Module 1R
Paper P2

Version 1·0
Date 11/1/25
Page 1

Q1 a)

$$s - m = b + r - 2j$$

$j = 4$
$r = 3$
$b = 6$

$$\rightarrow \quad s - m = 6 + 3 - 2(4)$$

$$= 1$$

(b) set $t_{II} = 0$

$$\begin{array}{c} \mathrm{I} \\ \mathrm{II} \\ \mathrm{III} \\ \mathrm{IV} \\ \mathrm{V} \\ \mathrm{VI} \end{array} \begin{bmatrix} 2/3 \\ 0 \\ 0 \\ -5/6 \\ 0 \\ -5/6 \end{bmatrix}$$

$$e = f\,t$$

$$\begin{bmatrix} e_{\mathrm{I}} \\ \mathrm{II} \\ \mathrm{III} \\ \mathrm{IV} \\ \mathrm{V} \\ \mathrm{VI} \end{bmatrix} = \frac{L}{AE} \begin{bmatrix} 2 & & & & & \\ & \sqrt{65}/4 & & & & \\ & & \sqrt{65}/4 & & & \\ & & & 5/4 & & \\ & & & & 1 & \\ & & & & & 5/4 \end{bmatrix} \cdot \left(\begin{bmatrix} 2/3 \\ 0 \\ 0 \\ -5/6 \\ 0 \\ -5/6 \end{bmatrix} + x_1 \begin{bmatrix} 0.664 \\ 1 \\ 1 \\ -1.45 \\ -1.74 \\ -1.45 \end{bmatrix} \right)$$

$$S_1 \cdot e = 0$$

$$\begin{bmatrix} 2/3 \\ 0 \\ 0 \\ -5/6 \\ 0 \\ -5/6 \end{bmatrix} \begin{bmatrix} 2 & & \\ & \ddots & \\ & & 5/4 \end{bmatrix} \begin{bmatrix} 0.664 \\ 1 \\ 1 \\ -1.45 \\ -1.74 \\ -1.45 \end{bmatrix}$$

$$\Rightarrow \quad 3.91 \frac{L}{AE} + 13.2 \frac{x_1 L}{AE} = 0$$

$$x_1 = -0.296$$

$t = t_0 + x_1 s_1$

$$t = \begin{bmatrix} 0.47 \\ -0.3 \\ -0.3 \\ -0.4 \\ 0.52 \\ -0.40 \end{bmatrix}$$

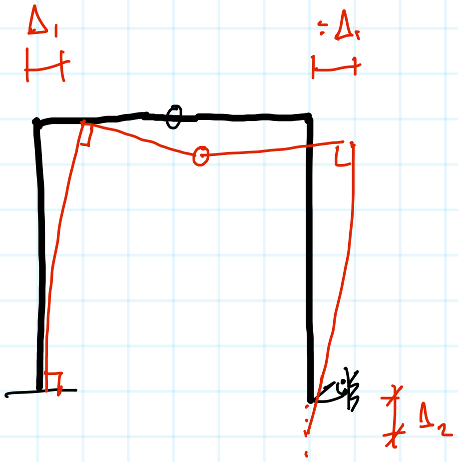
c) $e = ft$

$e_1 = 2(0.47) = 0.94 \frac{PL}{AE}$

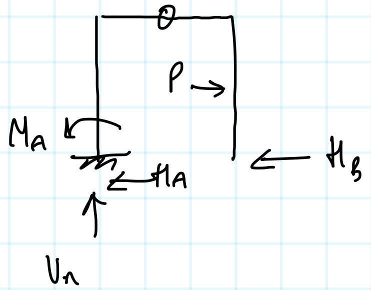
Module 2P2 Paper
Version 1·0 Date
Page 3

Q2

a)



b)



