Version TB/5

EGT1
ENGINEERING TRIPOS PART IB

Monday 9 June 2025 9.00 to 11.10

Paper 1

MECHANICAL ENGINEERING

Answer not more than ***four*** *questions.*

Answer not more than ***two*** *questions from each section. All questions carry the same number of marks.*

The ***approximate*** *number of marks allocated to each part of a question is indicated in the right margin.*

Answers to questions in each section should be tied together and handed in separately. Write your candidate number ***not*** *your name on the cover sheet.*

STATIONERY REQUIREMENTS
Single-sided script paper

SPECIAL REQUIREMENTS TO BE SUPPLIED FOR THIS EXAM
CUED approved calculator allowed
Engineering Data Book

10 minutes reading time is allowed for this paper at the start of the exam.

You may not start to read the questions printed on the subsequent pages of this question paper until instructed to do so.

You may not remove any stationery from the Examination Room.

SECTION A

1 Figure 1 shows in plan view a piston D connected to a crank AB of length a by a connecting rod BCD of length $2\sqrt{3}a$. Pinned joints at B and D are frictionless. The crank rotates at constant angular velocity ω in the clockwise direction. The centre of rotation of the crank is offset from the centreline of the piston. The piston has mass m_P and a friction force F opposes sliding motion of the piston in its cylinder. The connecting rod has mass m_C centred at C, which is halfway along BD, and has rotational inertia I about C. At the instant shown the angle ABD is 90° and the connecting rod makes an angle of 30° to the centreline of the piston. Orthogonal unit vectors **i** and **j** may be used to specify direction. For the instant shown:

(a) find the velocity of the piston, and the velocity and angular velocity of the connecting rod at C. The suggested scale of a velocity diagram is $\omega a = 100$ mm; [6]

(b) find the acceleration of the piston, and the acceleration and angular acceleration of the connecting rod at C. The suggested scale of an acceleration diagram is $\omega^2 a = 100$ mm; [9]

(c) find the torque at the crank necessary to maintain crank rotation at constant angular velocity ω. [10]

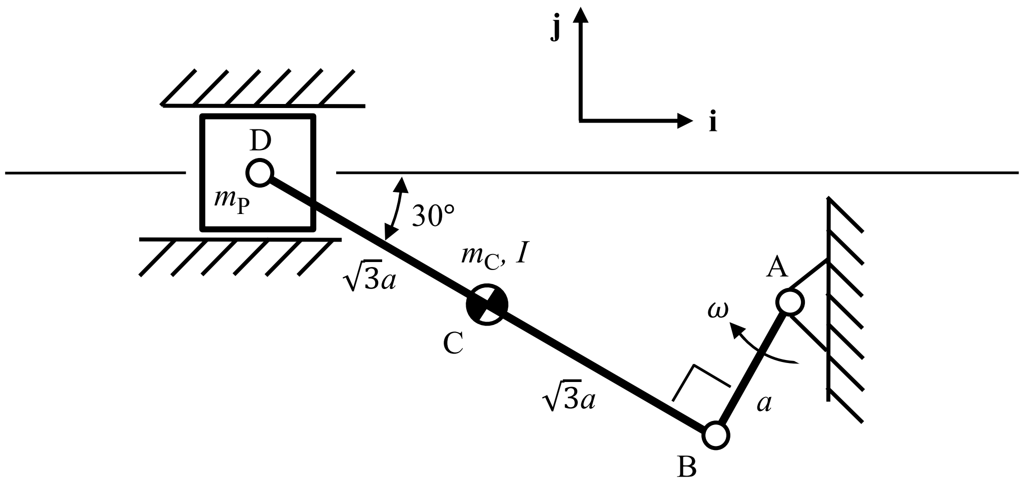

Fig. 1

2 Figure 2 shows a rigid beam AB of length L and mass m which is uniformly distributed. The beam is supported at an angle of 30° to the horizontal by two long, vertical, light and inextensible cables, one at each end of the beam. The beam is stationary when suddenly the cable that supports the beam at A breaks. For the instant immediately after the cable breaks:

(a) derive expressions for the instantaneous:

(i) angular acceleration of the beam; [4]

(ii) acceleration at the centre of the beam; [4]

(iii) force in the cable that supports the beam at B; [4]

(b) derive an expression for the instantaneous shear force in the beam in terms of distance along the beam and hence find the position of the maximum bending moment. [13]

