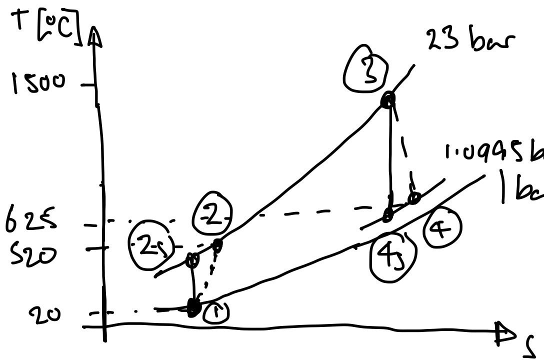
```
% Q1)

%
% Combined Cycle Gas-Turbine/Steam Rankine
%

% Question data
W_x_GT = 400;           % [MW]     - GT shaft power ouput
eta_T = 0.85;           % [-]
eta_C = 0.85;           % [-]

gamma = 1.4;            % [-]
cp = 1.01;              % [kJ/kg]

p_1 = 1e5;              % [Pa]     - compressor inlet
T_1 = 20 + 273.15;      % [K]      - compressor inlet
Pr = 23;                % [-]      - compressor pressure ratio
p_2 = p_1*Pr;

T_3 = 1500 + 273.15;    % [K]      - combustor exit
p_3 = p_2;              % [K]      - neglibible pressure drop in combustor
p_4 = 1.0995e5;         % [Pa]     - exit pressure

%
% (a) (i)
%
```




```
% Consider an isentropic compressor first
R = (p_2/p_1)^((gamma-1)/gamma)
```

R = 2.449

```
T_2s = T_1*R
```

T_2s = 718.1

```
% Now include efficiency to get actual compressor delivery temperature
T_2 = T_1 + 1/eta_C*(T_2s - T_1)
```

T_2 = 793.03

```
T_2/T_1
```

ans = 2.705

```
T_2_deg = T_2 - 273.15
```

T_2_deg = 520 deg. C

```
% Now conisder an ideal turbine expansion
T_4s = T_3*(p_4/p_3)^((gamma-1)/gamma);

% Now conisder the real turbine expansion
T_4 = T_3 - eta_T*(T_3 - T_4s);
T_4_degC = T_4 - 273.15
```

T_4_degC = 625 deg. C

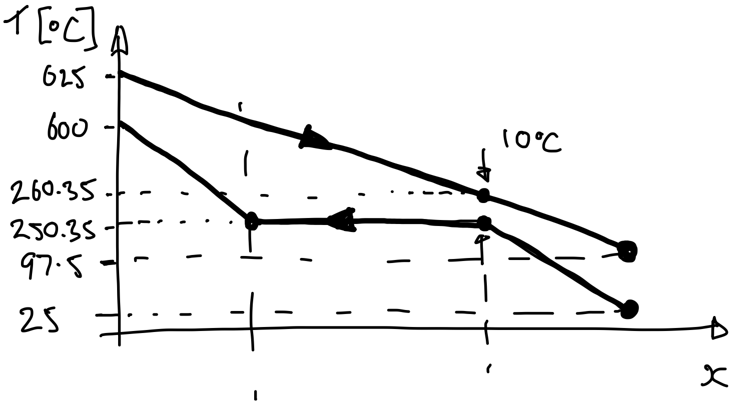
```
%
% (a) (ii)
%

% balance the shaft power between the turbine, the compressor and the
% output
m_dot_air = W_x_GT*1e6/(cp*1e3*(T_3-T_4-T_2+T_1))
```

```
m_dot_air = 1.0559e+03 kg/s
```

```
% heat input across combustor
Q_in = m_dot_air*cp*1e3*(T_3 - T_2)/1e6
```

```
Q_in = 1.0453e+03 MW
```

```
%
% (b) (i)
%
```




```
%
% (b) (ii)
%
% 40 bar steam has saturation temperature of 250.35 deg. C
% (Databook page 21)
T_p = 250.35 + 10;

% 40 bar, 600 deg. C superheat
% (Databook page 23)
h_out = 3674.9;

% assume pinch point is at 40 bar sat. liquid % (Databook page 21)
h_p = 1087.5;

% enthalpy balance across HSRG from air inlet to pinch point
% to get mass flow rate of water
m_dot_H20 = m_dot_air * cp*1e3 * (T_4_degC - T_p) / ((h_out - h_p)*1e3)
```

```
m_dot_H20 = 150.3 kg/s
```

```
% now consider the full HSRG to get air outlet temperature
% water inlet is sub-cooled liquid at 25 deg. C, 40 bar
% (Databook page 23)
h_in = 108.5;
T_out = T_4_degC - m_dot_H20 / (m_dot_air * cp*1e3) * ((h_out - h_in)*1e3)
```

```
T_out = 122.4 deg. C
```

```
% NB pinch point assumption OK as 97.5 - 25 deg. C > 10 deg. C

%
% (b) (iii)
%

% now consider the isentropic turbine expansion from 40 bar, 600 deg. to
% the condenser
% (Databook page 23)
s_exit = 7.371;
s_f_exit = 0.422;
```

```
s_g_exit = 8.473;
h_f_exit = 121.4;
h_fg_exit = 2432.3;

% compute dryness fraction at exit
x_exit = (s_exit - s_f_exit)/(s_g_exit - s_f_exit);

% compute exit enthalpy
h_exit = h_f_exit + x_exit * h_fg_exit
```

```
h_exit = 2.2208e+03
```

```
% compute the output power
W_x_Rankine = m_dot_H20 * (h_out - h_exit)/1e3
```

```
W_x_Rankine = 218.6 MW
```

```
%
% (c) (i)
%

% add the GT and Rankine power output and compare to the GT heat input to
% get efficiency
eta_CC = (W_x_Rankine + W_x_GT)/Q_in * 100
```