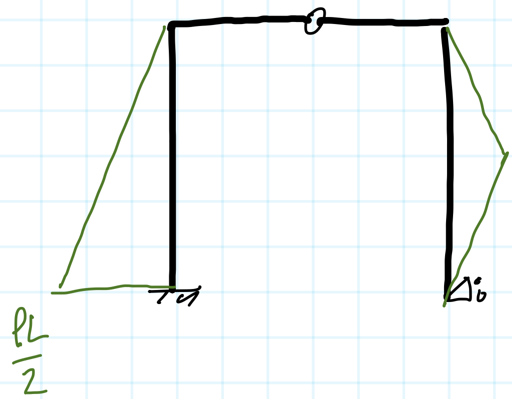
$V_A = 0$

$P \cdot \frac{L}{2} = H_B \cdot L \quad \therefore H_B = \frac{P}{2}$

$\therefore H_A = \frac{P}{2}$

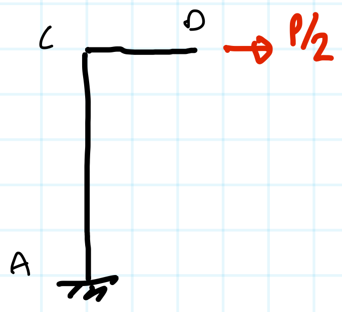
$M_A = \frac{PL}{2}$

c)



$\frac{P}{2} \cdot \frac{L}{2} = \frac{PL}{4}$

d) part A-C-D

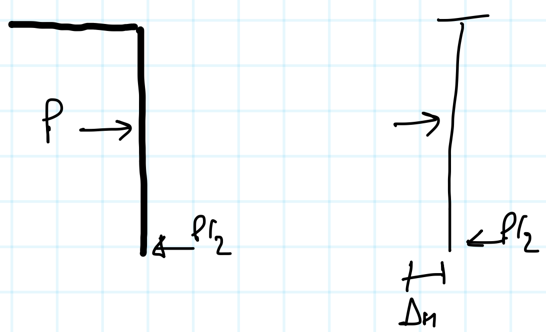


$$\delta_H = \frac{W.L^3}{3EI} = \frac{P}{2}\cdot\frac{L^3}{3EI} = \frac{PL^3}{6EI}$$

end rotation @ C $= \dfrac{(P/2)L^2}{2EI}$

$$\delta_v = \phi.\frac{L}{2} = \frac{P/2\,L^2}{2EI}\cdot\frac{L}{2} = \frac{PL^3}{8EI}$$

part D-E-B



$$\Delta_{H,B} = \frac{P(L/2)^3}{3EI} - \frac{P/2(L)^3}{3EI} = \frac{PL^3}{8EI}$$

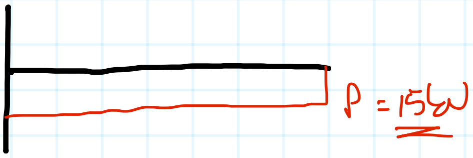
but $\Delta_{H,B} = 0$

$$\theta_2 . L = \frac{PL^3}{8EI} \quad \therefore \quad \theta_2 = \frac{PL^2}{8EI}$$

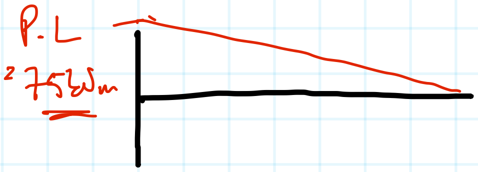
c) then, $\delta_E = \theta_2 \cdot \frac{L}{2} = \frac{PL^3}{16EI}$

$\Rightarrow$ total $\delta = \frac{PL^3}{8EI} + \frac{PL^3}{16EI} = \underline{\underline{\frac{3PL^3}{16EI}}}$

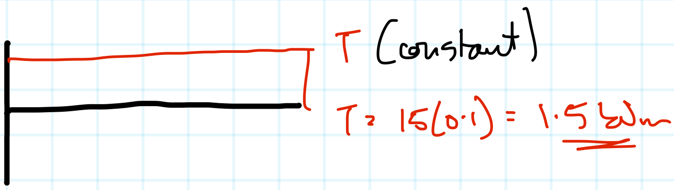
3a) SFD :



