Version JJO/7

EGT1
ENGINEERING TRIPOS PART IB

Wed 8 June 2022 14.00 to 16.10

Paper 2

STRUCTURES

Answer not more than ***four*** *questions, which may be taken from either section.*

All questions carry the same number of marks.

The ***approximate*** *number of marks allocated to each part of a question is indicated in the right margin.*

Answers to questions in each section should be tied together and handed in separately.

Write your candidate number ***not*** *your name on each cover sheet.*

STATIONERY REQUIREMENTS
Single-sided script paper

SPECIAL REQUIREMENTS TO BE SUPPLIED FOR THIS EXAM
CUED approved calculator allowed
Engineering Data Book

10 minutes reading time is allowed for this paper at the start of the exam.

You may not start to read the questions printed on the subsequent pages of this question paper until instructed to do so.

You may not remove any stationery from the Examination Room.

SECTION A

1 Figure 1 shows a weightless pin-jointed truss subjected to a horizontal point load H. The truss has a pin support at A and a roller support at B. The unloaded structure comprises members I to VI. All members have the same cross-sectional area A and are made of a material with Young's modulus E. All behaviour is linear elastic and the truss is initially stress-free.

(a) Find the number of redundancies. [2]

(b) Find the particular solution for bar tensions in equilibrium with the applied load H. [4]

(c) Find the possible states of self stress in the structure. [6]

(d) Find the elastic solution for the bar tensions under load H. [10]

(e) Find the horizontal displacement at support B. [3]

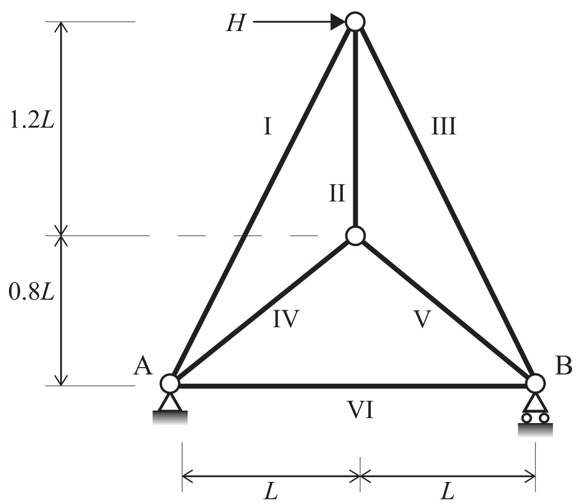


Fig. 1

2 The weightless rectangular frame shown in Fig. 2 is pinned at support points A and D and is continuous between them. All members have flexural rigidity EI, are axially rigid, and behave elastically. A vertical point load is applied to member BC. The unloaded frame is unstressed.

(a) Draw a feasible deflected shape for the frame under the applied load P. Annotate your diagram to highlight any salient points. [3]

(b) Derive expressions for the vertical reactions at A and D. [4]

(c) Derive expressions for the horizontal reactions at A and D. [12]

(d) Draw the bending moment diagram for the entire frame, annotating salient points. [4]

(e) Verify that the deflected shape drawn in (a) is valid, by comparing it to the bending moment diagram drawn in (d). [2]