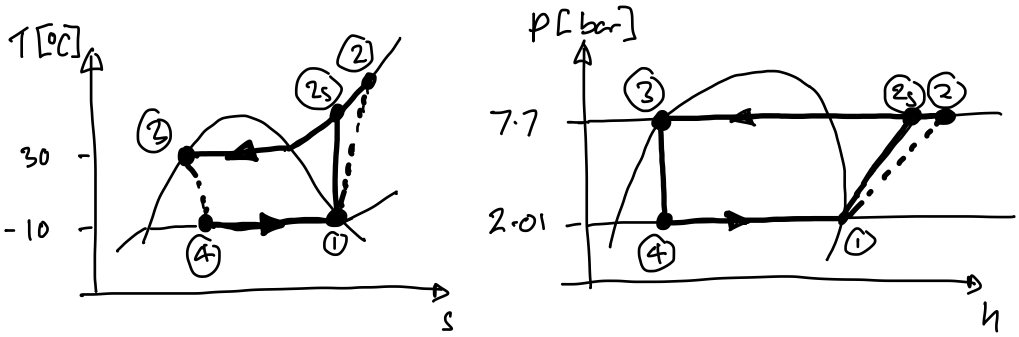
```
eta_CC = 59.2 %
```

```
%
% (c) (ii)
%

% The 1056 kg/s of air at 122.4 deg. C has huge power potential.
% Given the potentially high future cost of stored H2, it is perhaps
% reasonable to expect another, possibly organic fluid Rankine cycle at exit to recover
% more power. Also, adding re-heat, using a higher boiler pressure and using a
% lower condenser pressure are all valid answers.
%
% Additionally (not in the notes) it may become ecomincal to add a (perhaps) solid oxide fuel
% cell to the top of the cycle providing in-direct heating to the gas-turbine.
```

```
%
% Q2)
%

% R134a
% Databook page 37

% Saturation line at -10 deg. C is 2.01 bar
p_1_4 = 2.01;

% pressure ratio of 3.831 gives
p_2_3 = 2.01*3.831
```

```
p_2_3 = 7.7003
```

```
%
% (a)(i)
%
```




```
%
% (a)(ii)
%

%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
%                                                   %
%                       - 1 -                       %
%                                                   %
% vapour compressor inlet, sat. vapour at 2.01 bar  %
%                                                   %
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%

T_1 = -10;          % [deg. C]      - saturation temp.
h_1 = 392.7;        % [kJ/kg]       - h_g, saturated vapour
s_1 = 1.7331;       % [kJ/kg/K]     - s_g, saturated vapour

%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
%                                                   %
%                       - 2 -                       %
%                                                   %
%  vapour compressor outlet, superheated @ 7.7 bar  %
%                                                   %
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%

% Saturation line @ 7.7 bar is 30 deg. C
T_2_sat = 30;         % [deg. C]
s_2 = 1.7331;       % [kJ/kg/K]     - isentropic to - 1 -

% look at 20 K superheat @ 7.7 bar
s_plus_20 = 1.7803; % [kJ/kg/K]     - +20 K superheat
s_g = 1.7148;       % [kJ/kg/K]     - sat. vapour at 7.7 bar

h_plus_20 = 435.4;  % [kJ/kg]     - +20 K superheat
h_g = 414.8;        % [kJ/kg]     - sat. vapour at 7.7 bar

% interpolate to get actual superheat
```

```
DT_superheat = 20*(s_2 - s_g)/(s_plus_20 - s_g)
```

```
DT_superheat = 5.59 deg. C
```

```
T_2 = T_2_sat + DT_superheat
```

```
T_2 = 35.6 deg. C
```

```
h_2 = h_g + (h_plus_20 - h_g)*DT_superheat/20
```

```
h_2 = 420.5554
```

```
%
% ideal compressor specific work
%
w_comp_ideal = h_2 - h_1
```

```
w_comp_ideal = 27.9 kJ/kg
```

```
%
% 20 K superheat compressor specific work
%
w_comp_real = h_plus_20 - h_1
```

```
w_comp_real = 42.7 kJ/kg
```

```
%
% real compressor efficiency
%
eta_comp = w_comp_ideal/w_comp_real
```

```
eta_comp = 0.65
```

```
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
%                                                   %
%                       - 3 -                       %
%                                                   %
%     condenser outlet, saturated fluid @ 7.7 bar   %
%                                                   %
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%

h_3 = 241.7;        % [kJ/kg]       - h_f, saturated liquid

%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
%                                                   %
%                       - 4 -                       %
%                                                   %
%     throttle outlet, isenthalpic to - 3 -         %
%             saturated at 2.01 bar                 %
%                                                   %
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%

h_4 = h_3;
h_4_f = 186.7;      % [kJ/kg]       - h_f, saturated liquid
h_4_g = 392.7;      % [kJ/kg]       - h_f, saturated vapour

x_4 = (h_4 - h_4_f )/(h_4_g - h_4_f)
```

```
x_4 = 0.2670
```

```
q_condenser = h_plus_20 - h_3
```

```
q_condenser = 193.7  kJ/kg
```

```
q_condenser_comp_ideal = h_2 - h_3
```

```
q_condenser_comp_ideal = 178.86 kJ/kg
```

```
q_evaporator = h_1 - h_4
```

```
q_evaporator = 151. kJ/kg
```

```
%
% (b)
%
T_H = T_2_sat + 273.15
```

```
T_H = 303.1500
```

```
T_C = T_1 + 273.15
```

```
T_C = 263.15
```

```
CoP_Carnot_HP = T_H / (T_H - T_C)
```

```
CoP_Carnot_HP = 7.58
```

```
CoP_HP = q_condenser / w_comp_real
```

```
CoP_HP = 4.54
```

```
CoP_HP_reversible = q_condenser_comp_ideal / w_comp_ideal
```

```
CoP_HP_reversible = 6.42
```

```
Carnot_fraction = CoP_HP/CoP_Carnot_HP*100
```

```
Carnot_fraction = 59.9 %
```

```
Carnot_fraction_reversible = CoP_HP_reversible/CoP_Carnot_HP*100
```

```
Carnot_fraction_reversible = 84.7 %
```

```
%
% (c)
%

T_H_max = 20+273.15;
T_C_max = 10+273.15;

CoP_Carnot_HP_max = T_H_max / (T_H_max  - T_C_max)
```

```
CoP_Carnot_HP_max = 29.3
```

```
Maximum_fraction = CoP_HP/CoP_Carnot_HP_max*100
```

```
Maximum_fraction = 15.5 %
```

```
% The heat pump only achieves 15.5% of the theoretical limit.
%
% Achieving this theoretical performance implies infinite area heat
```

```
% exchangers which are obviously impractical and expensive to approximate.
%
% However, there is significant potential for improvement over and above the simple
% improvements in the vapour compressor efficiency.
%
% The practical trade-off of heat exchanger size will depend on the price of carbon
% neutral electricity, the regulatory framework around the burning of natural gas and the
% potential roll-out of piped hydrogen in different regions of the world.
```

```
%
% Q3
%

%
% Question data
%
P = 75e3;                       % [W]
T_wall = 40;                    % [deg. C]
T_infty = -40;                  % [deg. C]
T_in_coolant = 120;             % [deg. C]
T_out_coolant = 80;             % [deg. C]

D_tube = 2e-3;                  % [m]
N_tube = 150;                   % [-]

cp_coolant = 2142.8;            % [J /kg /K]
lambda_coolant = 0.18;          % [W /m /K]
mu_coolant = 9e-3;              % [Pa s]

%
% (a) (i)
%

% Overall thermal balance to find the mass flow rate of
% coolant needed to drop the power across the given
% temperature difference

m_dot_tube = P / ( cp_coolant * (T_in_coolant - T_out_coolant))/N_tube
```

```
m_dot_tube = 0.0058
```

```
m_dot_coolant = m_dot_tube*N_tube
```

