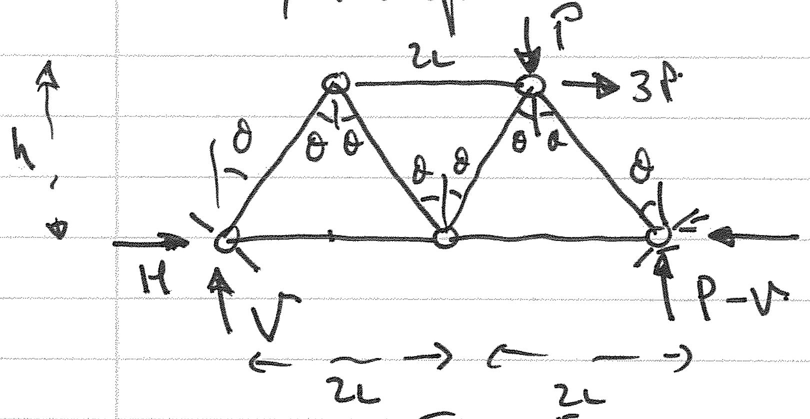
Q1 Bars III, IX ineffective: $t_{II} = t_{IV}$ and $t_{VIII} = t_{X}$

Thus, analyse

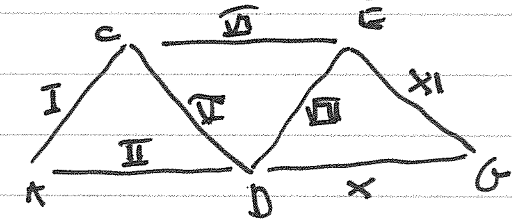


$h = 4\,m$: $L = 3\,m$

$\Rightarrow \tan\theta = 3/4$

$\Rightarrow \sin\theta = 3/5$: $\cos\theta = 4/5$

$P = 10\,kN$: retain as P.

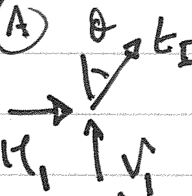


labelling scheme.

set t_{II} = redundant tension.

∴ remove for general solution. (P, $3P$ present)

(A)

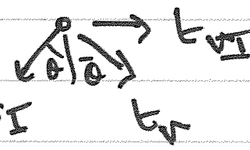


reactions → H_1, V_1 when t_{II} removed

$t_I \cos\theta = V$ $\Rightarrow$ $t_I = -V_1/\cos\theta$

$H_1 + t_I \sin\theta = 0$ $\quad H_1 = -t_I \sin\theta = V_1 \tan\theta$

(C)

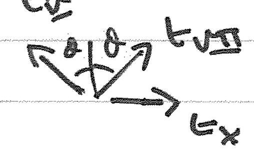


$R\uparrow \Rightarrow t_V = -t_I = V_1/\cos\theta$

$R\rightarrow$: $t_{VI} + (t_V - t_I)\sin\theta = 0$

$t_{VI} = 2t_I\sin\theta = -2V_1\tan\theta$

(D)

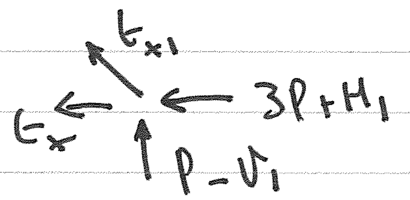


$R\uparrow \Rightarrow t_{VII} = -t_V = -V_1/\cos\theta$

$R\rightarrow$ $t_X + (t_{VII} - t_V)\sin\theta = 0$

$t_X = 2t_V\sin\theta = 2V_1\tan\theta$

(E)



$t_{XI}\cos\theta = V_1 - P \Rightarrow t_{XI} = \dfrac{V_1 - P}{\cos\theta}$

M_G ↺ for whole truss: $-PL + 3Ph + 4LV_1 = 0$

$$\Rightarrow \quad V_1 = \left[P - 3Ph/L\right]\cdot 1/4 = P/4\left[1 - 3/\tan\theta\right]$$

But $\tan\theta = 3/4 \Rightarrow V_1 = P/4 \cdot \left[1 - \cancel{3}\cdot 4/\cancel{3}\right] = \underline{\underline{-3P/4}}$

$$t_I = -V_1/\cos\theta = \frac{3P}{4}\,\frac{5}{4} = \frac{15P}{16} : \quad t_V = -t_I = -15P/16$$

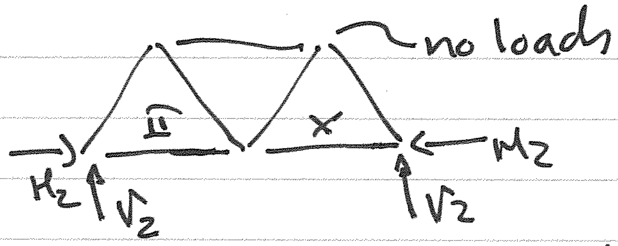
$$t_{VI} = -2V_1\tan\theta = \frac{6P}{4}\cdot\frac{3}{4} = \frac{18P}{16} ; \quad t_{VII} = -t_V = 15P/16$$

$$t_X = 2V_1\tan\theta = -18P/16 ; \quad t_{XI} = \frac{V_1 - P}{\cos\theta} = \frac{5}{4}\left[-\frac{3P}{4} - P\right] = \frac{-35P}{16}$$

$$\underset{\sim}{t_0} = \left[\, t_I \;\; t_{II} \;\; t_V \;\; t_{VI} \;\; t_{VII} \;\; t_X \;\; t_{XI} \,\right]^T$$

$$= P/16\left[\,+15 \;\; 0 \;\; -15 \;\; 18 \;\; 15 \;\; -18 \;\; -35\,\right]^T$$

Self-stress



set $t_{II} = +1 \Rightarrow$ eqlm at D gives $t_X = +1$: all other bar tensions zero and $\underline{\underline{V_2 = 0;\ H_2 = -1}}$

$$\underset{\sim}{s} = \left[\, t_I \ldots t_{XI} \,\right]^T = \underline{\left[\, 0 \;\; 1 \;\; 0 \;\; 0 \;\; 0 \;\; 1 \;\; 0 \,\right]^T}$$

Need $\underset{\sim}{e} = \underset{\sim}{F}\,\underset{\sim}{t} = \underset{\sim}{F}\left[\underset{\sim}{t_0} + x\,\underset{\sim}{s}\right]$

extns (vector)

flexibility matrix

unknown factor for self-stress.

