2P2 2022

Q1.

(a) $s - m = b + r - Dj$

$= 6 + 3 - 2(4)$

$= \underline{\underline{1}}$

(b) Bar V selected as redundant

$$t_0 = \begin{bmatrix} 1.118 \\ 0 \\ -1.118 \\ 0 \\ 0 \\ 0.5 \end{bmatrix}$$

(c)

$t_V = 1$

$$s_1 = \begin{bmatrix} -0.698 \\ 1.249 \\ -0.698 \\ 1.0 \\ 1.0 \\ -0.469 \end{bmatrix}$$

2P2 2022

Q1(d)

$$t = t_0 + x_1 s_1$$

$$f = \begin{bmatrix} 2.236 & & & & & \\ & 1.200 & & & & \\ & & 2.236 & & & \\ & & & 1.281 & & \\ & & & & 1.281 & \\ & & & & & 2.000 \end{bmatrix}$$

$$e = f.t$$

$$\Rightarrow e = \begin{matrix} 2.500 \\ 0.00 \\ -2.500 \\ 0 \\ 0 \\ 1.000 \end{matrix} \frac{HL}{AE} + \begin{pmatrix} -1.562 \\ 1.500 \\ -1.562 \\ 1.281 \\ 1.281 \\ -0.937 \end{pmatrix} \frac{x_1 L}{AE}$$

$$s_1 . e = 0$$

$$\Rightarrow -0.469 \frac{HL}{AE} + 7.055 \frac{x_1 L}{AE} = 0$$

$$\Rightarrow x_1 = \underline{\underline{0.066}}$$

Q1(d) cont...

$$t = t_0 + x_1 s_1$$

$$\Rightarrow t = \begin{bmatrix} 1.072 \\ 0.083 \\ -1.164 \\ 0.066 \\ 0.066 \\ 0.469 \end{bmatrix} \quad M$$

1(e)

apply unit load at B →

$$t^* = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 1.000 \end{bmatrix}$$

$$\delta = t^*.e \Rightarrow \quad e = F.t = \begin{bmatrix} 2.396 \\ 0.100 \\ -2.604 \\ 0.085 \\ 0.085 \\ 0.938 \end{bmatrix}$$

2P2 2022

4

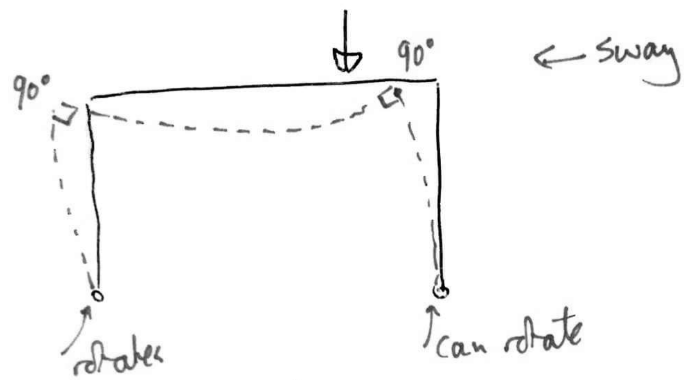
Q1(e) cont

$$t^{\kappa}.e \Rightarrow \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0.938 \end{bmatrix} \frac{HL}{AE}$$

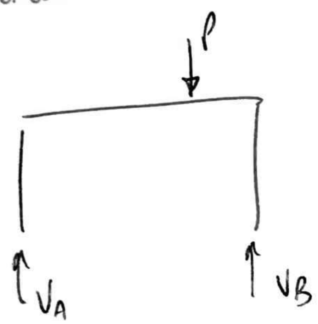
$\therefore$ displacement @ B = $0.938\ HL/AE$

2P2 Q2

Q2(a)



Q2(b)