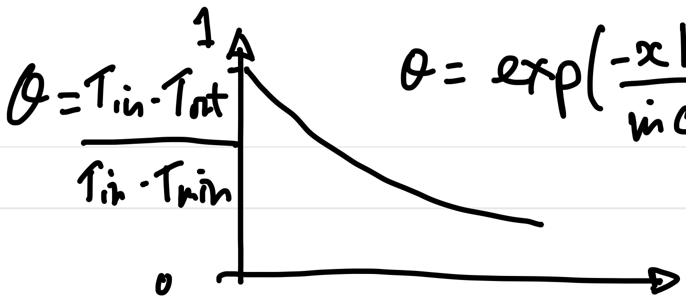
```
m_dot_coolant = 0.875 kg/s
```

```
%
% (a) (ii)
%

%
% Reynolds No.
%
Re_int_d = 4 * m_dot_tube / ( mu_coolant * pi * D_tube )
```

```
Re_int_d = 412.64
```

```
% -> Re_int_d < 2300 so laminar flow

% Thermofluids Databook for laminar flow in tube with constant wall
% temperature.
%
% NB the derivation is beyond scope of course and needs eigenvalue solution
% of differential equation.

Nu_d = 3.66;

h = Nu_d*(lambda_coolant)/D_tube
```

```
h = 329.4 W /m^2 /K
```