P.T.O.

$$\underset{\sim}{F} = \operatorname{diag}\left(L/EA\right) = \frac{1}{EA}\begin{bmatrix} \cancel{5} \\ \cancel{6} \\ \cancel{5} \\ \cancel{6} \\ \cancel{5} \\ \cancel{6} \\ \cancel{5} \end{bmatrix} \underbrace{\begin{bmatrix} 5 &&&&&& \\ & 6 &&&&& \\ && 5 &&&& \\ &&& 6 &&& \\ &&&& 5 && \\ &&&&& 6 & \\ &&&&&& 5 \end{bmatrix}}_{\text{members}}$$

L: L_I, L_{II} etc

$$\Rightarrow \underset{\sim}{e} = \frac{1}{EA}\begin{bmatrix} 5 &&&&&& \\ & 6 &&&&& \\ && 5 &&&& \\ &&& 6 &&& \\ &&&& 5 && \\ &&&&& 6 & \\ &&&&&& 5 \end{bmatrix}\left[\frac{P}{16}\begin{bmatrix} +15 \\ 0 \\ -15 \\ 18 \\ 15 \\ -18 \\ -35 \end{bmatrix} + x\begin{bmatrix} 0 \\ 1 \\ 0 \\ 0 \\ 0 \\ 1 \\ 0 \end{bmatrix}\right]$$

$$= \frac{1}{EA}\left[\frac{P}{16}\begin{bmatrix} +75 \\ 0 \\ -75 \\ 108 \\ 75 \\ -108 \\ -175 \end{bmatrix} + x\begin{bmatrix} 0 \\ 6 \\ 0 \\ 0 \\ 0 \\ 6 \\ 0 \end{bmatrix}\right]$$

To find x, perform $\underbrace{\underset{\sim}{S} \cdot \underset{\sim}{e}}_{\underset{\sim}{S}^T \underset{\sim}{e}} \Rightarrow \underbrace{[0100010] \times}_{=0}$

$$= P/16\left[0 \times 1 + -108 \times 1\right] + x\left[6 + 6\right] = 0$$

$$= \cancel{108} \; -\cancel{108}^{\,9}P/16 + \cancel{12}x = 0 \Rightarrow x = \frac{9P}{16}$$

$$\underset{\sim}{t} = \frac{P}{16}\begin{bmatrix} 15 \\ 0 \\ -15 \\ 18 \\ 15 \\ -18 \\ -35 \end{bmatrix} + \frac{9P}{16}\begin{bmatrix} 0 \\ 1 \\ 0 \\ 0 \\ 0 \\ 1 \\ 0 \end{bmatrix} = P/16\begin{bmatrix} 15 \\ 9 \\ -15 \\ 18 \\ 15 \\ -9 \\ -35 \end{bmatrix}$$

P.T.O.

must multiply by self-stress compatibility factor

$$H = H_1 + H_2 = V_1 \tan\theta - x . 1$$

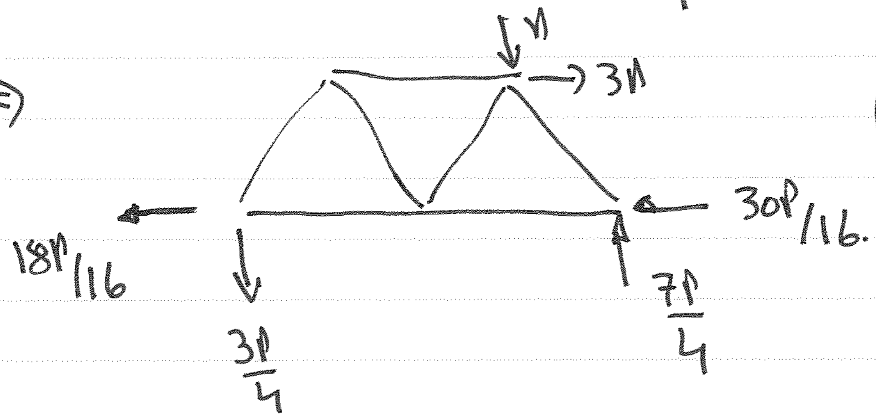
$$= -\frac{3P}{4} \cdot \frac{3}{4} - {}^{9P}/_{16} = \underline{\underline{-{}^{18P}/_{16}}} \text{ (left-side).}$$

(right-side) $3P + H_1 + H_2 = 3P + V_1 \tan\theta - x.$

$$= 3P - \frac{18P}{16} = \frac{30P}{16}.$$

Vertically : V_1 (left-side) $= -\frac{3P}{4}$: rightside $P - V_1 = \frac{7P}{4}$

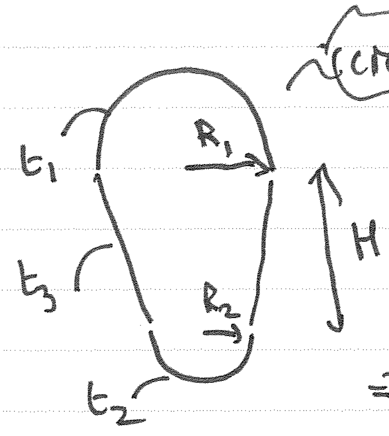
$\Rightarrow$



(all +ve directn.)