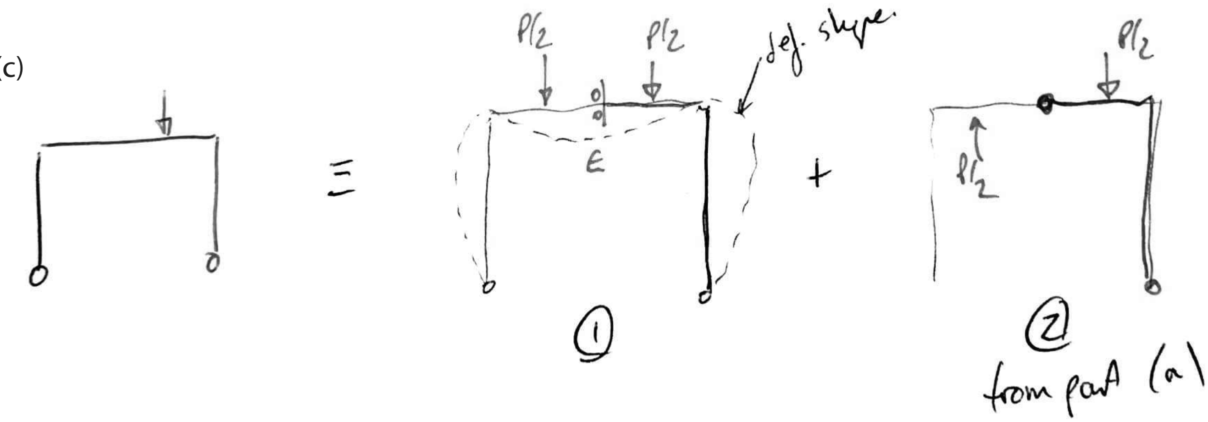


$P(3L/2) = V_B(2L)$

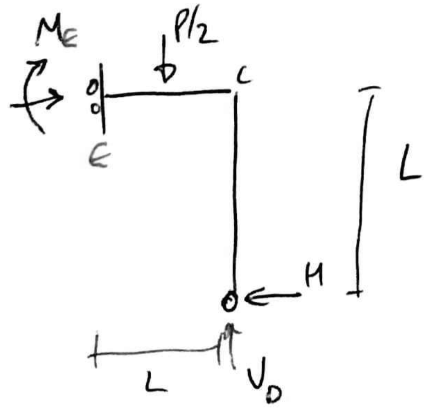
$V_B = 0.75P$

$\therefore V_A = 0.25P$

Q2(c)



1 - symmetric loading.



$V_D = P/2$ - since no vertical at midspan.

$M_C = H.L$

$\delta_C = 0$ (no sway)

2P2 Q2

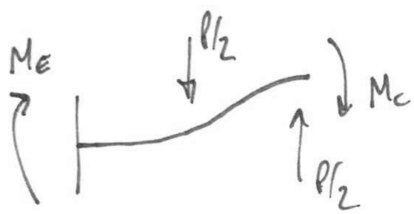
Q2(c)...

- take moments at E

$$: +(P/2)(L/2) + HL + M_E - (P/2)(L) = 0 \quad ①$$

$$M_E = \frac{PL}{4} - H.L$$

- θ at $E = 0$ (symmetric)



(beam)

- θ at C = sum of 3 components

$$\theta_C = \frac{(P/2)(L^2)}{2EI} - \frac{(P/2)(L/2)^2}{2EI} - \frac{HL^2}{EI}$$

