EGT1
ENGINEERING TRIPOS PART IB

Monday 11 June $2018 \quad 2$ to 4.10

## Paper 4

## THERMOFLUID MECHANICS

Answer not more than four questions.
Answer not more than two questions from each section.
All questions carry the same number of marks.
The approximate number of marks allocated to each part of a question is indicated in the right margin.

Answers to questions in each section should be tied together and handed in separately.

Write your candidate number not your name on the cover sheet.

## STATIONERY REQUIREMENTS

Single-sided script paper

## SPECIAL REQUIREMENTS TO BE SUPPLIED FOR THIS EXAM

CUED approved calculator allowed
Engineering Data Book

## 10 minutes reading time is allowed for this paper at the start of the exam.

You may not start to read the questions printed on the subsequent pages of this question paper until instructed to do so.

## Version WRG/4

## SECTION A

Answer not more than two questions from this section

1 (a) In a gas-turbine power plant, air is compressed from 1 bar and 288 K to 16 bar, then heated to 1400 K in a combustor, and finally expanded through a turbine back to 1 bar. The turbomachinery flows are adiabatic, with isentropic efficiencies of $80 \%$ for the compressor and $90 \%$ for the turbine. The mass flow rate of fuel is negligible and there is no pressure drop across the combustor. Assuming that both the air and the combustion products behave as perfect gases with specific heat capacity $c_{p}=1.005 \mathrm{~kJ} \mathrm{~kg}^{-1} \mathrm{~K}^{-1}$ and specific-heat ratio $\gamma=1.4$, calculate:
(i) the specific work input required by the compressor;
(ii) the net specific work output from the plant;
(iii) the thermal efficiency of the plant.
(b) The compressor work can be reduced by intercooling. The compression process is split between two compressors, each with pressure ratio 4 , and with a heat exchanger after the first to return the air temperature to 288 K . Both compressors are adiabatic with $80 \%$ isentropic efficiency, pressure loss in the heat exchanger is negligible, and entry conditions to the turbine are unchanged. No use is made of the energy extracted from the air flow.
(i) Calculate the total specific work required for the two-stage compression.
(ii) Find the new thermal efficiency of the plant.
(iii) Assess the merits of intercooling. Also discuss how your assessment would change if the energy supply came from heat exchange with waste gases rather than combustion.
(c) The work needed for compression can be lowered further by increasing the number of compressor/heat-exchanger steps over which it is carried out. The theoretical limit of this progression is isothermal compression. In this case, both the ideal (reversible) and actual processes have the same start and end states. If the ideal compressor work is $80 \%$ of the actual requirement, what now is the net specific work output from the plant?

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2 A waste-heat recovery system uses a Rankine cycle operating between pressures of 10 kPa and 5 bar. A $1 \mathrm{~kg} \mathrm{~s}^{-1}$ flow of saturated water from the condenser is first pumped to the upper pressure, and then heated to $200^{\circ} \mathrm{C}$ in a boiler. The resulting steam expands through a turbine and exits back to the condenser. The pump and turbine flows are both adiabatic, and the turbine isentropic efficiency is $85 \%$. Losses in the pump and pressure drops in the boiler and condenser can be neglected.
(a) (i) Sketch the cycle on a $T-S$ diagram.
(ii) Calculate the heat input to, and net power output from, the cycle.
(b) The heat supplied in the boiler comes from a constant-pressure, counter-flowing stream of waste gas which enters at temperature $250^{\circ} \mathrm{C}$ and leaves at $150^{\circ} \mathrm{C}$. The heat rejection from the condenser is to an environment at $20^{\circ} \mathrm{C}$.
(i) If the waste gas is effectively perfect, with specific heat capacity $c_{p}=1 \mathrm{~kJ} \mathrm{~kg}^{-1} \mathrm{~K}^{-1}$, what is its mass flow rate?
(ii) What is the maximum power available from a steady-flow device with the same waste-gas input and output streams, and exchanging heat with an environment at the same temperature (i.e. $20^{\circ} \mathrm{C}$ )?
(c) (i) What is the maximum power available from a steady-flow process whose input is the steam leaving the boiler, whose output is the water leaving the pump, and which exchanges heat with an environment at $20^{\circ} \mathrm{C}$ ?
(ii) Compare your answer to (i) with the power outputs calculated in parts (a)(ii) and (b)(ii), and identify the causes of the differences.

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3 The $T-Q$ plot for a counter-flow heat exchanger is shown in Fig. 1. Hot water enters at $60^{\circ} \mathrm{C}$ and leaves having been cooled to $55^{\circ} \mathrm{C}$. The other stream is air, entering at $15^{\circ} \mathrm{C}$ and exiting at $40^{\circ} \mathrm{C}$. The heat-transfer rate is 2 kW .
(a) The heat-transfer rate, $\dot{Q}$, is linked to the log-mean temperature difference, $\Delta T_{m}$, via the relation

$$
\dot{Q}=U A \Delta T_{m}
$$

(i) What do the symbols $U$ and $A$ in this formula represent?
(ii) Evaluate the product $U A$.
(b) What are the mass flow rates of the water and air streams?
(c) It is proposed to increase the heat-transfer rate by increasing $U A$, without altering the mass flow rates or the inlet temperatures. Explain why there is a limit to the heat transfer that can be achieved, and find the corresponding exit temperatures.
(d) An alternative to part (c) envisages increasing the air mass flow rate without altering $U A$ from its original value. The inlet temperatures and water mass flow rate are also to remain unchanged.
(i) If the new water exit temperature is $54^{\circ} \mathrm{C}$, confirm that the air exit temperature must be $29.4^{\circ} \mathrm{C}$ and find its required mass flow rate.
(ii) Explain why there is a lower bound on the water exit temperature achievable with this approach.