Q2. a)

cross-secn



$t_1 = 3\text{mm}$
$t_2 = 2\text{mm}$
$t_3 = 1\text{mm}$

$$A_e = \frac{\pi}{2}R_1^2 + \frac{\pi}{2}R_2^2 + \frac{1}{2}\left[2R_1 + 2R_2\right]H$$

$R_1 = 100\text{mm} : R_2 = 50\text{ mm} : H = 200\text{mm}$

$$\Rightarrow A_e = \frac{\pi}{2}100^2 + \frac{\pi}{2}50^2 + \frac{1}{2}\left[200+100\right]\cdot 200$$
$$= \underline{\underline{49635\text{ mm}^2}}$$

Torque applied $= 2 \times 10\text{kN} \times 1\text{m} = \underline{\underline{20\text{kNm}}}$

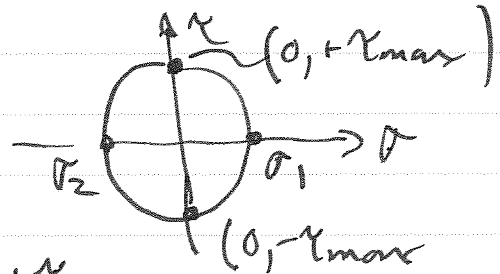
shear flow : $\underbrace{q = T/2A_e}_{\text{constant around cross-secn.}}$: ~~max~~ shear stress $\tau = q/t$

$\Rightarrow \tau_{max}$ when thickness smallest; i.e. $t_3 = 1\text{mm}$

$$\Rightarrow \tau_{max} = T/2A_e t = \frac{20 \times 10^3}{2 \times (49635 \times \underbrace{(1 \times 10^{-6})}_{\text{mm}^2 \to \text{m}^2}) \times \overbrace{0.001}^{\text{m}}}$$

$$\Rightarrow \underline{\underline{\tau_{max} = 201.5 \times 10^6 \text{ Pa.}}}$$

b) Mohr's Circle $\Rightarrow$



Principal stresses, $\sigma_1 = +\tau_{max}$
$\sigma_2 = -\tau_{max}$
$\sigma_3 = 0$ (thinwalled theory)

$\Rightarrow$ Von-Mises: $\sigma_1 - \sigma_2$ is largest differential: furthermore, multiply current stress state by λ to arrive at yield, ie.

$$\lambda\left[\sigma_1 - \sigma_2\right] = Y \Rightarrow \lambda = Y/2\tau_{max}$$

Y = 500×10^6 Pa.

$$\Rightarrow \lambda = \frac{500}{2 \times 201.5} = \underline{\underline{1.24}}$$

Q2c) $J = \frac{4A_e^2}{\oint \frac{ds}{t}}$ for torsional rigidity, GJ.

$J = 4 \times [49635 \times 10^{-6}]^2 / \oint \frac{ds}{t}$ $= \Sigma \, s/t$ for discrete sections

$\Sigma \frac{s}{t} \Rightarrow \pi R_1/t_1 + \pi R_2/t_2 + \underbrace{\sqrt{H^2 + (R_2 - R_1)^2} \times 2}_{\text{straight sides.}} / t_3$

$= \pi \frac{100}{3} + \pi \frac{50}{2} + 2 \times 206.2/1 =$

$\Rightarrow J = \frac{4 \times [49635 \times 10^{-6}]^2}{595.58} = 1.6546 \times 10^{-5}\ m^4.$

$GJ = \underbrace{81 \times 10^{9}}_{\text{steel}} \times 1.6546 \times 10^{-5} = \underline{1.3402 \times 10^6\ Nm^2}$

$\phi = T/GJ =$ rotation/length $= \frac{20 \times 10^3}{1.3402 \times 10^6} = 0.0149$ rad/m

Length of mast $= 5\,m \Rightarrow$ rotation $= 5 \times 0.0149 = 0.075$ rad
$= \underline{4.275^\circ}$

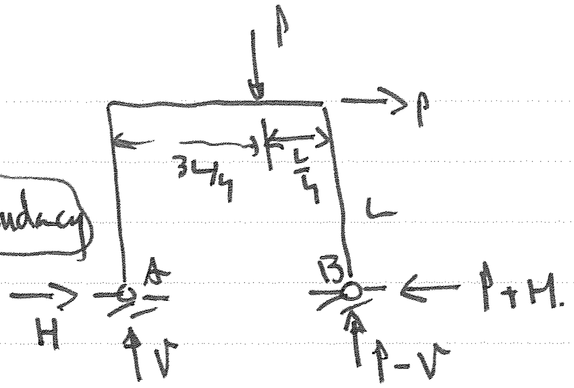
d). At yield, stresses have increased by 1.24 linearly $\Rightarrow$ new rotation at yield is

$1.24 \times 4.275 = \underline{5.30^\circ}$, which exceeds 5°

c) Increase $J \Rightarrow A_e \uparrow,\ t \uparrow,\ s \downarrow$

Q3.

(1 redundancy)

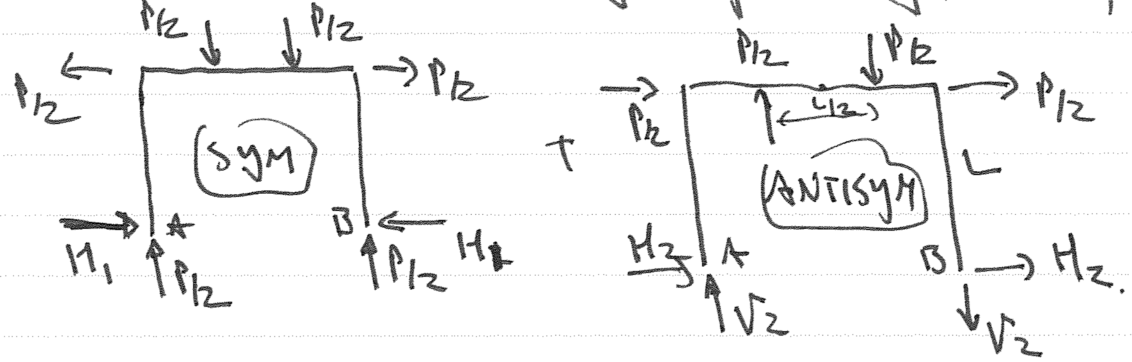


$M_B \circlearrowright \Rightarrow \quad -\frac{PL}{4} + PL + LV = 0$

$V = -P + P/4 = \underline{\underline{-3P/4}}$ (~~[illegible]~~ left)

$P - V = +7P/4$ (right)

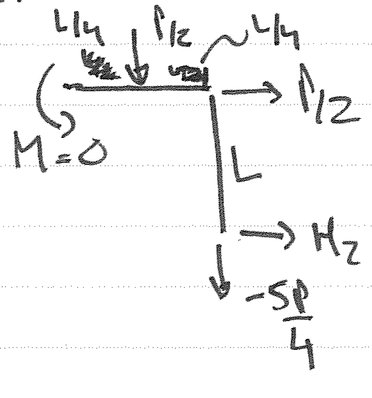
Horizontal reactions: tackled by symmetry | anti-symmetry.



