EGT1
ENGINEERING TRIPOS PART IB

Friday 10 June 2022   9.00 to 11.10

Paper 4

THERMOFLUID MECHANICS

Answer not more than ***four*** *questions.*

Answer not more than **two** *questions from each section.*

All questions carry the same number of marks.

The ***approximate*** *number of marks allocated to each part of a question is indicated in the right margin.*

Answers to questions in each section should be tied together and handed in separately.

Write your candidate number ***not*** *your name on the cover sheet.*

STATIONERY REQUIREMENTS
Single-sided script paper

SPECIAL REQUIREMENTS TO BE SUPPLIED FOR THIS EXAM
CUED approved calculator allowed
Engineering Data Book

10 minutes reading time is allowed for this paper at the start of the exam.

You may not start to read the questions printed on the subsequent pages of this question paper until instructed to do so.

You may not remove any stationery from the Examination Room.

SECTION A

*Answer not more than **two** questions from this section*

1 A power station uses a combined gas turbine - steam turbine cycle. The exhaust of the gas turbine is used to heat the Rankine steam cycle via a Heat Recovery Steam Generator (HRSG).

(a) The gas turbine uses air, which can be assumed to have specific heat ratio $\gamma = 1.4$, and specific heat at constant pressure $c_p = 1.01\,\mathrm{kJ\,kg^{-1}\,K^{-1}}$ throughout. The compressor inlet is $1\,\mathrm{bar}$, $20\,°\mathrm{C}$ with pressure ratio 23. The combustor has negligible pressure drop and an exit temperature of $1500\,°\mathrm{C}$. The turbine exhaust is at 1.0995 bar. The compressor and turbine have isentropic efficiency of 85%. Neglect the fuel mass and kinetic energy of the air.

(i) Sketch the T-s diagram, calculate the compressor delivery temperature and show that the turbine exit temperature is $625\,°\mathrm{C}$. [8]

(ii) For a gas turbine shaft power of $400\,\mathrm{MW}$, calculate the mass flow rate of air and the heat input. [4]

(b) The Rankine cycle side of the HRSG has a pressure of $40\,\mathrm{bar}$ and the condenser pressure is $0.04\,\mathrm{bar}$. The HRSG heats sub-cooled liquid at $25\,°\mathrm{C}$ to steam at $600\,°\mathrm{C}$.

(i) Given a pinch point of $10\,°\mathrm{C}$ sketch a T-X diagram for the HRSG. Calculate the mass flow rate of water and compute the exit temperature of the gas turbine exhaust at exit of the HRSG. [4]

(ii) Neglecting the feed pump work and assuming an isentropic turbine, calculate the power output from the steam turbine. [5]

(c) (i) Calculate the efficiency of the combined cycle. [2]

(ii) Comment on potential improvements to the combined cycle. [2]

2 A ground-source heat pump uses R-134a as working fluid. Saturated vapour enters the compressor at $-10\,°\mathrm{C}$. The vapour compressor is adiabatic, has pressure ratio 3.831 and a $50\,°\mathrm{C}$ exit temperature. The vapour passes to the condenser where it is cooled at constant pressure until it reaches the saturated liquid state. The saturated liquid then passes through a throttle to the evaporator inlet. As it flows through the evaporator, it is heated at constant pressure until it returns to the saturated vapour state at inlet to the compressor.

(a) (i) Sketch T-s and p-h diagrams of the cycle. [5]

(ii) Calculate the isentropic efficiency of the vapour compressor. [4]

(iii) Determine the dryness fraction at evaporator inlet. [2]

(iv) Calculate the heat rejected in the condenser and absorbed in the evaporator per unit mass of working fluid. [4]

(b) Determine the Coefficient of Performance (COP) of the heat pump. What is the potential improvement in COP that could be achieved with a reversible vapour compressor? Considering a Carnot cycle operating between the condenser and evaporator saturation temperatures, what fraction of the Carnot COP is achieved in each case? [5]

(c) The evaporator is installed in ground with a temperature of $10\,°\mathrm{C}$. The condenser is required to heat a room at $20\,°\mathrm{C}$. Considering the case of ideal heat exchangers such that the evaporator and condenser temperatures approach their surroundings, what fraction of the theoretical maximum performance does the heat pump achieve? Comment on the practical difficulties of achieving such ideal performance. [5]

3 A liquid-to-air heat exchanger provides 75 kW of cooling for an electric propulsor. The heat exchanger is integral to the aluminium alloy wall of the cowling, as shown in Fig. 1(a). The wall of the cowling can be assumed to have a uniform temperature, $T_{\text{wall}} = 40\,°\text{C}$.