Data: take the specific heat capacity of water as $4.184 \mathrm{~kJ} \mathrm{~kg}^{-1} \mathrm{~K}^{-1}$.

Version WRG/4


Fig. 1

## Version WRG/4

## SECTION B

Answer not more than two questions from this section

4 (a) The smooth contraction shown in Fig. 2(a) connects pipes with cross-sectional areas $A_{1}$ and $A_{2}$. An incompressible fluid of density $\rho$ flows steadily through the contraction in the direction shown. The volumetric flow rate is $Q$ and the gauge stagnation pressure is $p_{s}$. Viscous and gravitational effects are negligible. What are the gauge static pressures at entry to, and exit from, the contraction?
(b) The couplings between the contraction and the pipes are not load bearing, so the force required to hold the contraction stationary is entirely provided by its support bracket (not shown).
(i) Derive an expression for the force applied by the bracket to the contraction.
(ii) For what value of $p_{s}$ is your expression from part (i) equal to zero?
(iii) In what direction is the bracket force if $p_{s}$ is greater than the value from part (ii)?
(c) The smooth contraction is now replaced by the abrupt area reduction shown in Fig. 2(b). The stagnation-pressure loss due to this component is $k$ times the upstream dynamic pressure.
(i) If the flow rate is unchanged, and the upstream stagnation pressure is equal to the value found in part (b)(ii), what is the magnitude and direction of the supportbracket force?
(ii) Explain, with supporting sketches, why there is a stagnation-pressure loss in this case.


Fig. 2

## Version WRG/4

5 (a) The velocity profile in steady, parallel-streamline flow of an incompressible fluid between fixed plates at $y= \pm h$ is given by

$$
u(y)=\frac{1}{2 \mu}\left(-\frac{\mathrm{d} p}{\mathrm{~d} x}\right)\left(h^{2}-y^{2}\right)
$$

where $\mu$ is the fluid's dynamic viscosity, $\mathrm{d} p / \mathrm{d} x$ is the pressure gradient in the flow direction, and gravitional effects have been assumed negligible. Describe, step-by-step, the theoretical reasoning in the derivation of this expression from the general controlvolume momentum equation.
(b) Figure 3 shows a two-dimensional flow in which a piston pushes fluid from a reservoir of height $2 H$ through a narrow gap of length $L$ and height $2 h(\ll 2 H)$. The dynamic viscosity of the fluid is $\mu$. The piston velocity is $U_{p}$, and the reservoir gauge pressure is $p_{r}$. The gap flow exhausts to atmosphere. Assuming that the pressure at gap entry is equal to $p_{r}$, and that end effects are negligible, find the relation between $p_{r}$ and $U_{p}$.
(c) In reality, dynamic effects in the reservoir will influence the gap-entry pressure. A simplified model considers the entry flow as having two distinct stages: first, an effectively steady and inviscid acceleration from velocity $U_{p}$ in the main body of the reservoir to parallel-streamline flow at the gap entry; and second, a transition to fully-developed viscous flow which takes place over a negligible distance and with negligible pressure drop. Find the new relation between $p_{r}$ and $U_{p}$ implied by the model, and hence identify a dimensionless parameter which must be small for the approximation of part (b) to be valid. You should define any additional symbols that you introduce.


Fig. 3

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6 (a) Water is to be pumped between two large tanks at a rate of $0.01 \mathrm{~m}^{3} \mathrm{~s}^{-1}$. The flow leaves the supply tank without losses, passes through the pump, and then travels 50 m along a pipe of diameter 0.1 m to the delivery tank. The water level in each tank is the same, and the skin-friction coefficient of the pipe flow is 0.008 . If the pump efficiency is $100 \%$, what is its power requirement?
(b) A second, identical, delivery tank is now connected to the pump, via a valve and a pipe identical to that of part (a). The new system is shown schematically, in plan view, in Fig. 4. The flow at the junction J is loss-free. The valve introduces a stagnationpressure loss $\frac{1}{2} \rho V^{2} K$, where $\rho$ is the water density, $V$ the mean flow speed in the pipe, and $K$ a positive constant. The pump and valve settings are adjusted so the flow rate to the first delivery tank remains unchanged at $0.01 \mathrm{~m}^{3} \mathrm{~s}^{-1}$, and that to the second tank is $0.005 \mathrm{~m}^{3} \mathrm{~s}^{-1}$. Find the value of $K$. (You may assume that the skin-friction coefficient in the pipe flows is independent of Reynolds number.)
(c) The pump power is now reduced to the value you determined in part (a). Find the new delivery-tank flow rates.

Delivery tank 1
Supply tank


Delivery tank 2

Fig. 4

## END OF PAPER