Antisym first: moments about B: $\underbrace{-\frac{P}{2}\cdot\frac{L}{4} + \frac{P}{2}\cdot\frac{3L}{4}}_{\text{top}} + 2\frac{P}{2}\cdot L + V_2 L = 0.$

$\Rightarrow \frac{P}{2}\cdot\frac{L}{2} + PL + V_2 L = 0 \qquad \underline{\underline{V_2}} = -P \cancel{+} - P/4 = \underline{\underline{-5P/4}}$

Centre (top) B.M = 0 $\Rightarrow$ can find H_2 by eqlm alone.



$\cancel{\Rightarrow -P/2 \, L/2 + H_2 L + \frac{5P}{4} L = 0}$

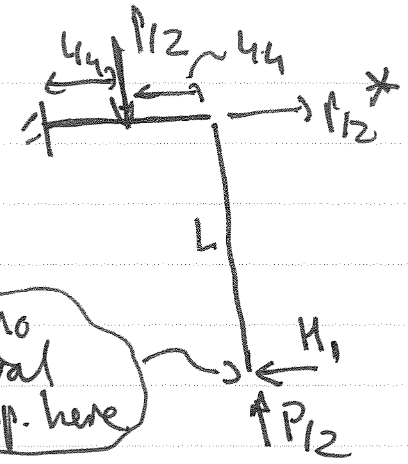
$\cancel{\underline{\underline{H_2}} = -\frac{5P}{4} + P/4 = \underline{\underline{-P}}}$

$-P/2\cdot\frac{L}{4} + \frac{5P}{4}\cdot\frac{L}{2} + H_2 L = 0$

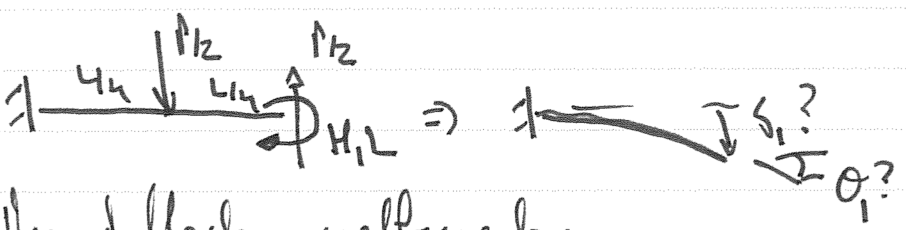
$\Rightarrow H_2 = -\frac{5P}{8} + \frac{P}{8} = -4P/8 = \underline{\underline{-P/2}}$

P.T.O.

Q3. Symmetry case: consider half of frame



$P/2^*$ does not affect displmt at foot.



Use deflectn coefficients:

$$\delta_1 = \underbrace{\frac{(P/2)(L/4)^3}{3EI} + \frac{L}{4}\cdot\frac{(P/2)(L/4)^2}{2EI}}_{P/2 \,@\, L/4} - \underbrace{\frac{P/2\,(L/2)^3}{3EI}}_{P/2\,@\,L/2} + \frac{(H_1L)(L/2)^2}{2EI}$$

$$= \frac{PL^3}{EI}\left[\frac{1}{3}\cdot\frac{1}{2}\cdot\frac{1}{64} + \frac{1}{4}\cdot\frac{1}{2}\cdot\frac{1}{2}\cdot\frac{1}{16} - \frac{1}{2}\cdot\frac{1}{3}\cdot\frac{1}{8}\right] + \frac{H_1L^3}{8EI}$$

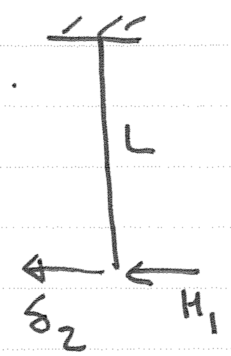
$$= \underline{\underline{\frac{-11PL^3}{768EI} + \frac{H_1L^3}{8EI}}}.$$ [not actually needed]

$$\theta_1 = \frac{(P/2)(L/4)^2}{2EI} - \frac{P/2\,(L/2)^2}{2EI} + \frac{(H_1L)(L/2)}{EI}$$

$$= \frac{PL^2}{EI}\left[\frac{1}{2}\cdot\frac{1}{2}\cdot\frac{1}{16} - \frac{1}{2}\cdot\frac{1}{2}\cdot\frac{1}{4}\right] + H_1L^2/2EI.$$

$$= \underline{\underline{\frac{-9PL^2}{192EI} + H_1L^2/2EI}}$$

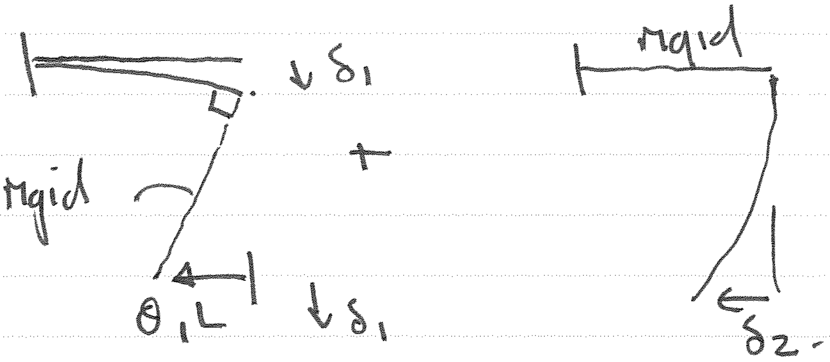
For column.



$$\delta_2 = H_1L^3/3EI.$$

P.T.O.