BMD



Torsion :

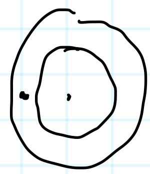


b) $\sigma_L = \frac{My}{I} = \frac{75\times10^6\left(300+5/2\right)}{\pi\, 300^3 . 5} = \underline{\underline{53.5\ \text{MPa}}}$

c) at root :

$\tau_s = \frac{SA\bar{y}}{It}$ $\quad \tau_t = \frac{T}{2A_e t}$

A



$$\tau_T = \frac{15\left(\pi\left(302.5^2 - 297.5^2\right)\right)\left(2\times300/\pi\right)}{2.\pi.300^3.5\ .\ 5}$$

$$+ \frac{1.5\times10^6}{2\times\left(\pi.300^2\right).5} = 6.36 + \underbrace{0.53}_{\tau_t} = \underline{\underline{6.89\ \text{MPa}}}$$

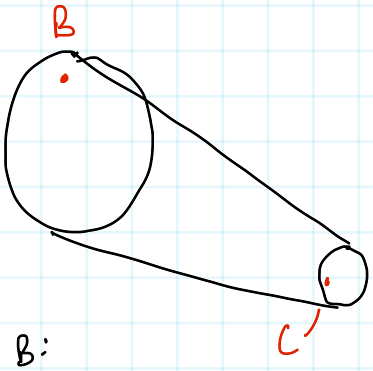
at tip :

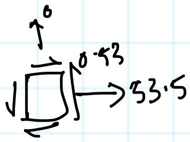
$A_e = \pi\left(100^2\right) \Rightarrow \tau_t = 4.8\ \text{MPa}$

$I = \pi\left(100^3\right)5 \quad \tau_s = 19.1\ \text{MPa}$

$\tau_T = \underline{\underline{23.9}}\ \text{MPa}$

3(d)





At B:

$\tau_t = 0.53\,\text{MPa} \approx 0\,\text{MPa}$
$\sigma_L = 53.5\,\text{MPa}$

$\sigma_1 = 53.5\,\text{MPa}$ (using Mohr's circle = 53.51 so σ_2)
$\sigma_2 = 0\,\text{MPa}$

$\Rightarrow$ Tresca : $\dfrac{275}{\sigma_1 - \sigma_2} = \underline{\underline{5.14}}$

At C:

$\tau_t = 23.9\,\text{MPa}$

$\sigma_1 = -\sigma_2 = 23.9$

$\Rightarrow$ Tresca $= \dfrac{275}{\sigma_1 - \sigma_2} = \underline{\underline{5.76}}$

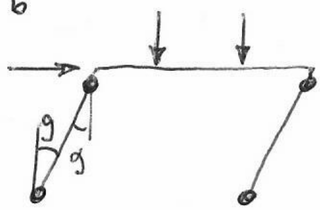
e) e.g. adjust dimensions, use thinner plate.

Module
Paper

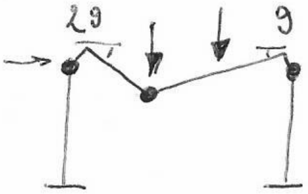
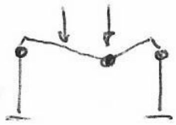
Version
Date
Page

4

a, b

(1) 

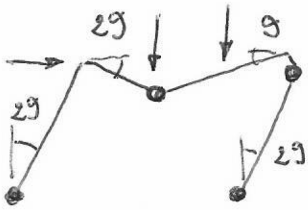
$$PL\theta = 4M_p\theta \Rightarrow P = \frac{4M_p}{L}$$