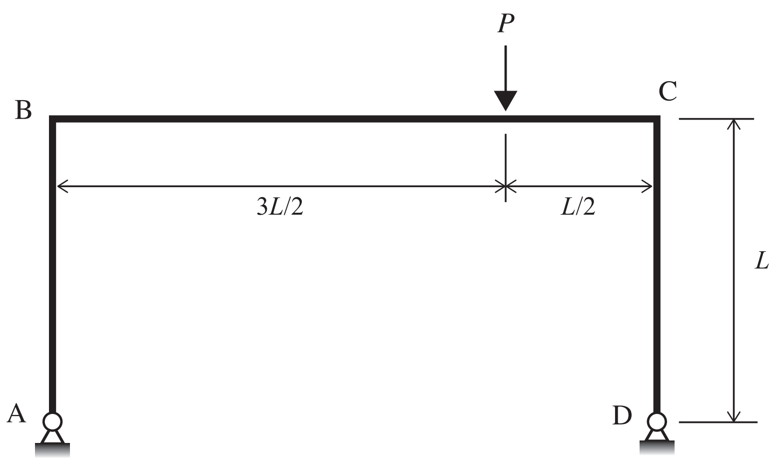


Fig. 2

3 Figure 3 shows a cable stayed footbridge with a main span of 75 m. The bridge deck is formed of a steel circular hollow section (CHS) that is fixed in translation and rotation at both ends. Geometrical properties of the CHS are given in Fig. 3. The CHS supports a 4 m wide cantilevered deck. Nine cables, which can carry only tension, are equally spaced along the longitudinal axis of the CHS and are connected to a rigid arch structure above. The arch is *not* connected to the CHS at the supports. The total vertical load on the bridge deck is $w = 12.5$ kNm^{-2}.

(a) Estimate the force carried by each cable. State your simplifying assumptions, and comment on your result. [3]

(b) Calculate the maximum torsional stress in the CHS and state where this occurs along the CHS. [5]

(c) To estimate the reactions at either end of the bridge, make the simplifying assumption that the section of the bridge from either fixed end support to the adjacent cable position may be analysed in isolation as a fixed end beam with a span of 7.5 m.

(i) Estimate the longitudinal stresses in the CHS at the fixed supports. [5]

(ii) Estimate the maximum shear stresses at the fixed supports. [5]

(d) The CHS is to be fabricated from steel with a yield strength of 355 MPa. Considering two suitable positions on the CHS cross-section, use the von Mises yield criterion to calculate the factor of safety against failure of the CHS at the fixed supports. [7]

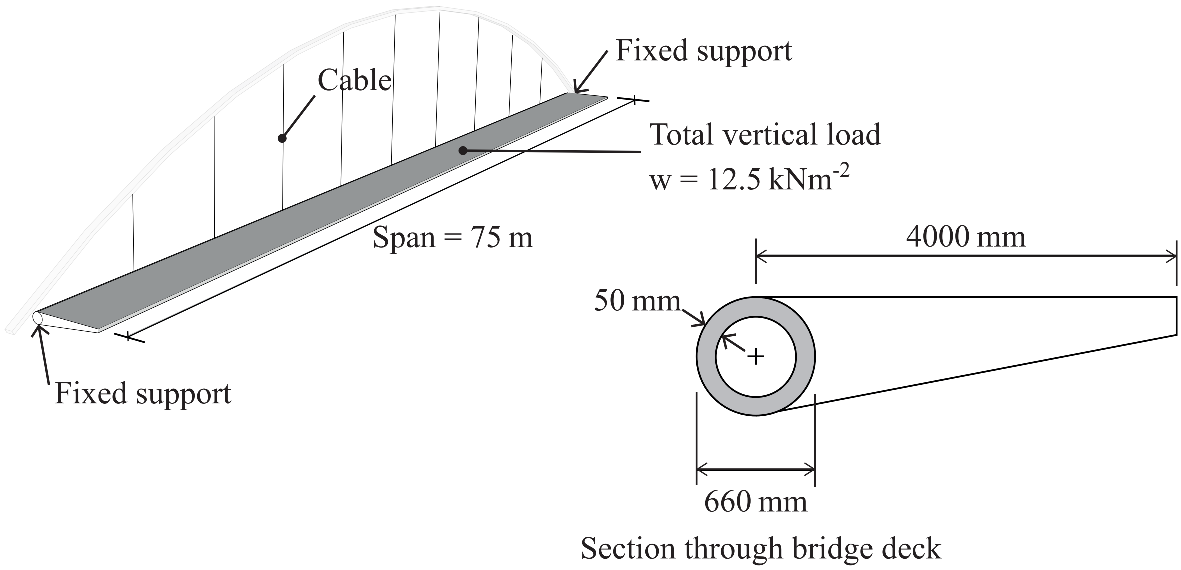

Cable
Fixed support
Total vertical load
w = 12.5 kNm-2
Span = 75 m
Fixed support
4000 mm
50 mm
660 mm
Section through bridge deck