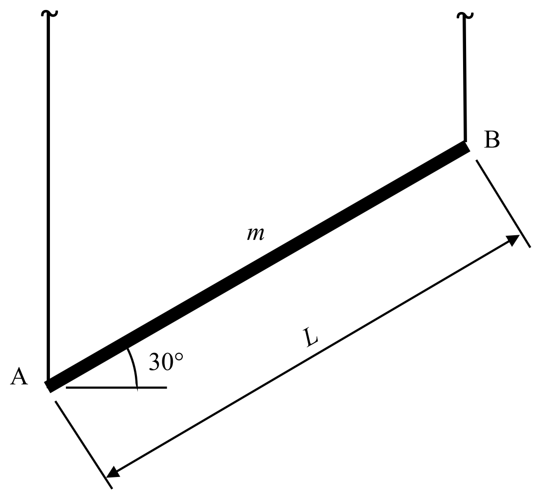

Fig. 2

3 Figure 3 shows a thin rigid ring of mass m, radius a and centre G. The ring is placed on a fixed cylinder of radius b and centre O. The axes of the ring and the cylinder are horizontal. When the ring is in static equilibrium, point A′ on the surface of the cylinder and point A″ on the ring are coincident. When the ring is displaced by angle ϕ the point of contact on the cylinder moves from A′ to B defined by angle θ. There is no slip between the ring and the cylinder.

(a) Find an expression for the potential energy of the ring as a function of θ. [6]

(b) Find an expression for the kinetic energy of the ring as a function of $\dot{\theta}$. [12]

(c) The ring is now released from a displaced position. Find an expression for the angular frequency of small amplitude oscillations. [7]

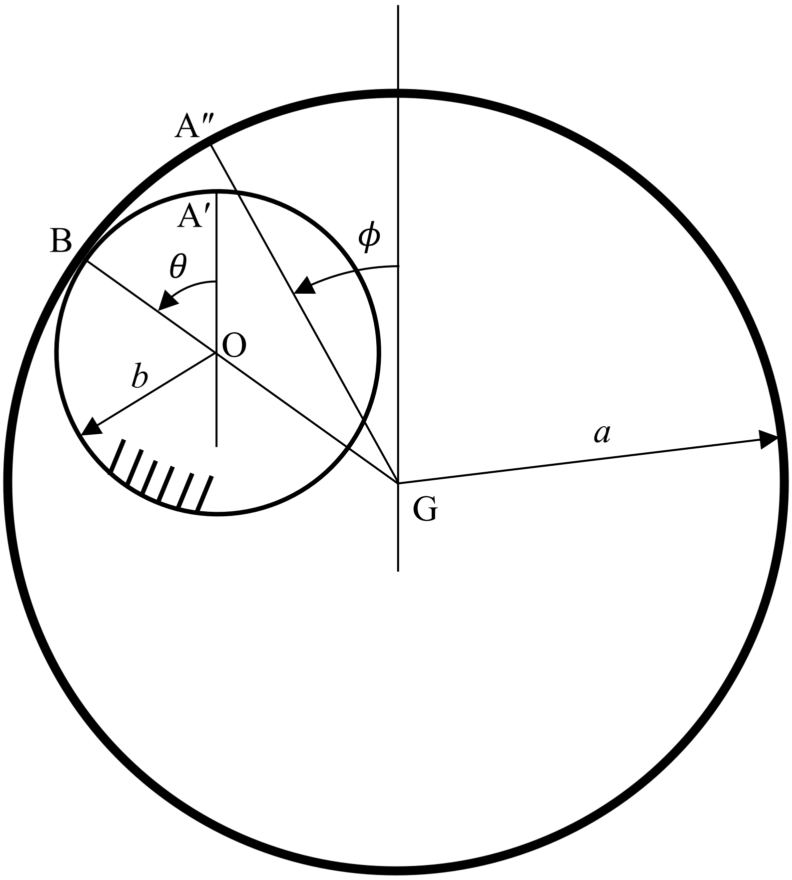

Fig. 3

SECTION B

4 A bead of mass m can slide without friction around a thin circular hoop of mass M and radius a. The hoop is free to spin around a vertical diameter, as illustrated in Fig. 4. The angular displacement of the bead from the bottom of the hoop is θ, and the angular displacement of the hoop about the vertical axis is ϕ. The effect of gravity should be included in this question, and the mass moment of inertia of the hoop about a vertical axis is $I_G = Ma^2/2$.