$$\theta_c = \frac{PL^2}{4} - \frac{PL^2}{16EI} - \frac{HL^2}{EI}$$

$$\delta_c = \theta_c . L = \frac{3PL^3}{16EI} - \frac{HL^3}{EI} \quad ②$$

$$\delta_{column} = \frac{HL^3}{3EI}. \quad ③$$

$$\therefore \frac{3PL^3}{16EI} - \frac{HL^3}{EI} = \frac{HL^3}{3EI} \quad ④$$

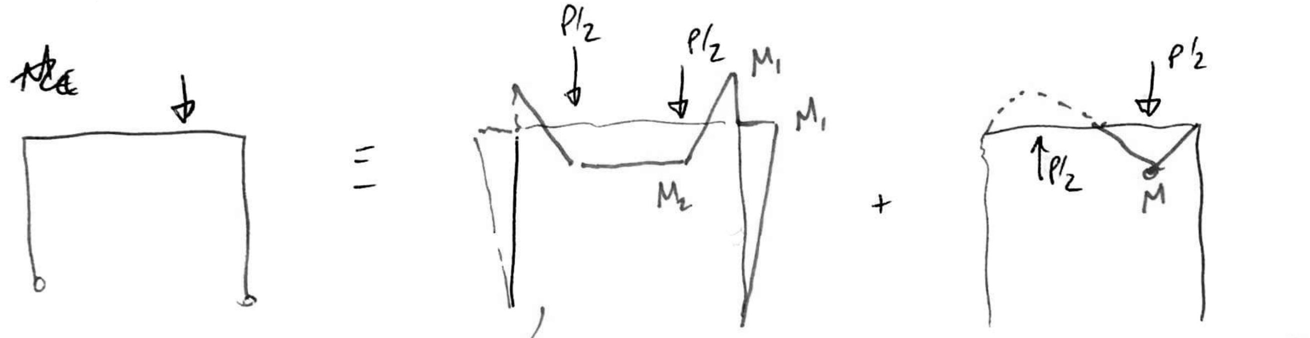
$$\underline{\underline{H = \frac{9}{64} P}} \quad ⑤$$

2P2 Q2

Q2(c)...

$\therefore \quad H_A = H_D = \dfrac{9}{64}P$

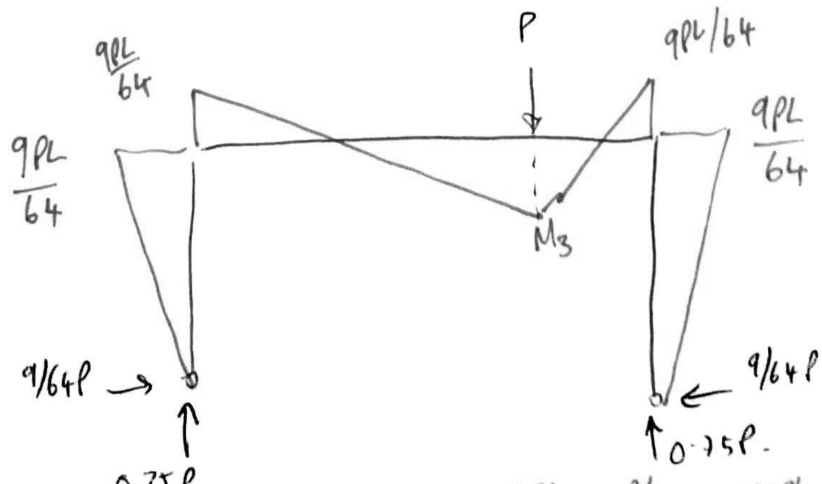
Q2(d)



$M = \dfrac{PL}{8}$ from (a)

$M_1 = H.L$
$= \dfrac{9PL}{64}$

$M_2 = M_E \; (1) = \dfrac{PL}{4} - HL = -\dfrac{7PL}{64}$ (note sign opposite to M_1)



$M_3 = \dfrac{7PL}{64} + \dfrac{PL}{8} = \dfrac{15PL}{64}$

Q2(e): compare BMD to deflected shape, demonstrating areas of tension are correctly shown

2P2 2022

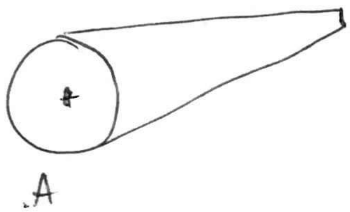
Q3(a)

Total load = $12.5 \times 4 \times 75 = 3750\,kN$

10 spaces = 375 kN/hanger.