Fig. 3

SECTION B

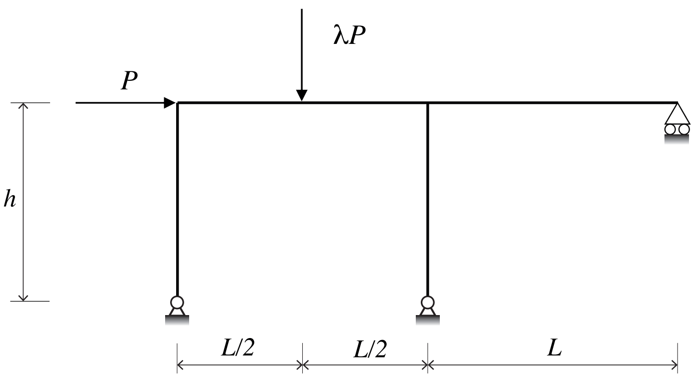
4 The steel frame in Fig. 4(a) is loaded with a horizontal force P and a vertical force which is equal to P multiplied by the parameter λ. The vertical members of the frame have a fully plastic moment capacity M_p, while that of the horizontal members is $2M_p$.

(a) Identify the critical locations where plastic hinges may potentially develop in the frame and sketch all compatible mechanisms. [8]

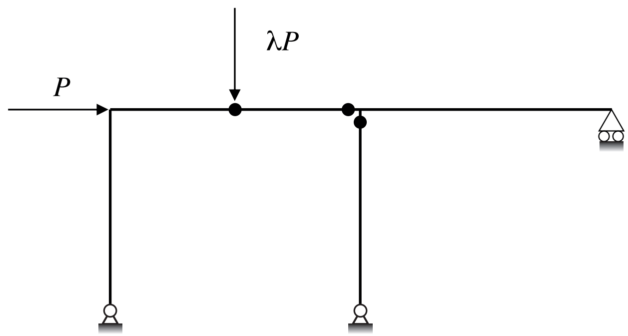
(b) Calculate the collapse load P associated with the plastic mechanism illustrated in Fig. 4(b). [5]

(c) Sketch the moment diagram associated with the mechanism in Fig. 4(b), indicating salient values. Use the Lower Bound theorem to prove that this mechanism results in the theoretically exact collapse load, provided that: $\lambda \geq 5h/L$. [10]

(d) Explain, without calculations, what you expect to happen for smaller values of λ. [2]



(a)



(b)

Fig. 4

5 (a) A plate extends to infinity in all directions and has a fully plastic moment per unit length indicated by m. Under a concentrated load transverse to the plate a circular yield line pattern develops, as depicted in Fig. 5(a). This mechanism can be seen as the limit case of the polygonal pattern shown in Fig. 5(b), whereby the number of vertices becomes infinite and the radial yield lines all merge into a continuously deformed shape. Calculate the collapse load associated with the mechanism shown in Fig. 5(a). [13]

(b) A plate with aspect ratio 2:1 has fully fixed boundary conditions along three of its four edges, with the remaining edge free, as shown in Fig. 5(c). The plate is loaded by a concentrated out-of-plane force P at point A. The fully plastic moment per unit length of the plate is m. Examine whether the semi-circular yield line mechanism in Fig. 5(c) is critical when compared to the yield line mechanism shown in Fig. 5(d). [12]

