecimal format:

Chip #	A19	A18	A17	A16	A0-A15	Hex
1	0	0	0	*0*	0	00000
	0	0	0	*1*	1	1FFFF
2	0	0	1	*0*	0	20000
	0	0	1	*1*	1	3FFFF
3	0	1	0	*0*	0	40000
	0	1	0	*1*	1	5FFFF
4	0	1	1	*0*	0	60000
	0	1	1	*1*	1	7FFFF
5	1	0	0	*0*	0	80000
	1	0	0	*1*	1	9FFFF
6	1	0	1	*0*	0	A0000
	1	0	1	*1*	1	BFFFF
7	1	1	0	*0*	0	C0000
	1	1	0	*1*	1	DFFFF
8	1	1	1	*0*	0	E0000
	1	1	1	*1*	1	FFFFF

[6]

9 (long)

(a)

X		Y		P			
X_2	X_1	Y_2	Y_1	P_4	P_3	P_2	P_1
0	0	1	0	0	0	1	0
		1	1	0	0	1	1
0	1	1	0	0	0	1	1
		1	1	0	1	0	0
1	0	1	0	0	1	1	0
		1	1	0	1	1	1
1	1	1	0	1	0	1	1
		1	1	1	1	0	0

P_4

	X_1			
Y_1	0	1	0	0
	0	1	0	0
		X_2		

P_1

0	0	1	1
1	1	0	0

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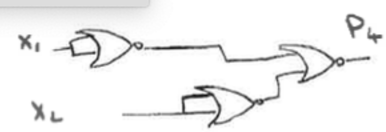
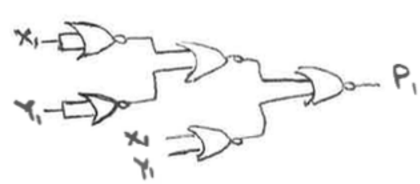
$\overline{P_4} = \bar{X}_1 + \bar{X}_2$

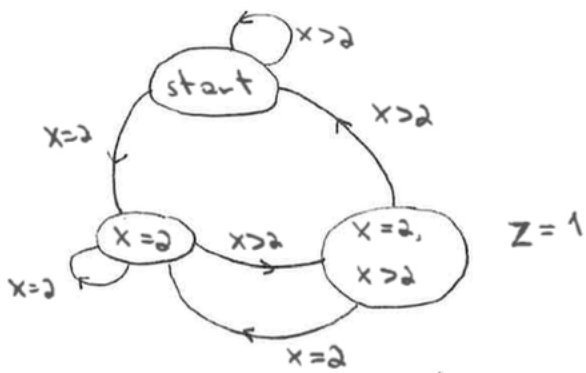


$\bar{P}_1 = X_1 Y_1 + \bar{Y}_1 \bar{X}_1 = \overline{\bar{X}_1 + \bar{Y}_1} + \overline{X_1 + Y_1}$



(b)

states	bistable allocation	
	Q_A	Q_B
start	0	0
$x = 2$	0	1
$x = 2, x > 2$	1	0



$x > 2$ $(x_1 = 1)$

$x = 2$ $(x_1 = 0)$

Current state Q_A	Current state Q_B	Input x_1	Next State Q_A	Next State Q_B	J_A	K_A	J_B	K_B
0	0	0	0	1	0	x	1	x
0	0	1	0	0	0	x	0	x
0	1	0	0	1	0	x	x	0
0	1	1	1	0	1	x	x	1
1	0	0	0	1	x	1	1	x
1	0	1	0	0	x	1	0	x

J_A (Q_A across top, x_1 rows, Q_B below)

x_1	x	x	1	0
	x	x	0	0

J_B (Q_A across top, x_1 rows, Q_B below)

x_1	0	x	x	0
	1	x	1	1

$J_A = x_1 Q_B$

$J_B = \bar{x}_1$

SECTION C

10 (short)

(a) Assume the radius of the sphere is R. By symmetry and Gauss's law, the electric field generated by uniformly distributed charges on sphere is:

$$\boldsymbol{E} = 0, \quad r < R$$

$$\boldsymbol{E} = \frac{Q}{4\pi\varepsilon_0 r^2}\hat{\mathbf{r}}, \quad r > R$$

(b) By definition, the capacitance is:

$$C = \frac{Q}{V}$$

where V is the potential of the sphere if the potential at infinity is set to be zero, which can be obtained by:

$$V = V(R) - V(+\infty) -= -\int_{+\infty}^{\boldsymbol{R}} \boldsymbol{E} \cdot d\boldsymbol{r} = -\int_{+\infty}^{\boldsymbol{R}} \frac{Q}{4\pi\varepsilon_0 r^2} dr = \frac{Q}{4\pi\varepsilon_0 r}\Big|_{+\infty}^{R} = \frac{Q}{4\pi\varepsilon_0 R}$$

Hence:

$$C = 4\pi\varepsilon_0 R$$

(c) From above, we have:

$$R = \frac{C}{4\pi\varepsilon_0} = \frac{1 \times 10^{-12}\ \mathrm{F}}{4\pi \times 8.854 \times 10^{-12}\ \mathrm{F\ m^{-1}}} = 8.988 \times 10^{-3}\ \mathrm{m} \approx 9\ \mathrm{mm}$$

11 (short)

(a) The surface charge densities for the inner and outer cylindrical surfaces are:

$$\sigma_1 = \frac{Q}{2\pi r_1 h}, \sigma_2 = \frac{-Q}{2\pi r_2 h}$$

The surface current density *i* formed by the rotation of each cylindrical surface is:

$$i_1 = \sigma_1 v = \sigma_1 \omega r_1 = \frac{Q}{2\pi r_1 h}\omega r_1 = \frac{Q\omega}{2\pi h}$$

$$i_2 = \sigma_2 v = \sigma_2 \omega r_2 = -\frac{Q}{2\pi r_2 h}\omega r_2 = -\frac{Q\omega}{2\pi h}$$