Total defln.

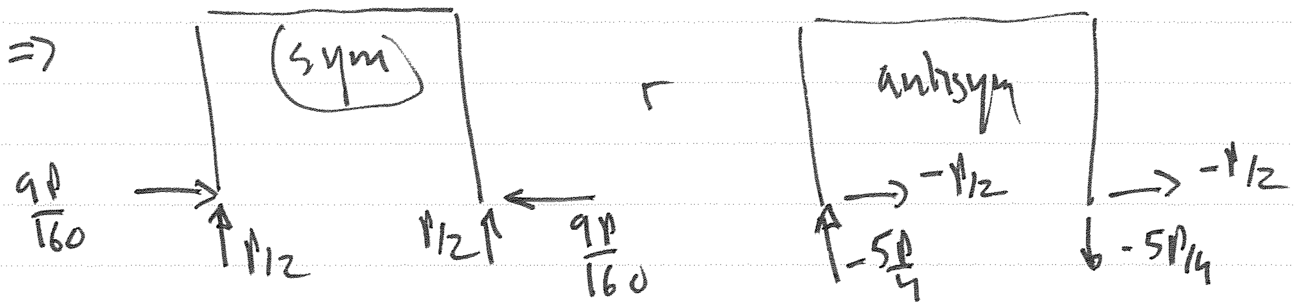


⇒ horizontally : $\theta_1 L + \delta_2 = 0$

$$\Rightarrow -\frac{9PL^{\cancel{3}}}{192E\cancel{I}} + \frac{H_1 L^{\cancel{3}}}{2\cancel{EI}} + \frac{H_1 L^{\cancel{3}}}{3\cancel{EI}} = 0$$

$$\frac{5}{\cancel{6}}H_1 = \frac{9P}{\cancel{192}\,32} \Rightarrow \underline{\underline{H_1 = \frac{9P}{160}}}$$

⇒



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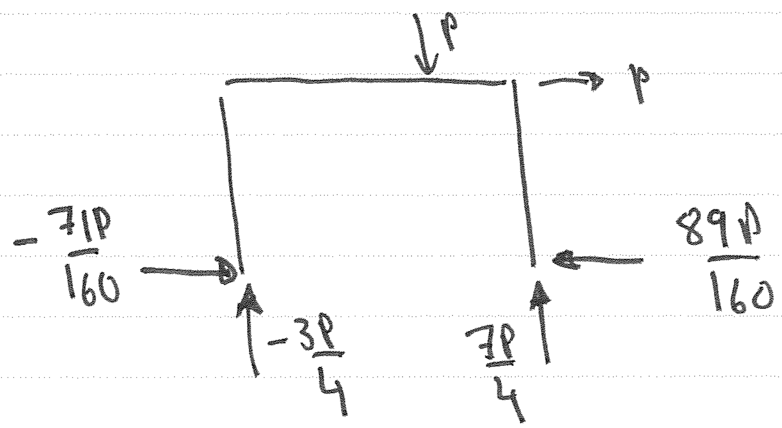
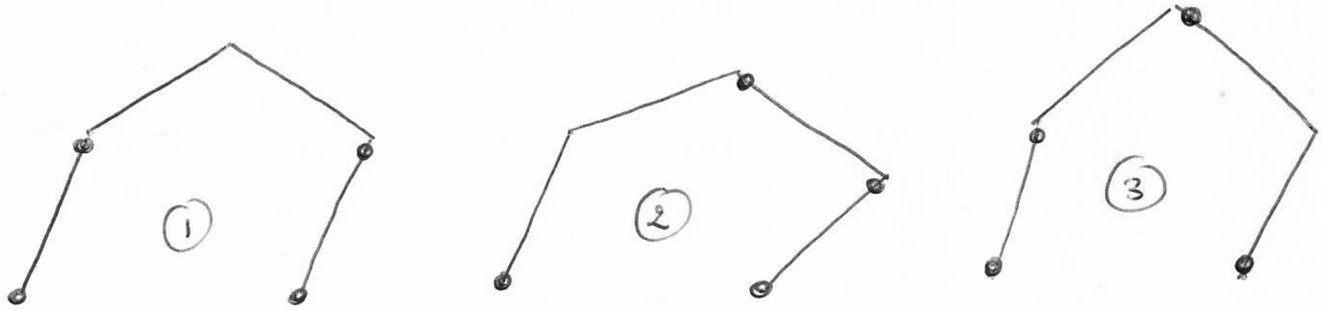
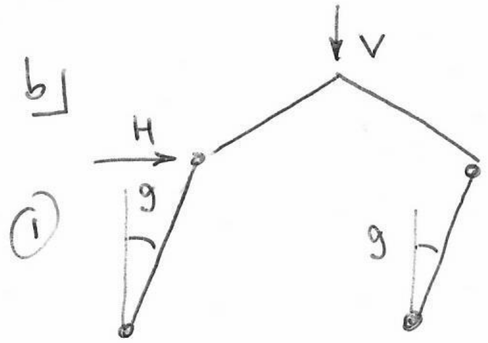


4.

a]

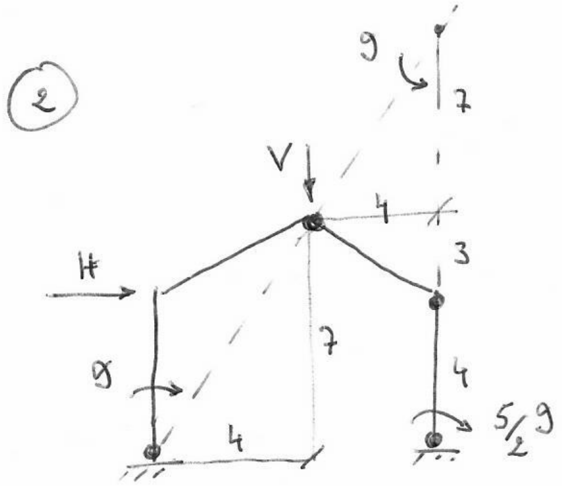


b]