$$\theta = \exp\left(\frac{-x h \pi D}{\dot{m} c_p}\right)$$

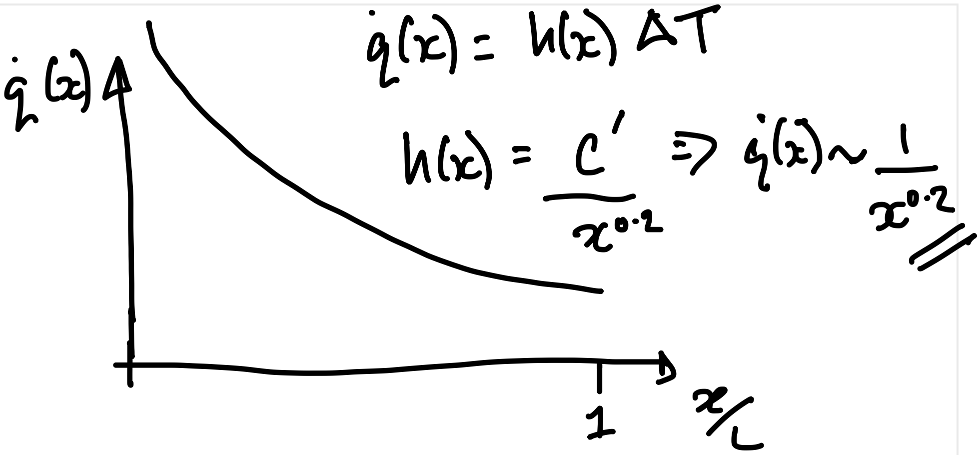
```
% Check Prandtl No.

Pr_coolant = mu_coolant*cp_coolant/lambda_coolant
```

Pr_coolant = 107.14

```
% -> Pr_coolant > 0.1 so OK

%
% (a) (iii)
%

% constant htc, so we can integrate along the tubes to get

theta = (T_in_coolant - T_out_coolant)/(T_in_coolant - T_wall);

W = (m_dot_tube) * cp_coolant / (h * pi * D_tube) * - log(theta)
```