(2)  $\equiv$ 

$$(2P)(2\theta)\left(\frac{2L}{3}\right) + (2P)(\theta)\left(\frac{2L}{3}\right) = M_p(3\theta) + \alpha M_p(3\theta)$$

$$4PL = 3M_p(1+\alpha)$$

$$P = \frac{3}{4}(1+\alpha)M_p$$

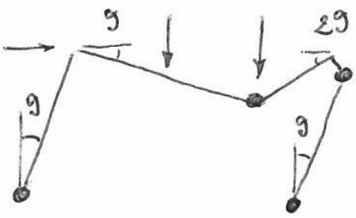
(3) 

$$P(2\theta)L + (2P)(2\theta)\left(\frac{2L}{3}\right) + (2P)(\theta)\left(\frac{2L}{3}\right)$$

$$= M_p(2\theta + 2\theta + 3\theta) + \alpha M_p(3\theta)$$

$$6PL = (7+3\alpha)M_p$$

$$P = \left(\frac{7}{6} + \frac{\alpha}{2}\right)\frac{M_p}{L}$$

(4) 

$$P(\theta)(L) + (2P)(\theta)\left(\frac{2L}{3}\right) + (2P)(2\theta)\left(\frac{2L}{3}\right)$$

$$= M_p(\theta + \theta + 3\theta) + \alpha M_p(3\theta)$$

$$5PL = (5+3\alpha)M_p$$

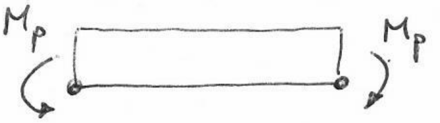
$$P = \left(1 + \frac{3}{5}\alpha\right)\frac{M_p}{L}$$

$$P_{(2)} < P_{(1)} \rightarrow \frac{3}{4}(1+\alpha) < 4 \rightarrow \alpha < \frac{13}{3}$$

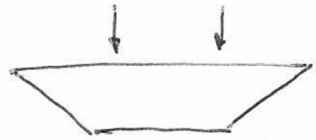
$$P_{(2)} < P_{(3)} \rightarrow \frac{3}{4}(1+\alpha) < \frac{7}{6} + \frac{\alpha}{2} \rightarrow \alpha < \frac{5}{3}$$

$$P_{(2)} < P_{(4)} \rightarrow \frac{3}{4}(1+\alpha) < 1 + \frac{3}{5}\alpha \rightarrow \boxed{\alpha < \frac{5}{3}}$$

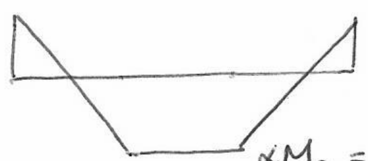
c) beam:



+



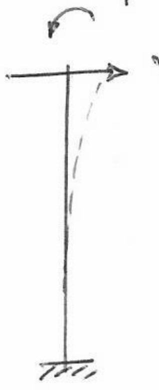
=



$\alpha M_p = \frac{3M_p}{2}$

$$P = \frac{3}{4}(1+\alpha)\frac{M_p}{L} = \frac{15}{8}\frac{M_p}{L}$$

columns:



$V_1 + V_2 = P$

1 redundancy left

→ compatibility

$$\Delta = \frac{M_p L^2}{2EI} + \frac{V_1 L^3}{3EI} \qquad \Delta = \frac{-M_p L^2}{2EI} + \frac{V_2 L^3}{3EI}$$

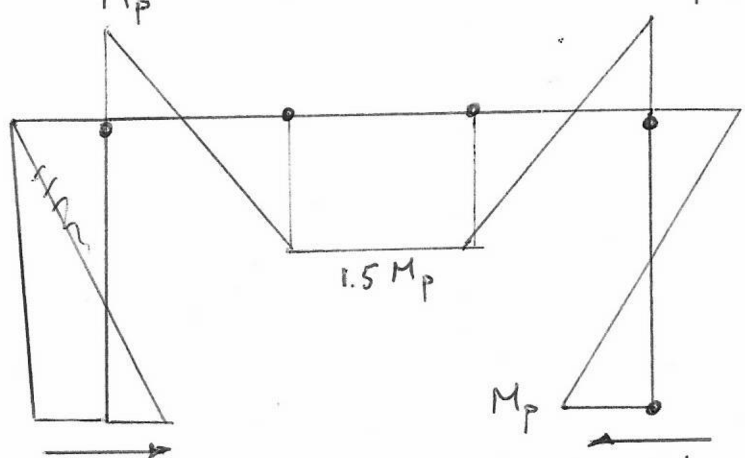
$$\Rightarrow 3M_p + 2V_1 L = -3M_p + 2V_2 L \quad \Rightarrow \quad V_2 - V_1 = \frac{3M_p}{L}$$