$$H(4\theta) = 4 M_p \theta$$

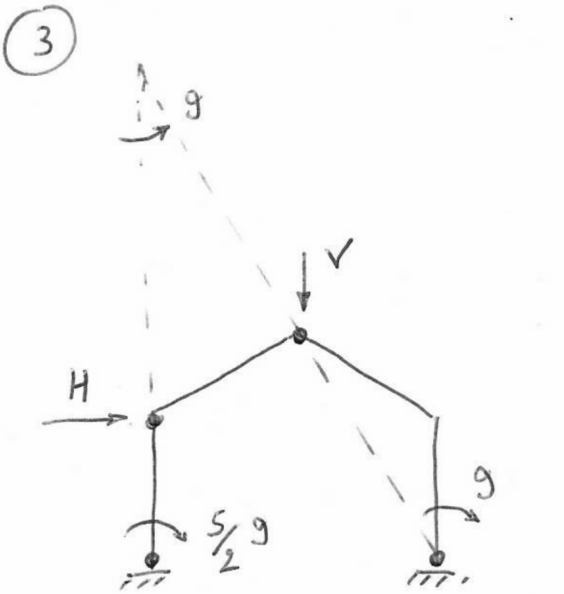
$$H = 10 \text{ kN}$$



$$(4\theta)H + (4\theta)V = M_p(\theta + 2\theta + \theta + 5\theta)$$

$$= M_p(9\theta)$$

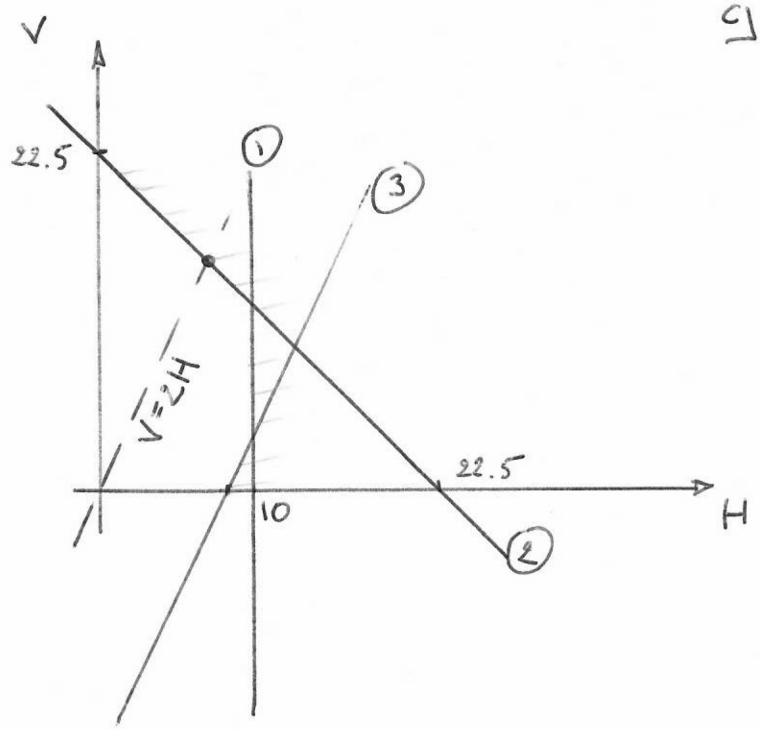
$$H + V = \frac{9}{4} M_p = 22.5$$



$$4\left(\frac{5}{2}\theta\right)H - (4\theta)V = M_p(9\theta)$$

$$10H - 4V = 90$$

$$H - 0.4V = 9$$



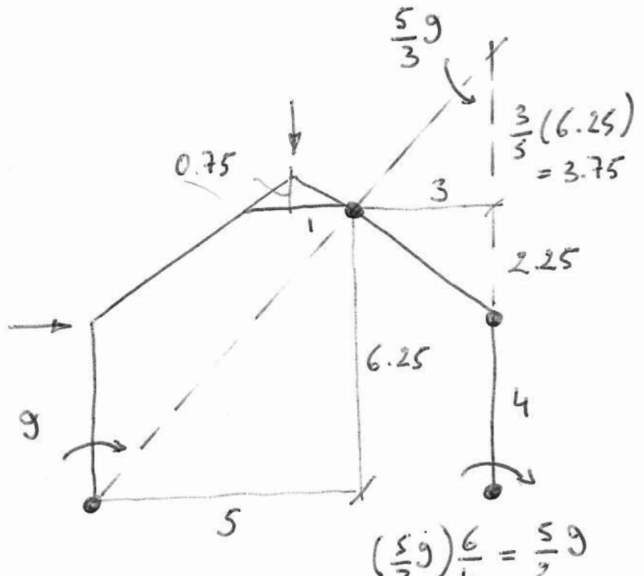
c]

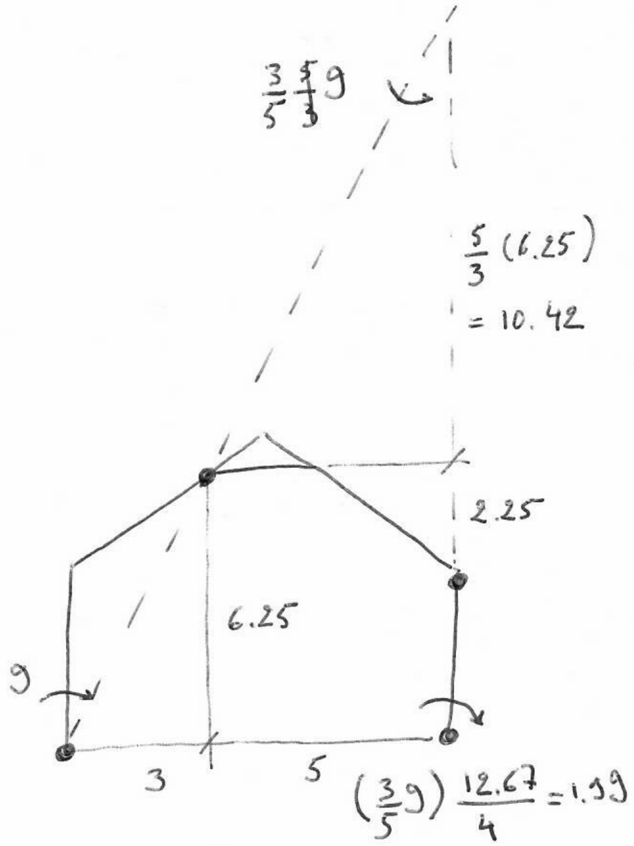
$$\begin{cases} H + V = 22.5 \\ V = 2H \end{cases}$$