– real tension depends on AE of cable, plus interaction between arch & deck

3(b)



$M = 12.5 \times 4 \times 2 = 100\,kNm/m$ span

$T = 100 \times \frac{75}{2} = \underline{\underline{3750\,kNm}}$

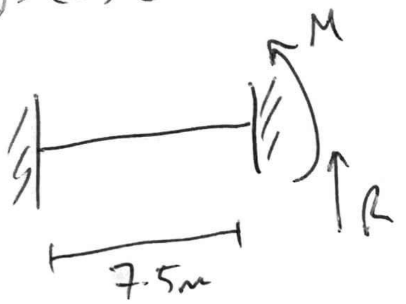
$\tau = \frac{Tr}{J}$

$J \approx 2\pi r_{av}^3 t = 8.91 \times 10^9\,mm^4$ (using avg radius)

[or $J = \pi \frac{(r_o^4 - r_i^4)}{2} = 8.97 \times 10^9\,mm^4$]

$\tau = \frac{3750 \times 10^6 \times 330}{8.91 \times 10^9} = \underline{\underline{139\,MPa}}$

Q3 (c) (i)



$$M = \frac{wl^2}{12} = \underline{\underline{234\,\text{kNm}}}$$

$$R = \frac{50 \times 7.5}{2} = \underline{\underline{187.5\,\text{kN}}}$$

$$I \approx \pi r^3 t = \pi (305)^3 (50) = 4.46 \times 10^9\ \text{mm}^4$$

$$\left[\text{or } I = \frac{\pi r_1^4}{4} - \frac{\pi r_0^4}{4} = 4.49 \times 10^9\ \text{mm}^4\right]$$

$$\sigma = \frac{My}{I} = \frac{234 \times 10^6 \times 330}{4.46 \times 10^9} = \underline{\underline{17.3\,\text{MPa}}}.$$

(c) (ii)

at mid-height:

$$A = \frac{\pi r_1^2 - \pi r_2^2}{2} = 47909\,\text{mm}^2$$

$$\bar{y}\ (\text{by integration}) = \frac{2r}{\pi} = \underline{\underline{194\,\text{mm}}}$$

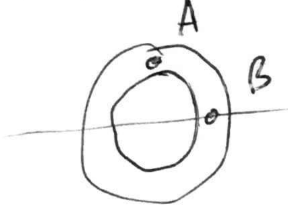
$$\tau = \frac{SA\bar{y}}{It} = \frac{187.5 \times 10^3 \times 47909 \times 194}{4.46 \times 10^9 \times (50 \times 2)} = \underline{\underline{3.91\,\text{MPa}}}$$

(small)

2P2 2012 Q3

(3)

(3)(d)



at A: $\sigma = +17.3\,MPa$

$\tau = 0 + 139\,MPa = \underline{139\,MPa}$

B $\sigma = 0\,MPa$

$\tau = 3.91 + 138 = \underline{143\,MPa}$

at A:

$\sigma_{avg} = 8.65\,MPa$

$\sigma_1 = 8.65 - \sqrt{8.65^2 + 139^2} = \underline{-131\,MPa}$

$\sigma_2 = 8.65 + (\quad " \quad) = \underline{148\,MPa}$

at B:

$R = \tau = 143\,MPa$

$\sigma_1 = -143\,MPa$ & $\sigma_2 = +143\,MPa$

von Mises = $\lambda^2 \left[(\sigma_1 - \sigma_2)^2 + \sigma_1^2 + \sigma_2^2 \right] = 2Y^2$

$Y = 355$

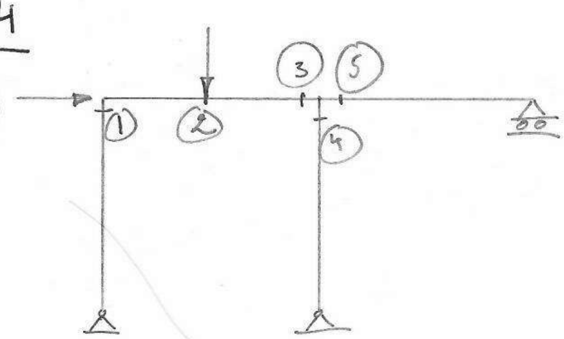
at A $\Rightarrow \lambda = \underline{\cancel{1.47}}$ 1.038