$$V_2 + V_1 = P = \frac{15}{8}\frac{M_p}{L}$$

$$V_2 = \frac{39}{16}\frac{M_p}{L}, \quad V_1 = -\frac{9}{16}\frac{M_p}{L}$$

$V_2 = \frac{39}{16}\frac{M_p}{L}$

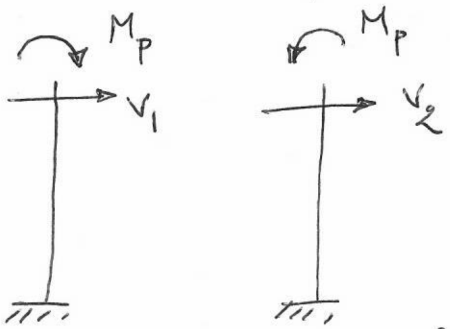
$M_p - \frac{39}{16}M_p < -M_p$ → hinge

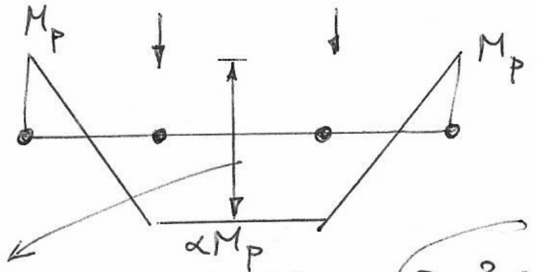


$\frac{M_p}{8L}$ $\qquad$ $\frac{2M_p}{L}$

$M_p - \frac{M_p}{8L}L = \frac{7}{8}M_p$

Shortcut (Solving b, c together)





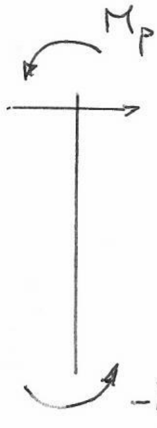
$(1+\alpha) M_p = (2P)\left(\frac{2L}{3}\right) \rightarrow P = \frac{3}{4}(1+\alpha) M_p$

$\Delta = \frac{M_p L^2}{2EI} + \frac{V_1 L}{3EI}$ $\quad \Delta = -\frac{M_p L^2}{2EI} + \frac{V_2 L^3}{3EI}$ $\Rightarrow V_2 - V_1 = \frac{3M_p}{L}$