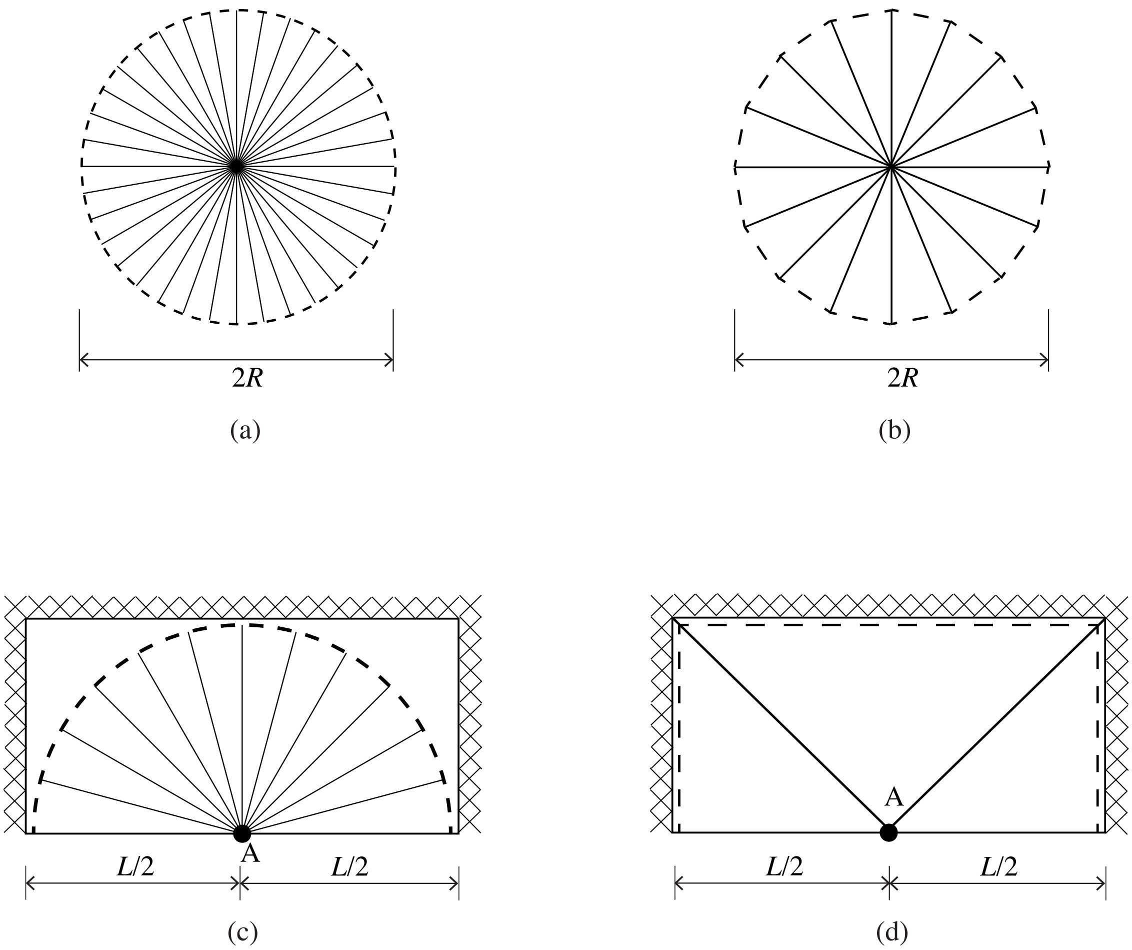


Fig. 5

6 Steel piles have been driven into a soil to stabilise a slope, as shown in Fig. 6. The piles consist of circular hollow sections with an outside diameter of 200 mm and a wall thickness of 10 mm. They are spaced at distances of 3 m in the out-of-plane direction. They are made of steel with a yield stress σ_y = 350 MPa which obeys the von Mises yield criterion. The soil can be modelled as a rigid-plastic material with a shear strength of 50 kPa.

(a) Assuming the plastic failure mechanism in Fig. 6(a), determine the maximum load P which can be sustained for a pile length L = 9 m. Note that P is a line load in the out-of-plane direction. Neglect the weight of the soil. [14]

(b) Investigate whether the slope stability can be significantly enhanced by applying a downward pressure of 2 kPa between points A and B. [5]

(c) Assess whether a straight failure surface shown in Fig. 6(b) is critical when compared to the mechanism shown in Fig. 6(a). Note that this failure surface crosses the piles, which are assumed to fail by yielding under shear stresses. Shear stresses may be assumed to be constant over the cross-section, in accordance with the Lower Bound theorem of plasticity. [6]