B $\Rightarrow \lambda = \underline{\cancel{1.43}}$ ← critical

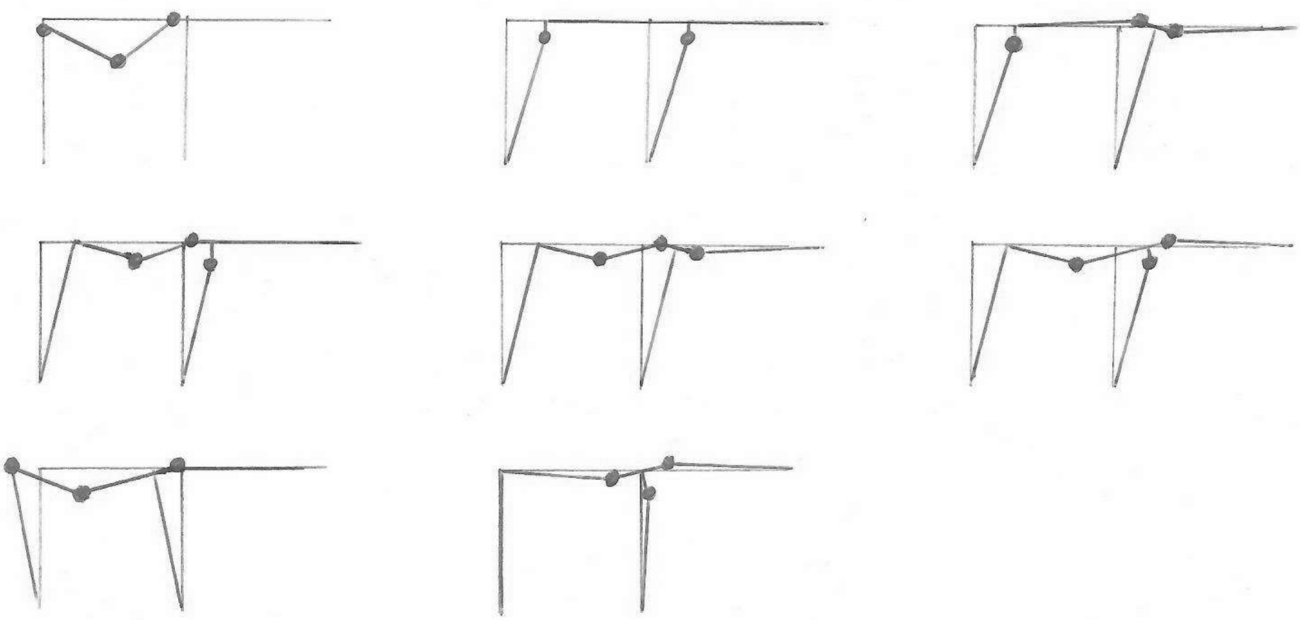
1.013

Q4

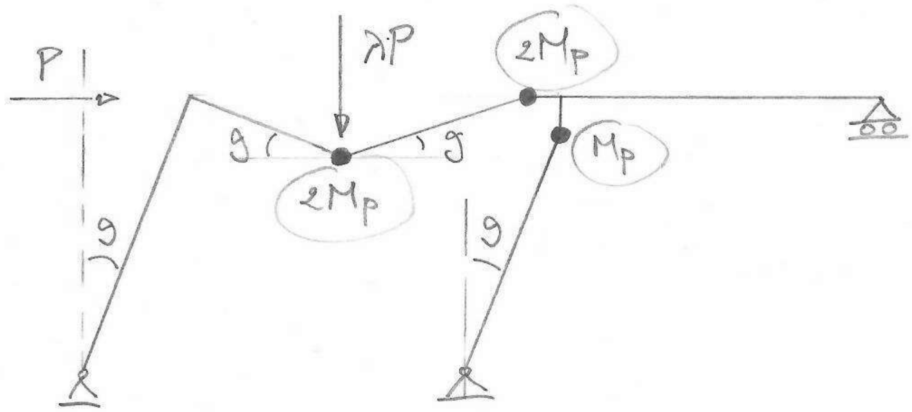
(a)