$V_2 + V_1 = P = \frac{3}{4}(1+\alpha) M_p$

$V_2 = \frac{3}{8}(5+\alpha)\frac{M_p}{L}$

$\rightarrow V_1 = \frac{3}{8}(-3+\alpha)\frac{M_p}{L}$

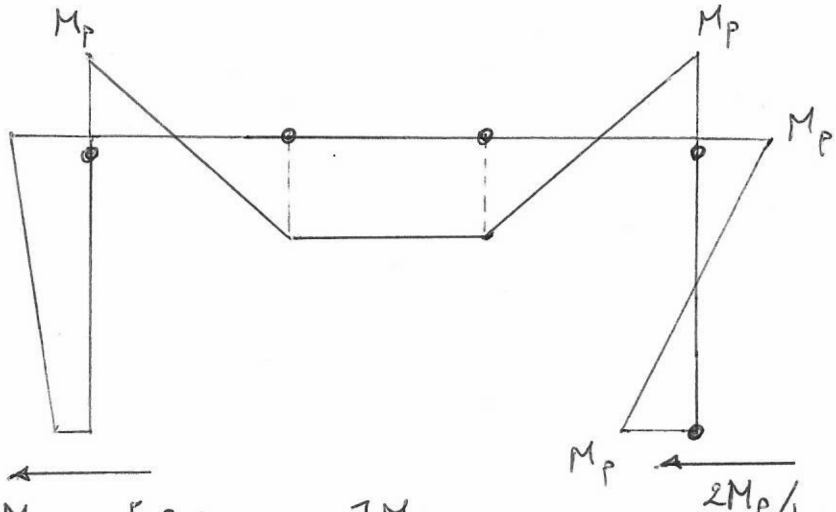


$V_2 = \frac{3}{8}(5+\alpha)\frac{M_p}{L}$

$-M_p + \frac{3}{8}(5+\alpha) M_p = \left(\frac{7}{8} + \frac{3}{8}\alpha\right) M_p < M_p$ if $\frac{7}{8} + \frac{3}{8}\alpha < 1$

$\rightarrow \alpha < 1/3$

but $\alpha > 1 \rightarrow$ certain range



$P - \frac{2M_p}{L} = \left[\frac{3}{4}(1+\alpha) - 2\right]\frac{M_p}{L}$

$= \left(-\frac{5}{4} + \frac{3}{4}\alpha\right)\frac{M_p}{L}$

$\left(\frac{5}{4} - \frac{3}{4}\alpha\right)\frac{M_p}{L}$

$M_p - \left(\frac{5}{4} - \frac{3}{4}\alpha\right) M_p = \left(-\frac{1}{4} + \frac{3}{4}\alpha\right) M_p$

$\left(-\frac{1}{4} + \frac{3}{4}\alpha\right) M_p < M_p$?

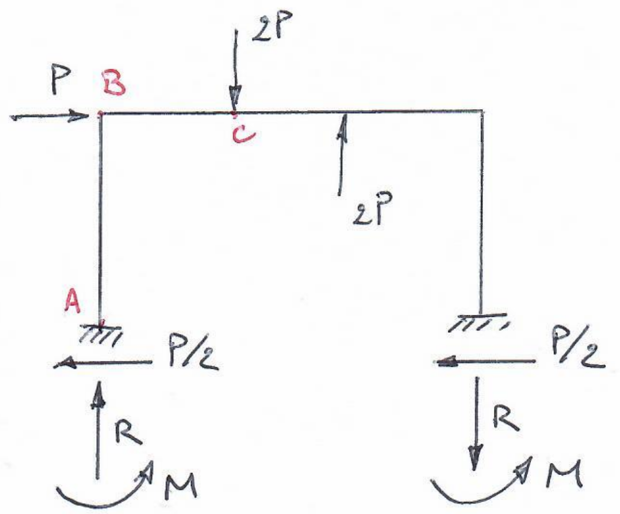
$-\frac{1}{4} + \frac{3}{4}\alpha < 1$

$-1 + 3\alpha < 4$

$\rightarrow \alpha < \frac{5}{3}$

5.

a)