$$\rightarrow H = 7.5 \text{ kN}$$

$$\gamma = \frac{7.5}{5} = 1.5$$

d]





$$(4\theta)H + (4\theta)V$$
$$= M_p\left(\theta + \theta + \frac{5}{3}\theta.2 + \frac{5}{2}\theta.2\right)$$
$$= M_p\,\theta\left(\frac{31}{3}\right)$$

$$\begin{cases} H + V = 25.83 \\ V = 2H \end{cases} \rightarrow H = 8.61$$

$$\gamma = \frac{8.61}{5} = 1.72$$

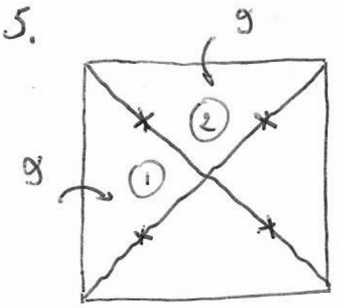
$$(4\theta)H + \frac{3}{5}\theta.4V$$
$$= M_p\left(\theta + \theta + \frac{3}{5}\theta.2 + 2(1.99)\right)$$
$$= 7\theta M_p$$

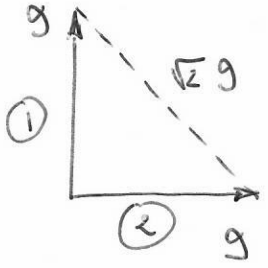
$$\begin{cases} 4H + \frac{12}{5}V = 70 \\ V = 2H \end{cases} \rightarrow H = 7.95$$

critical

$$\gamma = \frac{7.95}{5} = 1.59$$

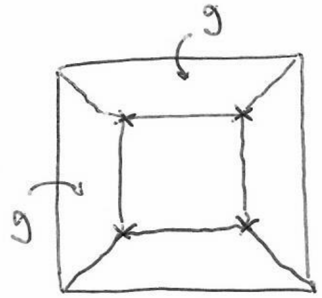
5.





$$4(wga) = 4(\sqrt{2}\cdot 2a\cdot\sqrt{2}g)m$$

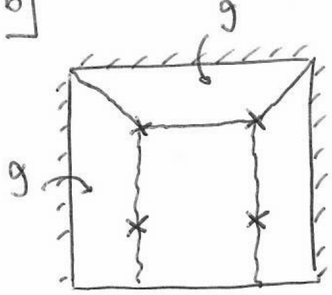
$$\underline{w = 4m}$$



$$4(wag) = 4(\sqrt{2}g\,a\sqrt{2} + g\,2a)m$$

$$\underline{w = 4m}$$

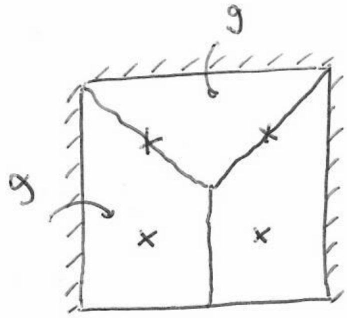
b)



$$4(wag) = 2(\sqrt{2}g\,a\sqrt{2})m + g(2a + 3a + 3a)m$$

$$4w = 12m$$

$$\underline{w = 3m}$$

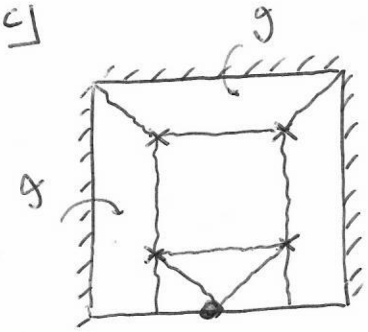


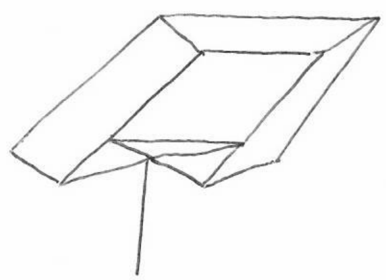
$$4(wag) = 2(\sqrt{2}g\sqrt{2}\cdot 2a)m + (2g\,2a)m$$

$$4w = 12m$$

$$\underline{w = 3m}$$

c)





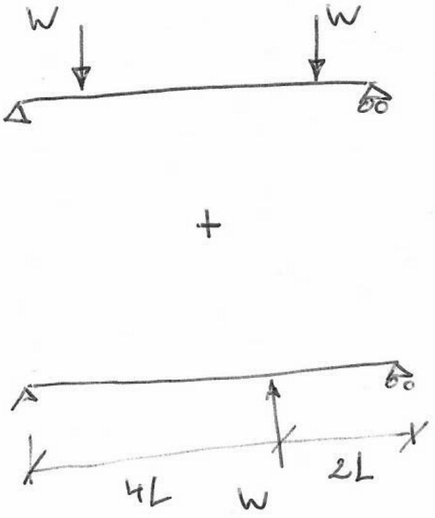
$$4(wag) = 2(\sqrt{2}g\,a\sqrt{2})m + g(2a + 4a)m + (g\,2a)m + 2(\sqrt{2}g)(\sqrt{2}a)m + 2(2g)a\cdot m \rightarrow \underline{w = 5m}$$