W = 4.19 m

```
%
% b(i)
%
```




```
%
% b(ii)
%

% Question data
U_inf_air = 100;            % [m/s]

% Databook transport properites for air at 0 deg. C
Pr_air = 0.72;
mu_air = 1.7e-5;
lambda_air = 0.024;

rho_air = 0.3e5/(287.1*(273+1/2*(T_infty+T_wall)));

%
%  bring all the correlation constants together
%
% h(x) = C_dash * x^-0.2
%
C_dash = 0.0296 * (rho_air * U_inf_air / mu_air)^0.8 * lambda_air * Pr_air^(0.4)
```

C_dash = 75.2345

```

%
% integrate to get average h for length L,
%
% h_average_L = 1/L * integral from 0 to L (C_dash * x^-0.2) dx
% -> h_average_L = C_dash / 0.8 * L^-0.2 = h(L)/0.8

DT = (T_wall - T_infty);
%
%      P = ( W * L ) * h_average_L * DT
% =>   P = ( W * L ) * C_dash / 0.8 * L^-0.2 * DT
%
L = ( P * 0.8 / (C_dash * W * DT) )^(1/0.8)
```

```
L = 2.96 m
```

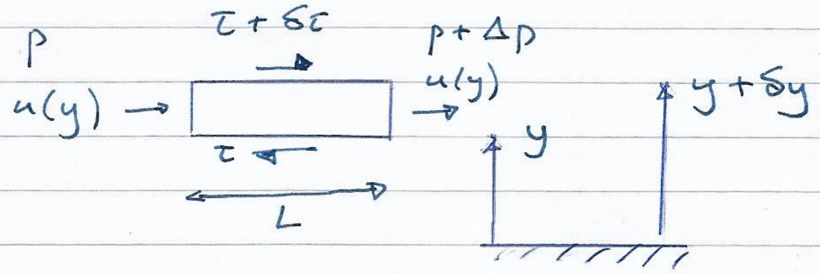
```
%
% (c)
%

% Reynolds' analogy suggests that c_f = 0.5*(Nusselt/(Re*Pr)), showing that we need skin
% friction to get heat transfer. We've already paid for the skin friction on the cowling
% so it makes sense to use it for auxiallry heat rejection as well :-).
%
% An additional heat exchanger will require the same order of aditional skin friction to
% acheive the heat transfer. Efforts to reduce the skin friction
% inside the heat exchanger by reducing the internal velocity will cost the propulsor in
% terms of weight, likely additional external area (and so even more skin friction) and
% potentially increased profile drag.
%
% However, the ducted heat exchanger would simplify complex issues like cost,%
manufacturing, maintenance and certification issues.
%
```

a) Fully-developed $\rightarrow$ velocity field steady and independent of x

Hence via conservation of volume for incompressible flow, vertical velocity is zero. (Strictly, $\partial v/\partial y = 0$ and $v = 0$ on $y = 0$.)

$\therefore$ Newton's law of viscosity applies: $\tau = \mu \frac{du}{dy}$

Control-volume momentum eq'n applied to infinitesimal element:



$$L\,\delta\tau - \Delta p\,\delta y = 0, \quad \text{with } \delta\tau = \frac{d\tau}{dy}\delta y$$

$$\frac{d\tau}{dy} = \frac{\Delta p}{L} \qquad (1)$$

For zero pressure gradient, integrates to $\tau = \tau_0$, a const.

Integrating Newton's law of viscosity $\quad u = \frac{\tau_0}{\mu} y + u_0$

$u = 0$ @ $y = 0$; $u = U$ @ $y = h$ $\Rightarrow$ $\quad u = U\frac{y}{h}$

b) (i) Due to the presence of the retaining wall, there can be no net volume flow in the horizontal direction. Therefore the plate-induced flow of part (a) must be balanced by an opposing stream. A <u>positive</u> pressure gradient is required to drive this stream.