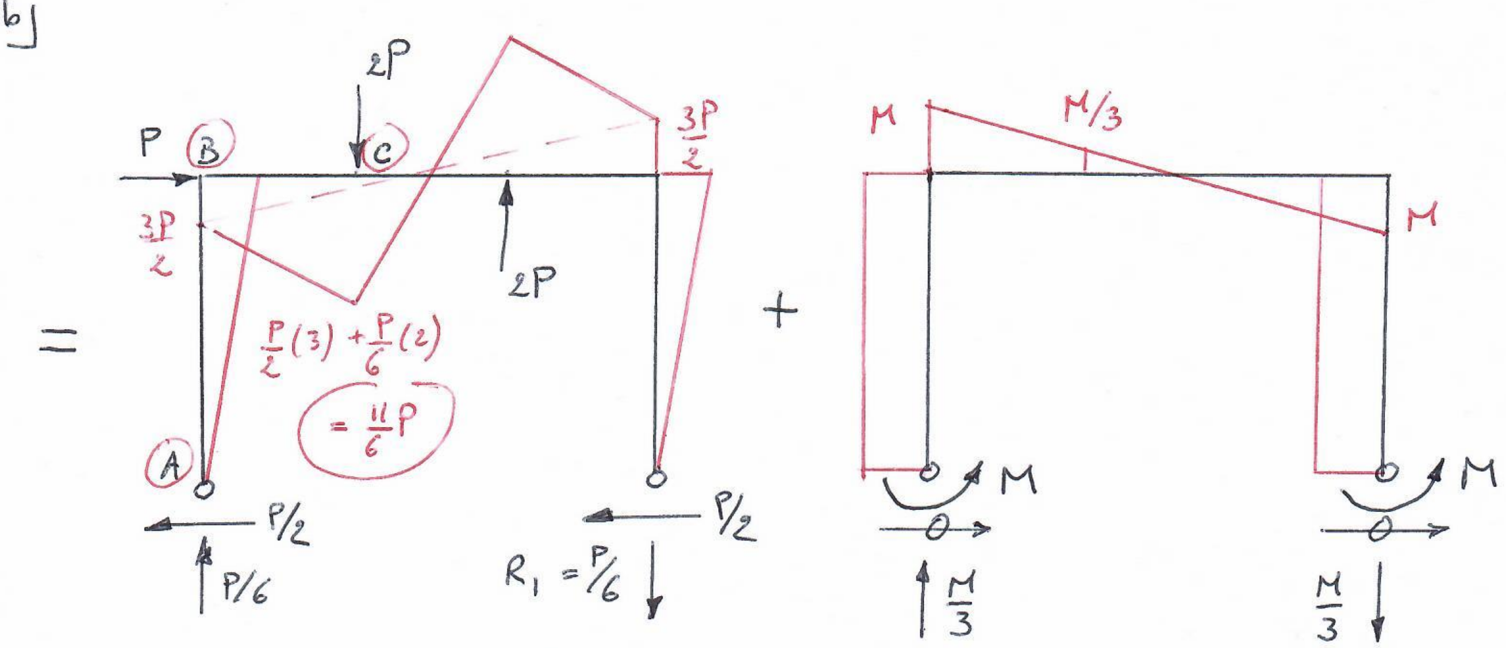
$M_A = 0$

$3P - (2P)(2) + 6R - 2M = 0$

$\rightarrow M = 3R - \frac{P}{2}$

3 → 1 redundancy left

b)



$M_{A} = -6R_1 + (2P)(2) - (3)P = 0$

$\rightarrow R_1 = P/6$

At C : $11\frac{P}{6} - \frac{M}{3} = M_p$

At A : $M = M_p$

$\frac{11P}{6} = \frac{4M_p}{3} \rightarrow P = \frac{8}{11} M_p$

Check B :

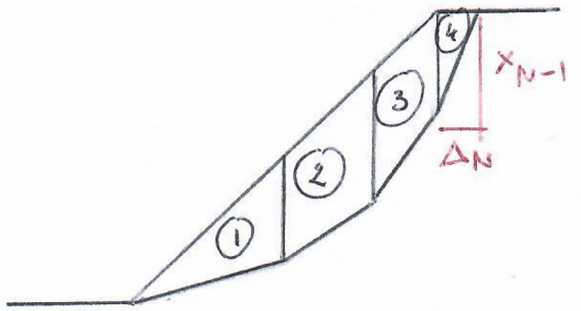
$\frac{3P}{2} - M = \frac{3}{2}\left(\frac{8}{11} M_p\right) - M_p = \left(\frac{12}{11} - 1\right) M_p = \frac{M_p}{11}$ ok