(a) The liquid side of the heat exchanger is formed by 150 tubes of internal diameter 2 mm, formed within the wall of the cowling, as indicated in Fig. 1(b). The coolant enters the tubes at 120 °C and exits at 80 °C. The coolant has specific heat capacity $2142.8\,\text{J}\,\text{kg}^{-1}\,\text{K}^{-1}$, thermal conductivity $0.18\,\text{W}\,\text{m}^{-1}\,\text{K}^{-1}$ and viscosity $9 \times 10^{-3}\,\text{Pa}\,\text{s}$.

(i) Determine the mass flow rate of coolant. [2]

(ii) Calculate the Reynolds number inside the tubes and find an appropriate heat transfer coefficient using the Databook. [3]

(iii) Ignoring entrance effects, sketch the temperature profile along the tubes and determine the required length W. [5]

(b) The outer surface can be considered as a flat plate of width W and length L beneath a uniform flow of air, as illustrated in Fig. 1(c). The air has a pressure of 0.3 bar, temperature $-40\,°\text{C}$ and freestream velocity $U = 100\,\text{m}\,\text{s}^{-1}$. The *local* Nusselt number at a distance x along the plate is given by

$$Nu_x = 0.0296\, Re_x^{0.8} Pr^{0.4}, \quad \text{where} \quad Re_x = \rho U x/\mu$$

The density, ρ, viscosity, μ and the Prandtl number, Pr, should be approximated at the mean of the wall and freestream temperatures using the Databook where appropriate.

(i) Without calculation, sketch the form of the variation of local heat flux with distance x along the plate. [5]

(ii) Determine the required plate length L. [5]

(c) An engineer suggests a separate heat exchanger, mounted in a duct beneath the cowling, instead of the integral design. Discuss the pros and cons of this idea with a particular reference to Reynolds' analogy between skin friction and heat transfer. [5]

