Version CSG/4

EGT1
ENGINEERING TRIPOS PART IB

Tuesday 12 June 2018 2 to 4:10

Paper 1

MECHANICAL ENGINEERING

*Answer not more than **four** questions.*

*Answer not more than **two** questions from each section. All questions carry the same number of marks.*

*The **approximate** number of marks allocated to each part of a question is indicated in the right margin.*

Answers to questions in each section should be tied together and handed in separately.

*Write your candidate number **not** your name on the cover sheet.*

STATIONERY REQUIREMENTS
Single-sided script paper

SPECIAL REQUIREMENTS TO BE SUPPLIED FOR THIS EXAM
CUED approved calculator allowed
Engineering Data Book

10 minutes reading time is allowed for this paper at the start of the exam.

You may not start to read the questions printed on the subsequent pages of this question paper until instructed to do so.

SECTION A

*Answer not more than **two** questions from this section.*

1 A rubber ball, with radius a and mass m, is traveling with horizontal velocity u_1, vertical velocity v_1 and rotating with angular velocity ω_1, as shown in Fig. 1. The ball impacts a horizontal surface and bounces in a completely elastic collision, after which its horizontal velocity is u_2, its vertical velocity is v_2, and its angular velocity is ω_2.

(a) Show that

$$u_1 + \frac{2}{5}a\omega_1 = u_2 + \frac{2}{5}a\omega_2$$

[8]

(b) Derive a relationship between u_1, u_2 and ω_1. [9]

(c) Derive an expression for ω_1 in terms of u_1 for the case where the ball bounces straight up after the impact. [8]

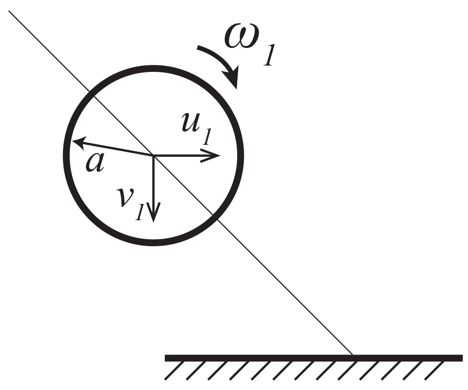

Fig. 1

2 A weight is supported at B and C by two light rods AB and CD, as shown in Fig. 2. The rods are free to rotate at all joints. The midpoint of BC is at E and the center of mass of the weight, G, is at distance d below E.

(a) At the moment depicted in the figure, BC is horizontal, and the left supporting rod AB is rotating with constant angular velocity ω around its anchor point A. Determine the angular velocity of the right support rod CD around its anchor point D. [6]

(b) Show that the acceleration of E is $\sqrt{13}a\omega^2$. [6]

(c) Calculate the acceleration of G for the case $d = a$. [8]

(d) By considering the vertical component of the acceleration of G, determine the condition relating d to a in order for the support to be stable with respect to small displacements. [5]

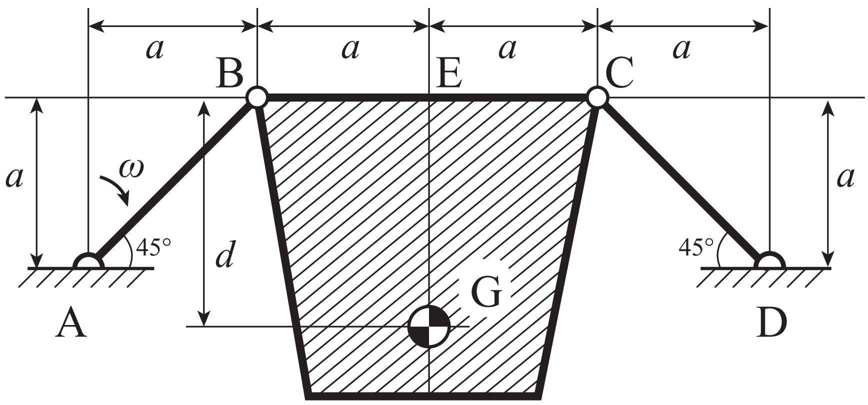

Fig. 2

3 A ladder of mass m and length $2a$ is standing on a smooth floor, held vertically against a smooth wall, when its bottom is slightly displaced. Figure 3 shows the ladder some time later when its angle to the vertical is θ.

(a) Determine the relationship between the angle of the ladder, θ, and its angular acceleration, $\ddot{\theta}$. [7]

(b) Explain why the horizontal acceleration of the centre of mass, G, is zero at the instant at which the ladder loses contact with the wall, and find the angle θ at this instant. [7]

(c) Show that the vertical acceleration of P, at a distance z along the ladder from B, is

$$\frac{3z}{4a}g$$

[6]

(d) At the moment of losing contact, determine the distance from B along the ladder at which the ladder experiences the greatest bending moment. [5]

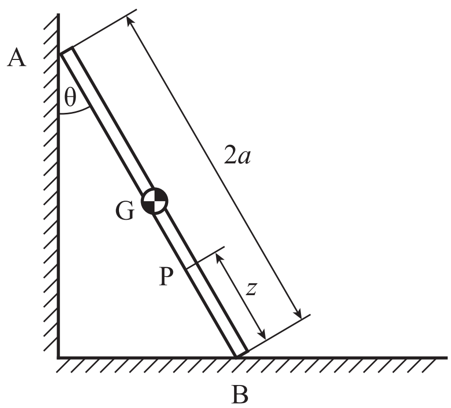

Fig. 3

SECTION B

*Answer not more than **two** questions from this section.*

4 A hoop of mass m and radius a has a mass m fixed to its inner face. The hoop is initially at rest on a horizontal table with its centre vertically above the point of contact with the table. The radial line passing through the centre of the hoop and the mass subtends an angle θ to the vertical, as shown in Fig. 4. The hoop begins to move with the mass at its highest point, i.e. when $\theta = 0$. You may assume that the hoop rolls without slip.