8 mechanisms



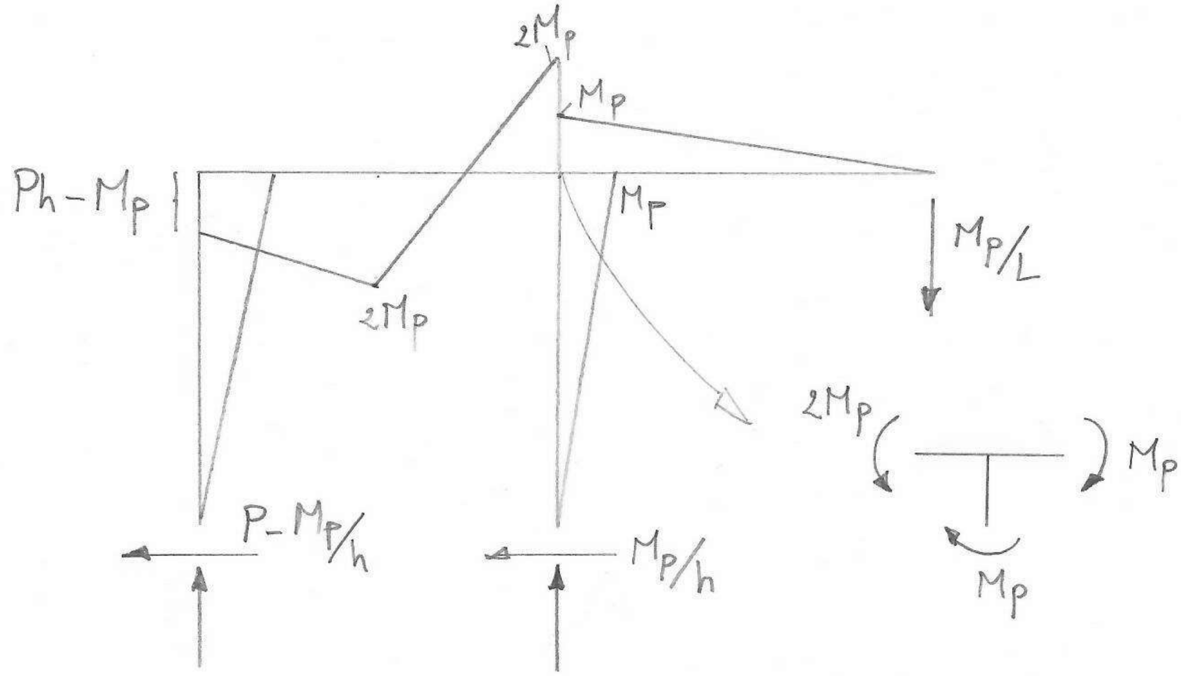
(b)



$$Ph\theta + \lambda P \frac{L}{2}\theta = (2\theta)(2M_p) + \theta(2M_p) + \theta(M_p)$$

$$Ph + \lambda P \frac{L}{2} = 7M_p$$

$$P = \frac{14M_p}{2h + \lambda L}$$



lower bound ?