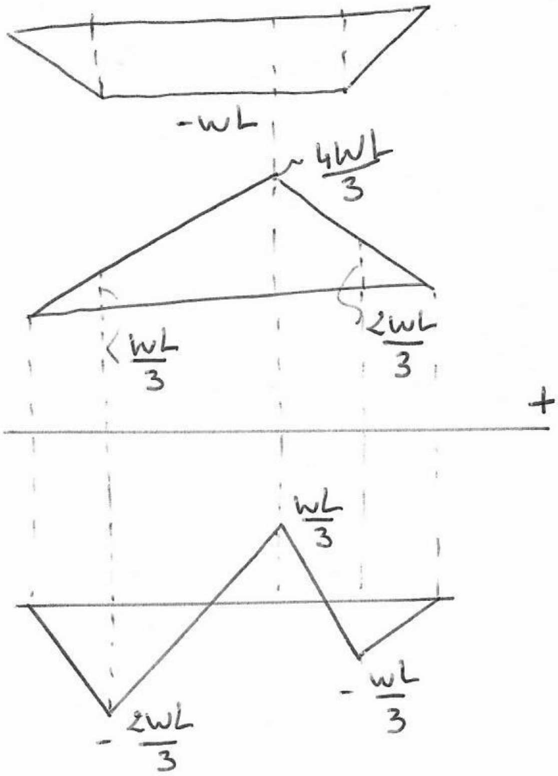
Not critical

→ A prop is as effective as a line support.

6.

a)





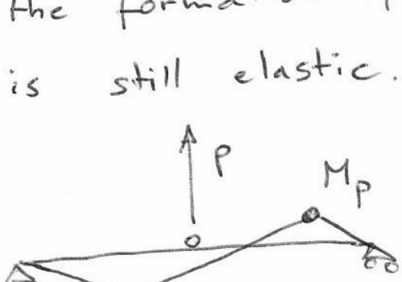
$$\frac{2WL}{3} = M_p$$

$$\Rightarrow \quad W = \frac{3M_p}{2L}$$

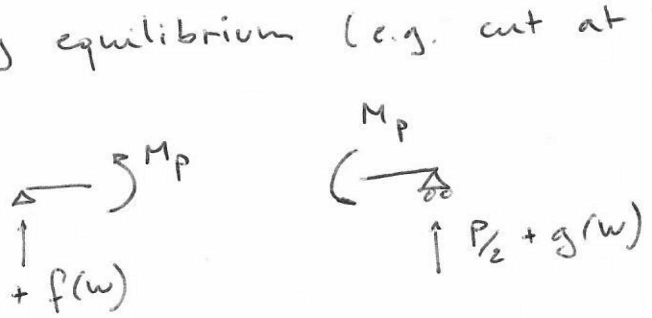
b) (i) The beam will collapse by either:

* the formation of one hinge and yielding of the cable. In this case, the force in the cable is $P = \frac{\pi d^2}{4} \cdot \sigma_y$ (thus independent of H).

* the formation of two hinges while the cable is still elastic.



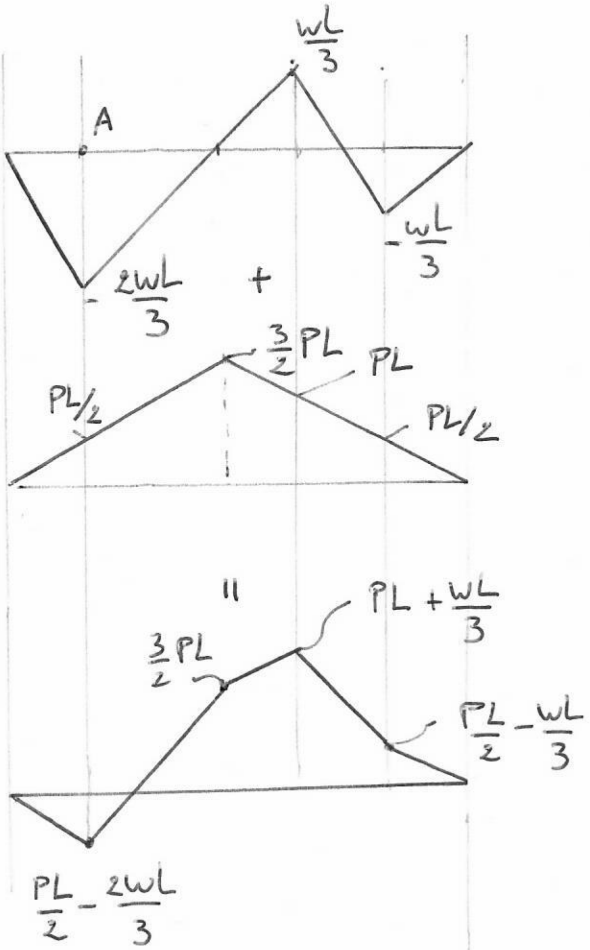
In this case, the moment line is entirely determined by equilibrium (e.g. cut at the plastic hinges:)



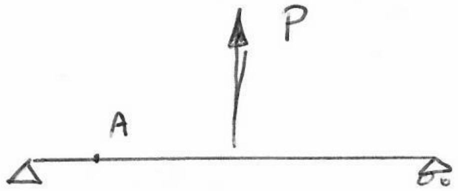
and P will follow from this (independently from H)

The beam is now a mechanism and will always be able to adjust to the necessary elongation of the cable, ensuring compatibility.

(ii)



The added moment from the force $P > 0$ (tension) will reduce the moment at A, increasing the capacity of the beam.



(iii)

Set $PL + \frac{wL}{3} = M_p \rightarrow P = \frac{M_p}{L} - \frac{w}{3}$

Set $\frac{PL}{2} - \frac{2wL}{3} = -M_p \rightarrow \frac{M_p}{2} - \frac{wL}{6} - \frac{2wL}{3} = -M_p$

$\rightarrow w = \frac{9}{5}\frac{M_p}{L} \rightarrow P = \frac{M_p}{L} - \frac{3}{5}\frac{M_p}{L} = \frac{2}{5}\frac{M_p}{L}$

if $P = \sigma_y \cdot \frac{\pi d^2}{4} = \frac{2}{5}\frac{M_p}{L}$

$\rightarrow d = \sqrt{\frac{8}{5\pi}\frac{M_p}{L}\cdot\frac{1}{\sigma_y}}$