(a) Show that

$$\dot{\theta}^2 = \frac{g(1-\cos\theta)}{a(2+\cos\theta)}$$

[12]

(b) Show that $\ddot{\theta} = \dfrac{3g}{8a}$ when $\theta = \dfrac{\pi}{2}$. [8]

(c) Find the coefficient of friction necessary for no slip when $\theta = \dfrac{\pi}{2}$. [5]

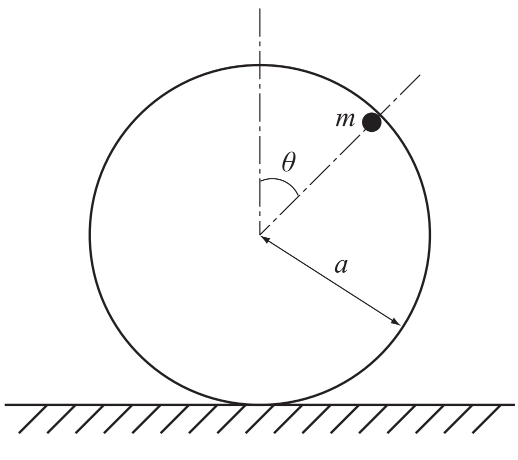

Fig. 4

5 A small, lightweight, single-seat aircraft is fitted with a front-mounted, large rotary engine. The propeller is rigidly fixed to the front face of the engine and the whole assembly is free to rotate on a cantilevered shaft mounted in an appropriate bearing. The assembly is sketched in Fig. 5 and rotates at 1,250 rpm.

(a) The propeller and engine are out of balance by 0.05 kg m and 0.01 kg m respectively.

(i) Write down the angular offset between the propeller and engine that will minimise the static out-of-balance. [3]

(ii) For this condition, calculate the dynamic moment on the bearing when the aircraft is flying straight and level. [7]

(b) The engine can be modelled as a uniform disc of mass 180 kg, diameter 1 m and thickness 200 mm. The propeller can be modelled as a uniform disc of mass 15 kg, diameter 2.6 m and thickness 200 mm. The effective centre of the bearing is 100 mm behind the rear engine face. Assuming that the aircraft axis is horizontal:

(i) calculate the reaction torque in the bearing when the aircraft rolls (i.e. rotates around the rotational axis of the engine) at a rate of $20^\circ\ s^{-1}$; [3]

(ii) calculate the reaction torque in the bearing when the aircraft yaws (i.e. rotates around a vertical axis) at a rate of $20^\circ\ s^{-1}$. [7]

(c) Briefly explain why the yawing manoeuvre is more likely to result in a loss of control of the aircraft. [5]

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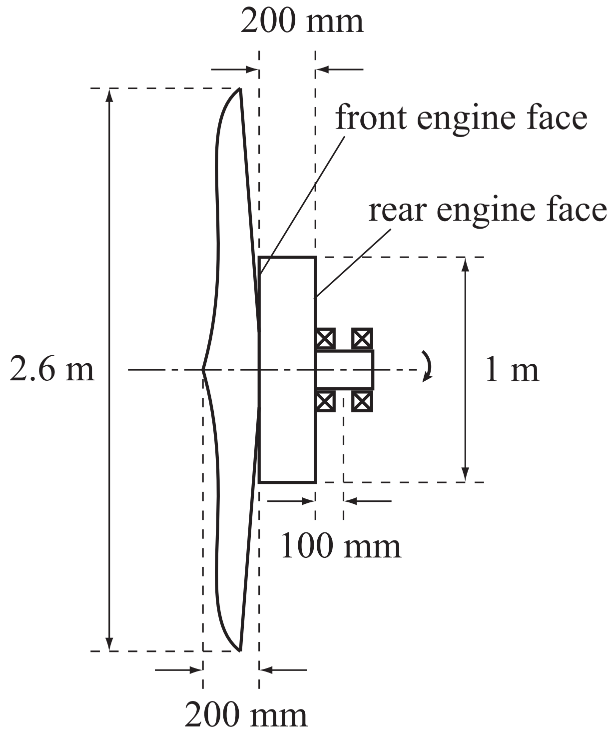

Fig. 5

(TURN OVER

6 Two steel bars AB and BC are welded together at B to form a rigid right-angle joint and AB is freely hinged to a wall at A. Both bars, AB and BC, have mass m and length $2a$. The right angle ABC is released from rest in the position shown in Fig. 6, with bar BC vertical.

(a) Show that the initial angular acceleration of bar AB is $\dfrac{9g}{20a}$. [8]

(b) Find the bending moment and shear force in bar BC at B. [10]

(c) Sketch the shear-force diagram for bar BC. [7]

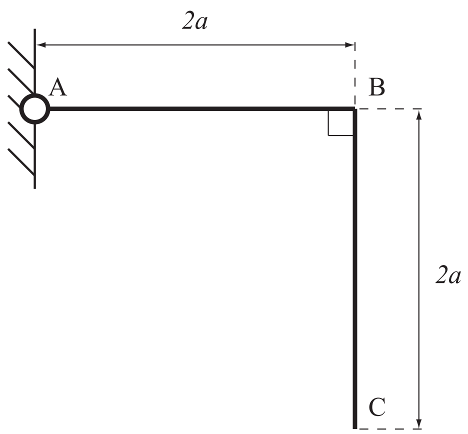

Fig. 6

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