$$-M_p < Ph - M_p < M_p$$

① $Ph - M_p < M_p$

$$\Rightarrow \frac{14 \cancel{M_p}}{2h + \lambda L} h < 2\cancel{M_p}$$

$$\Rightarrow 2h + \lambda L > 14h/2$$

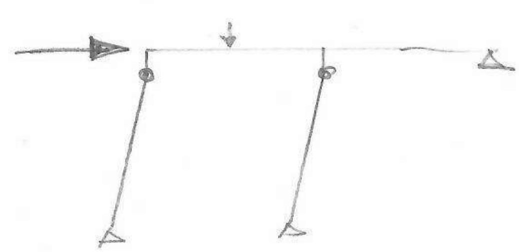
$$\Rightarrow \lambda > \frac{5h}{L}$$

② $-M_p < Ph - M_p$

$Ph > 0$

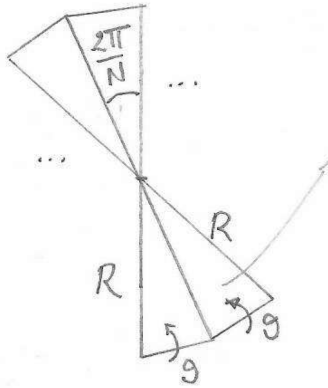
Ok

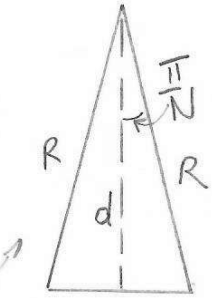
For $\lambda < \frac{5h}{L}$ sway mechanism will become critical.



Q5

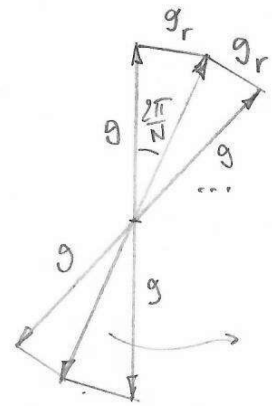
(a)

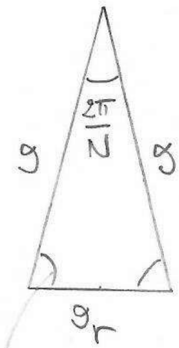




$d = R\cos\frac{\pi}{N}$

rotations ~~deformation~~ diagram:





$$\frac{\pi - \frac{2\pi}{N}}{2} = \frac{\pi}{2} - \frac{\pi}{N}$$

central deflection : $v = \theta d$

$$v = \theta R \cos\frac{\pi}{N}$$

$$\theta_r ? \quad \frac{\theta_r}{\sin\frac{2\pi}{N}} = \frac{\theta}{\sin\left(\frac{\pi}{2} - \frac{\pi}{N}\right)} \Rightarrow \theta_r = \theta\,\frac{\sin\frac{2\pi}{N}}{\cos\left(\frac{\pi}{N}\right)} = 2\theta \sin\left(\frac{\pi}{N}\right)$$

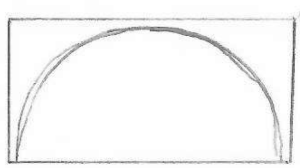
$$P\,\theta R\cos\frac{\pi}{N} = m\Big(\underbrace{N\theta\, 2R\sin\frac{\pi}{N}}_{\text{perimeter}} + \underbrace{N R\, 2\theta\sin\left(\frac{\pi}{N}\right)}_{\text{radial}}\Big)$$

$$\Rightarrow \quad P = 4m\, N\sin\left(\frac{\pi}{N}\right) \Big/ \cos\left(\frac{\pi}{N}\right)$$

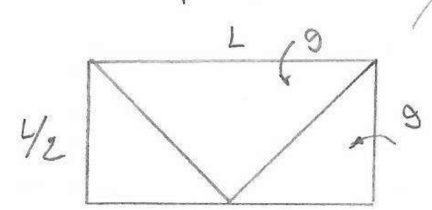
$$\lim_{N\to\infty} 4m\, N\sin\left(\frac{\pi}{N}\right) \Big/ \underbrace{\cos\left(\frac{\pi}{N}\right)}_{\to 1} = \lim_{\frac{\pi}{N}\to 0} 4m\,\frac{\sin\left(\frac{\pi}{N}\right)}{\frac{\pi}{N}}\cdot\pi$$

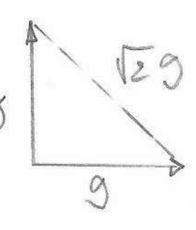
$$\Rightarrow \quad P = 4m\pi$$

(b)