(b) The surface currents of the two cylindrical surfaces are equivalent to that of two solenoids, and the corresponding $nI/l=i$, Ignoring the fringe effect, the solenoid only generates a magnetic field inside, so that:

$$\boldsymbol{B} = \boldsymbol{B}_1 + \boldsymbol{B}_2 = (\mu_0 i_1 + \mu_0 i_2)\hat{\boldsymbol{z}} = \left(\frac{\mu_0 Q\omega}{2\pi h} - \frac{\mu_0 Q\omega}{2\pi h}\right)\hat{\boldsymbol{z}} = 0, \quad r < r_1$$

$$\boldsymbol{B} = \boldsymbol{B}_1 + \boldsymbol{B}_2 = 0 + \mu_0 i_1 \hat{\boldsymbol{z}} = -\frac{\mu_0 Q\omega}{2\pi h}\hat{\boldsymbol{z}}, \quad r_1 < r < r_2$$

$$\boldsymbol{B} = \boldsymbol{B}_1 + \boldsymbol{B}_2 = 0 + 0 = 0, \quad r > r_2$$

12 (long)

(a) The capacitance of the capacitor with the dielectric plate is:

$$C = \frac{\varepsilon_0 \varepsilon_r A}{\varepsilon_r (d - t) + t} = \frac{\varepsilon_0 \varepsilon_r A}{\varepsilon_r d - (\varepsilon_r - 1)t}$$

(b) After disconnecting the power supply, the amount of charge Q on the capacitor plate remains the same before and after the dielectric plate is pulled out.

The electrostatic energy stored in the capacitor is:

$$W = \frac{Q^2}{2C}$$

where $Q=CV$ is the amount of charge on the capacitor plate.

After the dielectric plate is pulled out, the capacitance and electrostatic energy stored become:

$$C_0 = \frac{\varepsilon_0 A}{d}, \quad W_0 = \frac{Q^2}{2C_0}$$

Therefore, the increase of the electrostatic energy is:

$$\Delta W = W_0 - W = \frac{Q^2}{2}\left(\frac{1}{C_0} - \frac{1}{C}\right) = \frac{Q^2}{2}\left[\frac{d}{\varepsilon_0 A} - \frac{\varepsilon_r d - (\varepsilon_r - 1)t}{\varepsilon_0 \varepsilon_r A}\right] = \frac{Q^2}{2}\frac{(\varepsilon_r - 1)t}{\varepsilon_0 \varepsilon_r A}$$
$$= \frac{1}{2}\left[\frac{\varepsilon_0 \varepsilon_r A V}{\varepsilon_r d - (\varepsilon_r - 1)t}\right]^2 \frac{(\varepsilon_r - 1)t}{\varepsilon_0 \varepsilon_r A} = \frac{\varepsilon_0 \varepsilon_r (\varepsilon_r - 1) A t V^2}{2[\varepsilon_r d - (\varepsilon_r - 1)t]^2}$$

$\Delta W > 0$ shows the increase of the stored electrostatic energy, hence the work done by the external force to pull out the dielectric plate is:

$$U = \Delta W = \frac{\varepsilon_0 \varepsilon_r (\varepsilon_r - 1) A t V^2}{2[\varepsilon_r d - (\varepsilon_r - 1)t]^2}$$

(c) If the power supply is not disconnected, the potential difference between the two capacitor plates remains the same *V*. After pulling out the dielectric plate, the amount of charge on the capacitor plate becomes:

$$Q_0 = C_0 V$$

Therefore, the amount of the increased charge is:

$$\Delta Q = Q_0 - Q = (C_0 - C)V = \left[\frac{\varepsilon_0 A}{d} - \frac{\varepsilon_0 \varepsilon_r A}{\varepsilon_r d - (\varepsilon_r - 1)t}\right] V = -\frac{\varepsilon_0 (\varepsilon_r - 1) A t V}{[\varepsilon_r d - (\varepsilon_r - 1)t]d}$$

$\Delta Q < 0$ shows that charges flow from the capacitor to power supply when the dielectric plate is pulled out, therefore there is corresponding energy ΔW_Q flows from the capacitor to the power supply:

$$\Delta W_Q = -\Delta Q\, V = \frac{\varepsilon_0 (\varepsilon_r - 1) A t V^2}{[\varepsilon_r d - (\varepsilon_r - 1)t]d}$$

At the same time, the change of the electrostatic energy stored in the capacitor is:

$$\Delta W_C = W_0 - W = \frac{1}{2}(C_0 - C)V^2 = \frac{1}{2}\left[\frac{\varepsilon_0 A}{d} - \frac{\varepsilon_0 \varepsilon_r A}{\varepsilon_r d - (\varepsilon_r - 1)t}\right] V^2$$
$$= -\frac{\varepsilon_0 (\varepsilon_r - 1) A t V^2}{2[\varepsilon_r d - (\varepsilon_r - 1)t]d}$$

$\Delta W_C < 0$ shows that the electrostatic energy stored in the capacitor reduces after the dielectric plate is pulled out.

Considering the capacitor and power supply as a whole, the work done by the external force, which equals to the total energy change, is:

$$U = \Delta W = \Delta W_C + \Delta W_Q = \frac{\varepsilon_0 (\varepsilon_r - 1) A t V^2}{2[\varepsilon_r d - (\varepsilon_r - 1)t]d}$$

$\Delta W > 0$ shows the total energy in the system increases, as the result of the work done by pulling out the dielectric plate.