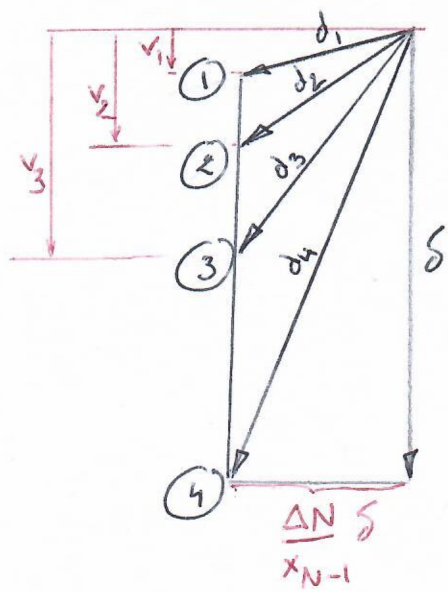
$P = 200$ kN $= \frac{8}{11} M_p \rightarrow M_p = 275$ kNm $= W_{pl} \cdot (275$ MPa$)$

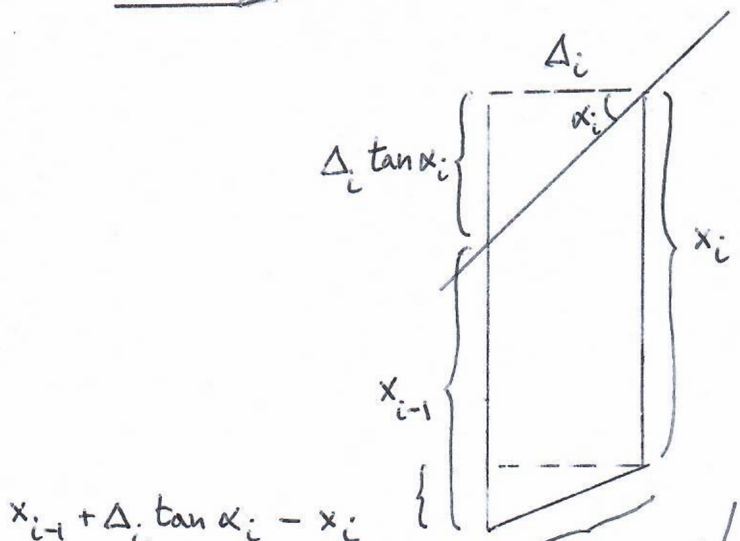
$\rightarrow W_{pl} = 10^6$ mm$^3 = 10^3$ cm^3

UB 457 × 152 × 52

6.







$$L_i = \sqrt{(x_{i-1} + \Delta_i \tan\alpha_i - x_i)^2 + \Delta_i^2}$$

$$\frac{d_i}{L_i} = \frac{\Delta_N / x_{N-1}\ \delta}{\Delta_i} \rightarrow d_i = L_i \frac{\Delta_N}{\Delta_i} \frac{\delta}{x_{N-1}}$$

(A) Energy dissipated along bottom slip surface:

$$E_A = k \sum L_i \cdot d_i = k\delta \frac{\Delta_N}{x_{N-1}} \sum_{i=1}^{N} \frac{(x_{i-1} - x_i + \Delta_i \tan\alpha_i)^2 + \Delta_i^2}{\Delta_i}$$

(B) Energy dissipated along vertical slip surfaces:

$$E_B = \sum_{i=1}^{N-1} x_i \, \| v_{i+1} - v_i \|$$

$$\frac{v_i}{\frac{\Delta_N}{x_{N-1}}\delta} = \frac{x_{i-1} - x_i + \Delta_i \tan\alpha_i}{\Delta_i}$$

$$E_B = k\delta \frac{\Delta_N}{x_{N-1}} \sum_{i=1}^{N} x_i \left\| \frac{x_i - x_{i+1} + \Delta_{i+1} \tan\alpha_{i+1}}{\Delta_{i+1}} - \frac{x_{i-1} - x_i + \Delta_i \tan\alpha_i}{\Delta_i} \right\|$$

Total: $E = E_A + E_B$

c) External Work:

$$P\delta + \sum_{i=1}^{N} \rho\left(\frac{x_i + x_{i-1}}{2}\right)\Delta_i \cdot v_i$$

$$= P\delta + \rho\delta\frac{\Delta_N}{2x_{N-1}} \sum_{i=1}^{N} (x_i + x_{i-1})(x_{i-1} - x_i + \Delta_i \tan\alpha_i)$$