(ii) In eq (1), we now have $\Delta p = L \frac{dp}{dx}$, so

$$\mu \frac{d^2 u}{dy^2} = \frac{dp}{dx}$$

$$\Rightarrow \quad u = \frac{1}{\mu}\frac{dp}{dx}\frac{y^2}{2} + Ay + B$$

$u = 0$ @ $y = 0 \Rightarrow B = 0$

$u = U$ @ $y = h \Rightarrow A = \frac{U}{h} - \frac{1}{\mu}\frac{dp}{dx}\frac{h}{2}$

so $$u = \frac{U}{h}y - \frac{1}{2\mu}\frac{dp}{dx}(hy - y^2)$$

(iii) Now $\int_0^h u\,dy = 0$ for zero net horizontal volume flux

$$\frac{Uh}{2} = \frac{1}{2\mu}\frac{dp}{dx}\left[\frac{h^3}{2} - \frac{h^3}{3}\right] = \frac{1}{2\mu}\frac{dp}{dx}\cdot\frac{h^3}{6}$$

$$\frac{dp}{dx} = 6\mu U / h^2$$

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Question number 4 Sheet number 3

C) cont'd

(i) $\frac{dp}{dx}$ is a fn of ρ, μ, h, U

4 independent variables, 3 dimensions → 1 dimensionless group

Non-dimensionalisation of dependent parameter:

$p \sim \rho U^2$, $x \sim h$ ⇒ $\frac{h}{\rho U^2}\frac{dp}{dx}$ is dimensionless

∴ $\frac{h}{\rho U^2}\frac{dp}{dx}$ is a function of $\frac{\rho U h}{\mu}$, the Reynolds no.

∴ $\frac{h^2}{\mu U}\frac{dp}{dx}$ is a function of $\rho\frac{Uh}{\mu}$

(iv) From (ii), $\frac{h}{\rho U^2}\frac{dp}{dx} = \frac{6\mu}{\rho U h} = 6\left(\frac{\rho U h}{\mu}\right)^{-1}$

- Consistent with the expression in (iii), but ...
- (ii) can also be written $\frac{h^2}{\mu U}\frac{dp}{dx} = 6$

alternative dimensionless form for dependent variable

With this form, the dimensional analysis has

$\frac{h^2}{\mu U}\frac{dp}{dx}$ is a function of $\rho\frac{Uh}{\mu}$; exact solution shows

that this function takes a very simple form (a constant).

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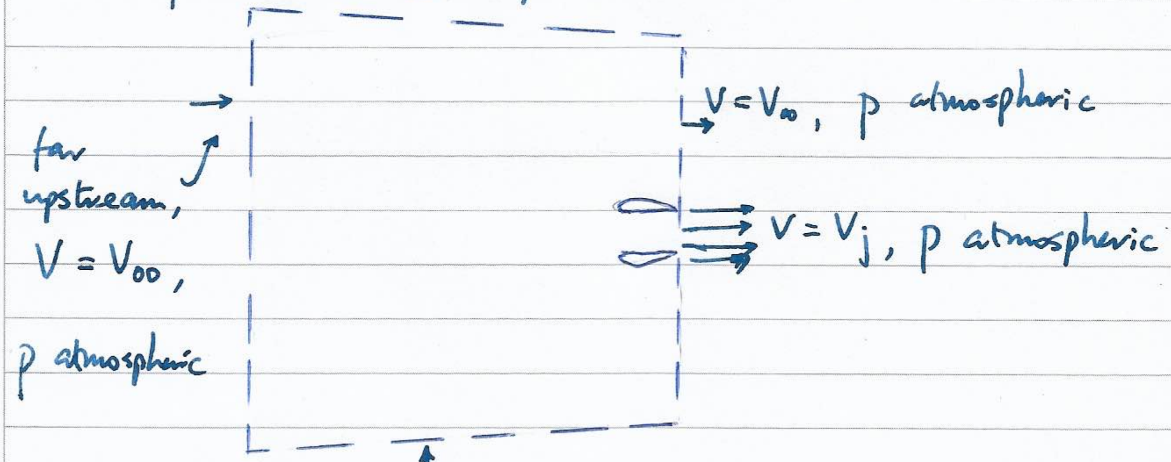
Question number 5 Sheet number 1

a) (i) Parallel s/lines → pressure uniform ... and since external-flow streamlines are also taken to be uniform, equal to atmospheric.

(ii) Choose a CV where we know velocities, and pressure is atmospheric:



distant streamline, p atmospheric

N.B. Also possible to take top and bottom parallel to V_{∞}, with incoming mass flux $\rho A_j (V_j - V_{\infty})$

(iii) Net force on fluid in control vol is $T \rightarrow$

SFME : $T = \dot{m} \Delta V$

$\dot{m}$ consists of ... external component $\dot{m}_e$, with $\Delta V = 0$
plus internal component $\rho A_j V_j$ with $\Delta V = V_j - V_{\infty}$

hence $\underline{\underline{T = \rho A_j V_j (V_j - V_{\infty})}}$

b) (i) Flow in duct is steady and inviscid
$\Rightarrow$ Bernoulli's eqn applies either side of fan.