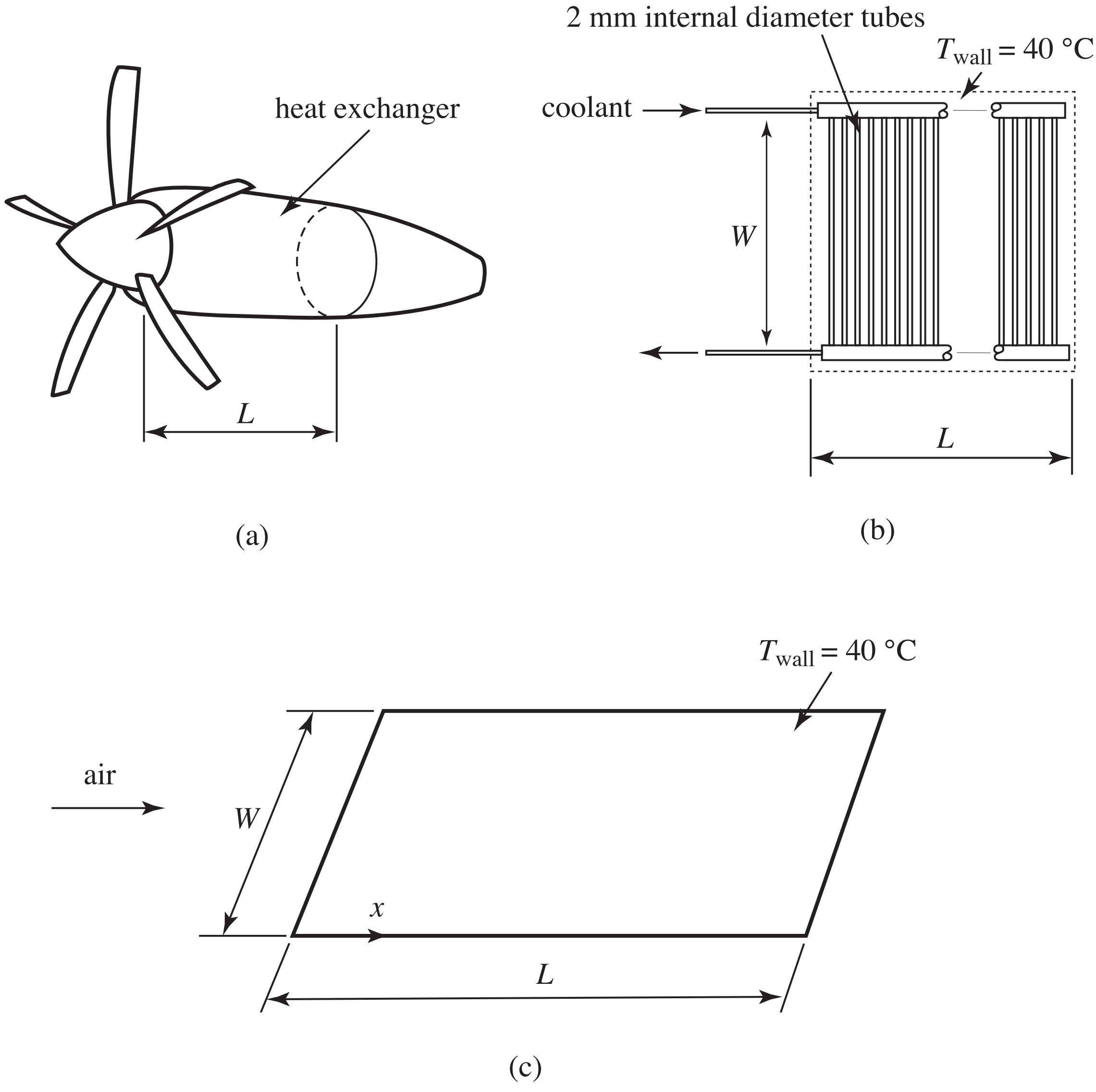
heat exchanger
L
(a)
2 mm internal diameter tubes
T_{wall} = 40 °C
coolant
W
L
(b)
T_{wall} = 40 °C
air
W
x
L
(c)

Fig. 1

SECTION B

*Answer not more than **two** questions from this section*

4 (a) Figure 2(a) shows a depth h of viscous incompressible fluid lying over a stationary horizontal surface which occupies the plane $y = 0$. A flat plate at $y = h$ is dragged horizontally over the fluid with speed U. In the region of interest, the resulting flow is fully developed and there is no pressure gradient in the x direction. Derive, from first principles, the horizontal velocity profile $u(y)$. You should assume that the flow is two-dimensional, i.e. variations perpendicular to the plane of the diagram are negligible. [8]

(b) Figure 2(b) shows a two-dimensional situation in which a flat plate is dragged at speed U over fluid of depth h in a basin with a vertical retaining wall. The fluid has density ρ and dynamic viscosity μ. Far away from the vicinity of the vertical wall, the flow is fully developed with horizontal pressure gradient $\mathrm{d}p/\mathrm{d}x$.

(i) Explain why the pressure gradient arises, and state whether it is positive or negative. [3]

(ii) Derive an expression for the pressure gradient, $\mathrm{d}p/\mathrm{d}x$. [7]

(c) A Physicist argues that $\mathrm{d}p/\mathrm{d}x$ should depend on ρ, μ, h and U.

(i) Derive two possible dimensionless forms for this relationship. The first should use a dynamic pressure and the second should use a viscous term to normalise the pressure gradient. [4]

(ii) Compare your answers to parts (b)(ii) and (c)(i), and suggest functional forms for the relationships. Comment on your answers in light of the assumptions made. [3]