(a) Show that the Lagrangian L for this system can be written in the form:

$$L = \alpha\dot{\theta}^2 + [f(\theta) + \beta]\,\dot{\phi}^2 + \gamma\cos\theta$$

and find the constants α, β, γ and the function $f(\theta)$. [4]

(b) Derive the two coupled equations of motion corresponding to the generalised coordinates θ, ϕ. [6]

(c) Show that one of the generalised forces is zero, and use that to identify a quantity C that is conserved for this system. [4]

(d) Find an effective potential in the form:

$$V_{\text{eff}} = -B\cos\theta - \frac{C^2}{D\sin^2\theta + E}$$

and find the constants B, D, E, noting that C is the same conserved quantity found in Part (c). [7]

(e) Without further calculation, comment on whether or not the bead is constrained to remain within one half of the hoop, and justify your answer. [4]

 (cont.

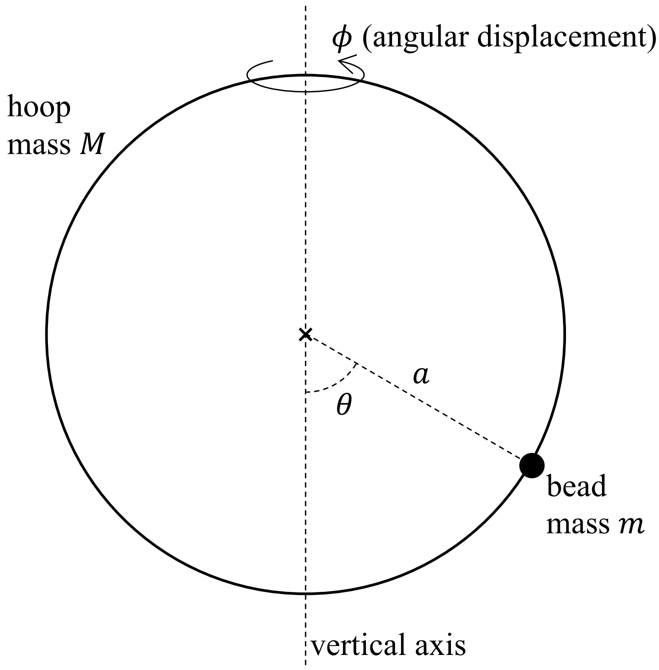
ϕ (angular displacement)
hoop
mass M
a
θ
bead
mass m
vertical axis

Fig. 4

5 A uniform cube of mass m and edge length $\sqrt{2}R$ pivots from edge to edge along a surface without slipping or losing contact. A side view is illustrated in Fig. 5. Assume that the motion is planar, that the cube rotates about an edge at a given time, and that no energy is lost each time the edge in contact with the ground changes. The angular displacement of the cube is denoted θ, with $\theta = 0$ defined when the centre of mass is vertically above the first edge in contact with the ground. The mass moment of inertia about an axis through the centre of the cube is $I_G = mR^2/3$.

(a) During rotation about an edge:

(i) show that the kinetic energy T and potential energy V can be written in the form:

$$T = \alpha\dot{\theta}^2 \ , \ V = \beta \cos\theta$$

and find the constants α and β; [2]

(ii) find an expression for the Lagrangian of the cube and derive the equation of motion for θ; [4]

(iii) identify the range of θ for which these equations can be applied without the cube switching edges. [2]

(b) Carefully consider the motion of the cube over the range $0 \leq \theta \leq \pi$, periodically repeating your results from Part (a) as needed to include the cube switching edges as it rotates. Detailed consideration of the instant when two edges are in contact is not needed.