Thus, with p_a as atmospheric pressure ...

$$\left(\tfrac{1}{2}\rho V_j^2 +\right) p_f + \Delta p = p_a \left(+ \tfrac{1}{2}\rho V_j^2\right)$$

$$p_f + \tfrac{1}{2}\rho V_j^2 = p_a + \tfrac{1}{2}\rho V_\infty^2$$

$$\Rightarrow \quad \Delta p - \tfrac{1}{2}\rho V_j^2 = -\tfrac{1}{2}\rho V_\infty^2$$

$$\Delta p = \underline{\underline{\tfrac{1}{2}\rho}} \left(V_j^2 - V_\infty^2\right)$$

(ii) Power required $= Q\Delta p_0$, with Q the vol. flux
p_0 stag'n pressure

Here we have $\Delta p_0 = \Delta p = \frac{1}{2}\rho(V_j^2 - V_\infty^2)$

and $Q = A_j V_j$

so required power: $\underline{\underline{\frac{1}{2}\rho A_j V_j \left(V_j^2 - V_\infty^2\right)}}$

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Question number 5 Sheet number 3

c) (i) Constant thrust, so $\rho A_j V_j (V_j - V_\infty) = \rho A_j V_{j0}^2$

$$V_j^2 - V_\infty V_j - V_{j0}^2 = 0$$

$$V_j = \frac{V_\infty}{2} \pm \underbrace{\sqrt{\frac{V_\infty^2}{4} + V_{j0}^2}}_{> V_\infty/2}$$

Hence the appropriate root is $V_j = \frac{V_\infty}{2} + \sqrt{\frac{V_\infty^2}{4} + V_{j0}^2}$

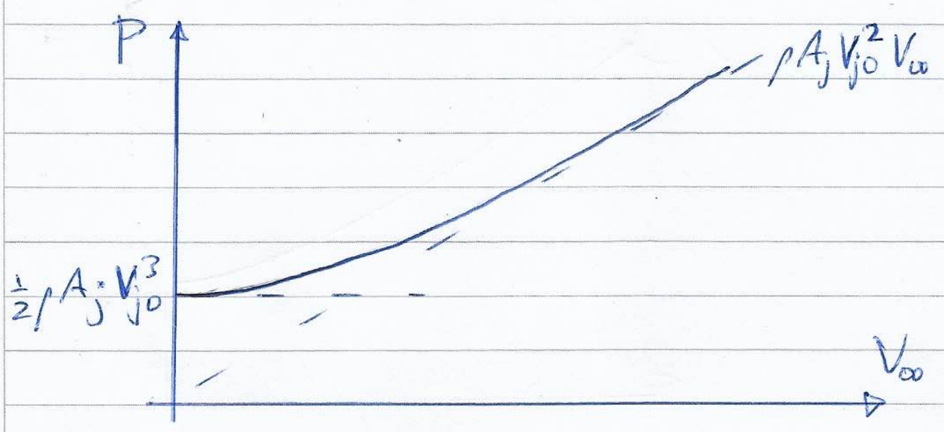
(ii) Req'd power $P = \frac{1}{2}\rho A_j V_j (V_j^2 - V_\infty^2)$

$$= \frac{V_j + V_\infty}{2} \rho A_j V_j (V_j - V_\infty)$$

$$= \frac{V_j + V_\infty}{2} \rho A_j V_{j0}^2$$

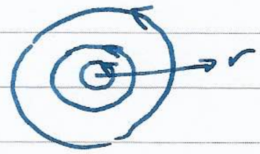
For $V_\infty \ll V_{j0}$, $V_j \simeq V_{j0}$ and $P \simeq \frac{1}{2}\rho A_j V_{j0}^3$, constant

For $V_\infty \gg V_{j0}$, $V_j \simeq V_\infty$ and $P \simeq \rho A_j V_{j0}^2 V_\infty$, $\propto V_\infty$



a) • Consider a horizontal plane at some height.