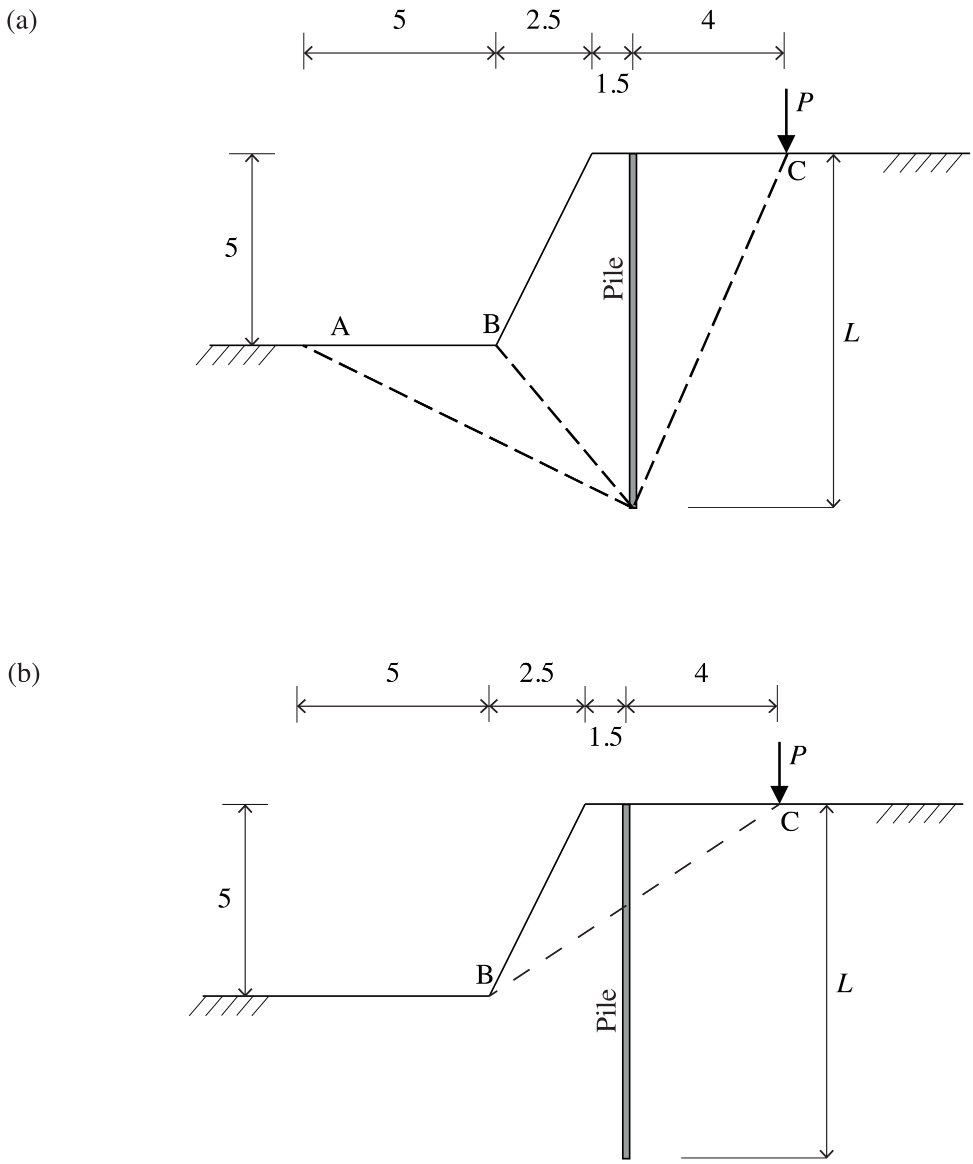


Fig. 6
(all dimensions in metres)

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2P2 2022
Numerical Answers

Q1(a) 1
Q1(e) 0.938HL/AE

Q2(b) $V_D = 0.75P$; $V_A = 0.25P$
Q2(c) $H_A = H_D = 9P/64$

Q3(b) τ = 139MPa
Q3(c)(i) σ = 17.3MPa
Q3(c)(ii) τ = 3.91MPa
Q3(d) $\lambda_A = 1.038$ and $\lambda_B = 1.013$

Q6(a) P = 2800kN/m
Q6(b) P = 2812kN/m
Q6(c) P = 1313kN