(i) Sketch the potential energy V as a function of θ. [4]

(ii) Find the equilibrium states of the cube and identify their stability. Comment on why the stable equilibria cannot be found using the derivative of the potential energy $V'(\theta)$ in this case. [4]

(iii) Sketch the phase portrait $(\theta, \dot{\theta})$ for the cube, labelling significant features. [4]

(c) For convenience, the angle ϕ is defined to be the angle of a cube face from horizontal, as shown in Fig. 5. If the cube is released from rest with an initial angle $\phi = \phi_0$, then the subsequent angular velocity can be taken to be given by:

$$\dot{\phi} = -D\sqrt{\sin\phi_0 + \cos\phi_0 - \sin\phi - \cos\phi}$$

where D is a constant. Assuming ϕ_0 is small, integrate this result from ϕ_0 to 0, and find the period of oscillation T_0 (the time taken to return to the original position) in terms of D and ϕ_0. Use small angle approximations to first-order accuracy. [5]

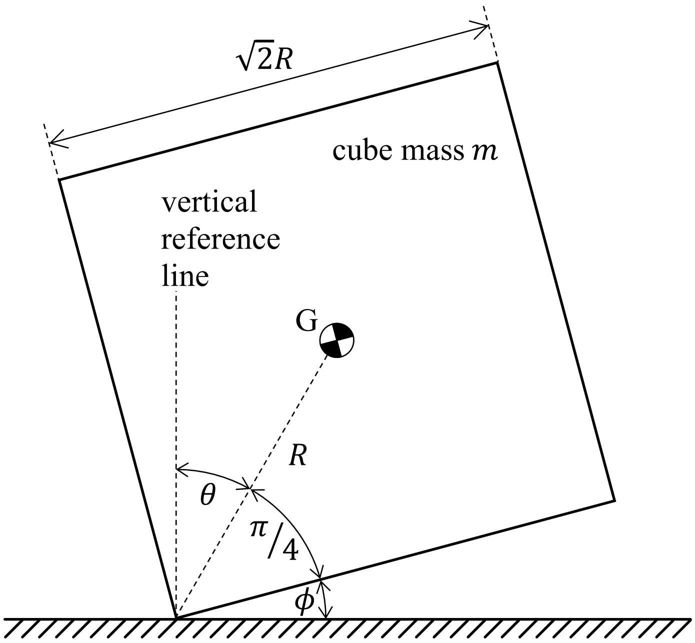

Fig. 5

6 Two identical magnets of mass m are connected to fixed walls by linear springs of stiffness k and are constrained to move along a frictionless horizontal surface, as illustrated in Fig. 6. The walls are separated by a distance $2a$, and the natural lengths of the springs are both a. Let x_1 and x_2 denote the absolute horizontal displacements of the two masses from the left wall, with $x_2 \geq x_1$. The North (N) and South (S) poles of the magnets are oriented as shown, and the potential energy associated with the magnets is modelled as:

$$V_{\text{mag}} = \frac{C}{x_2 - x_1}$$

where C is a positive constant representing the strength of the magnetic interaction. It can be assumed that the magnets are point masses with negligible size.

(a) Show that the Lagrangian L of the system can be written:

$$L = A\left(\dot{x}_1^2 + \dot{x}_2^2\right) - B\,(a - x_1)^2 - B\,(x_2 - a)^2 - \frac{C}{x_2 - x_1}$$

and find the constants A and B. [4]

(b) Find the equilibrium positions of the system in terms of a, C, k. [8]

(c) The spring stiffness is chosen such that the equilibrium positions are

$$x_{1,\text{eq}} = \frac{a}{2} \text{ and } x_{2,\text{eq}} = \frac{3a}{2}$$

Derive the linearised equations of motion about this equilibrium position. Write your answer in matrix form, in terms of a, C, k, m. You do not need to find the relationship between C and k for this equilibrium. [8]

(d) Consider the case where the linearised equation of motion has mass and stiffness matrices given by:

$$\mathbf{M} = m\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}, \quad \mathbf{K} = k\begin{bmatrix} 2 & -1 \\ -1 & 2 \end{bmatrix}.$$

Use the trial mode shape $\mathbf{u} = [1 \quad \alpha]^T$, or otherwise, to find the natural frequencies and mode shapes of the system for small oscillations around the equilibrium positions. [5]

(cont.

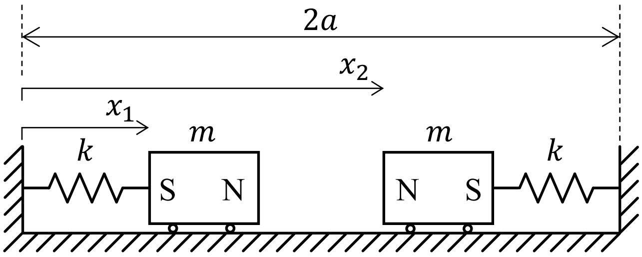

Fig. 6

END OF PAPER

THIS PAGE IS BLANK