Plan view of streamlines:

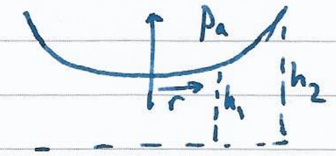


Hence (via streamline-normal equation), pressure must increase with r.

• In the vertical direction, pressure varies hydrostatically.

• At the liquid surface, pressure is constant.

Hence, to achieve radial pressure increase, surface looks like



$p_1 = \rho g h_1 + p_a$ $\quad$ $p_2 = \rho g h_2 + p_a$

$p_2 > p_1 \Rightarrow h_2 > h_1$.

b) Let the depth of liquid be $h(r)$, so (gauge) pressure at base is $\rho g h(r)$

Streamline/normal eqn: $\dfrac{dp}{dr} = \rho \dfrac{V^2}{r} = \rho \Omega^2 r$ (rigid-body rotation)

Substitute for p: $\rho g \dfrac{dh}{dr} = \rho \Omega^2 r$;

$$h = \frac{\Omega^2 r^2}{2g} + A$$

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Question number 6 Sheet number 2

To find A, apply conservation of volume:

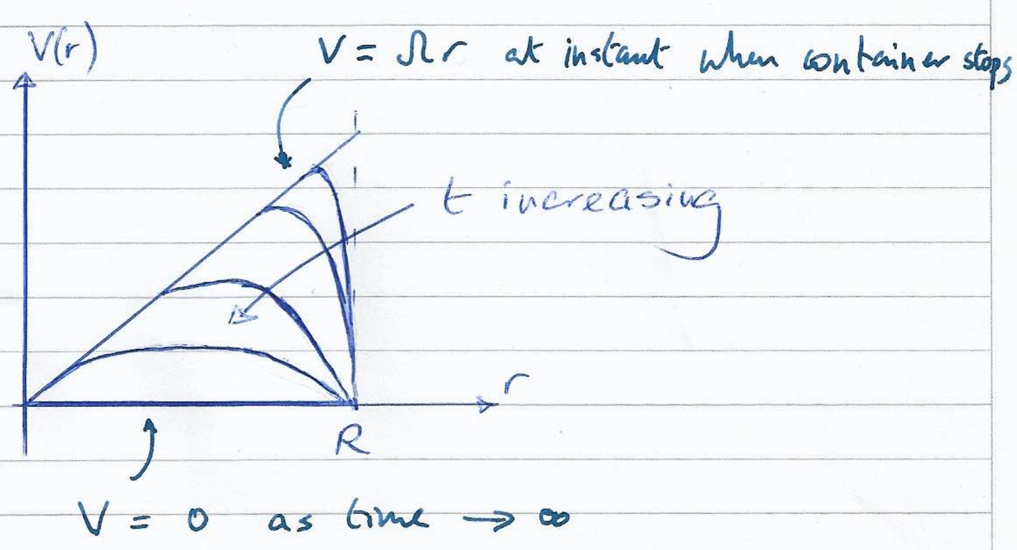
$$\int_0^R h(r).\,2\pi r\,dr = \pi R^2 h_0$$

$$\frac{\pi\Omega^2}{g}\int_0^R r^3\,dr + \pi R^2 A = \pi R^2 h_0$$

$$A = h_0 - \frac{1}{\pi R^2}\frac{\pi\Omega^2}{g}\frac{R^4}{4} = h_0 - \frac{\Omega^2 R^2}{4g}$$

and thus $$h = h_0 + \frac{\Omega^2}{4g}(2r^2 - R^2)$$

c) (i) Far away from base and surface, expect velocity field to remain azimuthal; effect of no-slip condition at container wall will 'diffuse' inwards via action of viscosity:



b) cont'd

(ii) Units: $d : L$ $\nu : L^2 T^{-1}$

so $\tau = \frac{d^2}{\nu}$ has dimensions of time, and is thus the required scale.

(iii) Rotation time T_R is of order $\frac{2\pi}{\Omega}$

For many rotations before fluid comes to rest (which occurs over the viscous time-scale*)

$$\tau \gg T_R$$

*with $d = R$ here: $\frac{R^2}{\nu} \gg \frac{2\pi}{\Omega}$

$$\frac{\Omega R^2}{2\pi \nu} \gg 1 \quad \left(\text{or } \frac{\Omega R^2}{\nu} \gg 1\right)$$

The quantity on the LHS is a Reynolds number, since ΩR is a velocity.