$P_1 = 2m\pi$





$$P.\theta.\frac{L}{2} = m\left(2L\theta + \sqrt{2}\,\theta.2.\frac{L}{2}\sqrt{2}\right)$$

$$\Rightarrow \quad P = 8m > 2\pi m$$

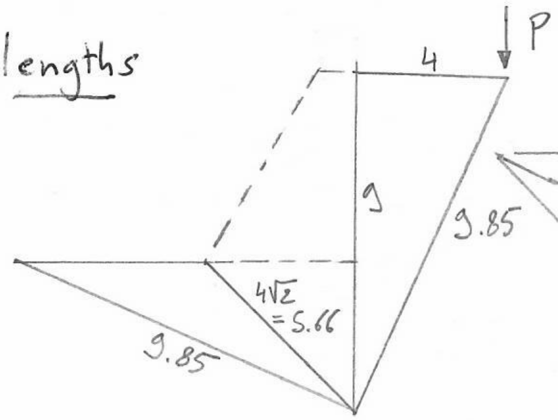
P_1 is more critical

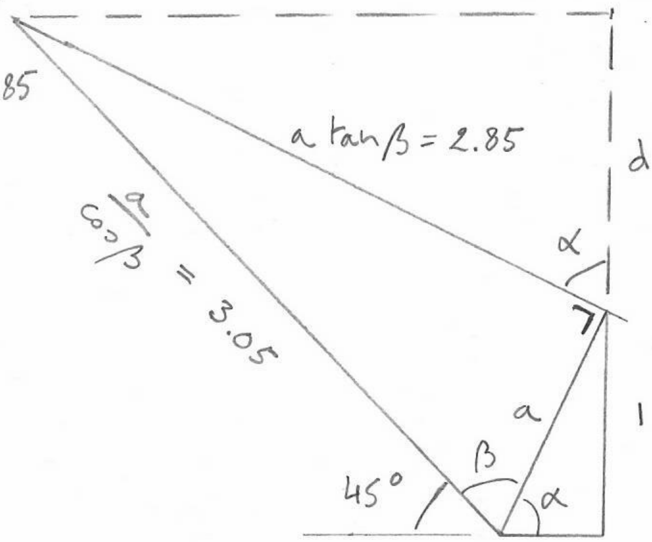
Q6

(a)

lengths

deformations





$$P.1 = k\left[(9.85)(1.09) + (9.85)(2.85) + (5.66)(3.05)\right]$$

$$= 2800 \text{ kN/m}$$

$$a = \frac{9.85}{9} = 1.09$$

$$\alpha = \operatorname{asin}\frac{1}{1.09} = 66^\circ$$

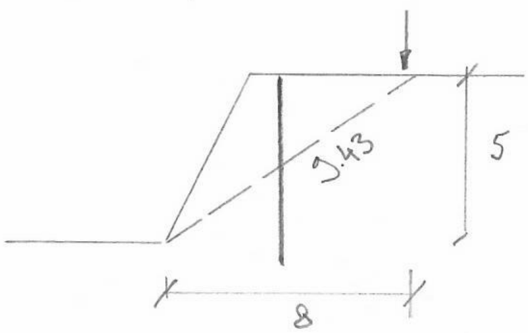
$$\beta = 180^\circ - 45^\circ - 66^\circ = 69^\circ$$

(b) $d = (2.85)\cos\alpha = 1.16$

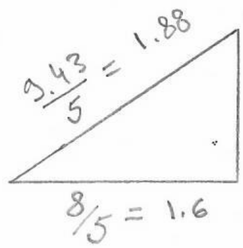
$$P.1 - (2 \text{ kPa})(1.16)(5) = 2800 \text{ kN/m}$$

$\Rightarrow P = 2812$ kN/m (negligible effect)

(c)



deformations:



Shear through pile: $A.\frac{f_y}{\sqrt{3}} = \pi(200)(10)\frac{(350)}{\sqrt{3}} = 1270$ kN

$$P.1 = k(9.43)(1.88) + \frac{1270 \text{ kN}}{3\text{m}}\frac{(1.88)}{1.6}(1.88) = 1822 \text{ kN}$$

$\rightarrow$ critical