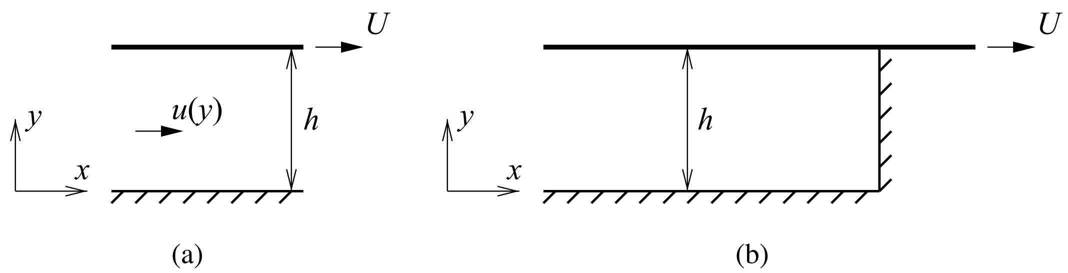

Fig. 2

5 Figure 3 shows an idealisation of a ducted-fan propulsor on a vehicle moving at speed V_∞. The ambient fluid is effectively incompressible, and has density ρ. Relative to the propulsor, the flow is steady, with upstream velocity V_∞. Inside the duct, it can be considered one-dimensional and inviscid. Across the fan (represented by a dashed line), the static pressure rises from p_f to $p_\mathrm{f} + \Delta p$. The duct has constant area A_j, and at its exit a parallel-streamline jet emerges with velocity V_j.

(a) (i) Given the parallel streamlines, what can be assumed about the pressure in the jet and its surroundings at exit? [2]

(ii) Sketch a control volume suitable for calculation of the thrust delivered by the propulsor. [2]

(iii) Show that the thrust is given by $\rho A_\mathrm{j} V_\mathrm{j} (V_\mathrm{j} - V_\infty)$. [6]

(b) (i) Find the pressure rise across the fan. [4]

(ii) Hence, or otherwise, derive an expression for the power that the fan must supply to the fluid. [3]

(c) The propulsor is required to provide a constant thrust $\rho A_\mathrm{j} V_\mathrm{j0}^2$ as the vehicle speed varies.

(i) Find how the jet velocity V_j varies with the vehicle speed V_∞. [4]

(ii) Sketch how the power that must be supplied to the fluid varies with V_∞, indicating in particular the behaviour at small and large values (relative to V_j0). [4]

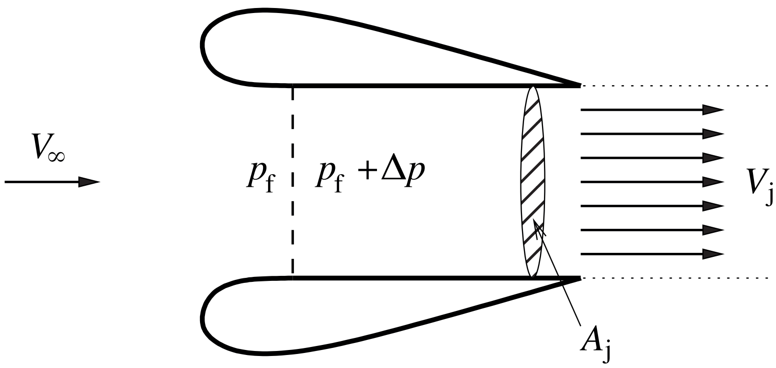

Fig. 3

6 A cylindrical container of radius R is filled with liquid to a depth h_0, as shown in Fig. 4. The container is now spun about its cylindrical axis with angular velocity Ω. After some time, the liquid spins in rigid-body motion with the container.

(a) Describe, qualitatively, the shape of the liquid surface in the spinning condition. The physical reasoning behind your answer should be carefully explained. [6]

(b) The liquid's depth in the spinning condition is a function of radial distance r from the cylinder axis. Derive the mathematical formula for this function. (You may assume that the liquid is incompressible.) [8]

(c) If the container is suddenly stopped, the liquid continues rotating for a while until viscous effects gradually bring it to rest.

(i) For a horizontal plane, well away from both the base of the container and liquid surface, sketch the variation of fluid velocity with radius at several time instants spanning the deceleration process. [4]

(ii) A time-scale for viscous phenomena to take effect over a distance d can be derived (via dimensional reasoning) from d and the kinematic viscosity ν. Give the expression for this time-scale. [2]

(iii) On the basis of part (c)(ii), find a dimensionless parameter that must be large in order for the fluid in the cylinder to undergo many rotations before coming to rest. Comment on your answer. [5]

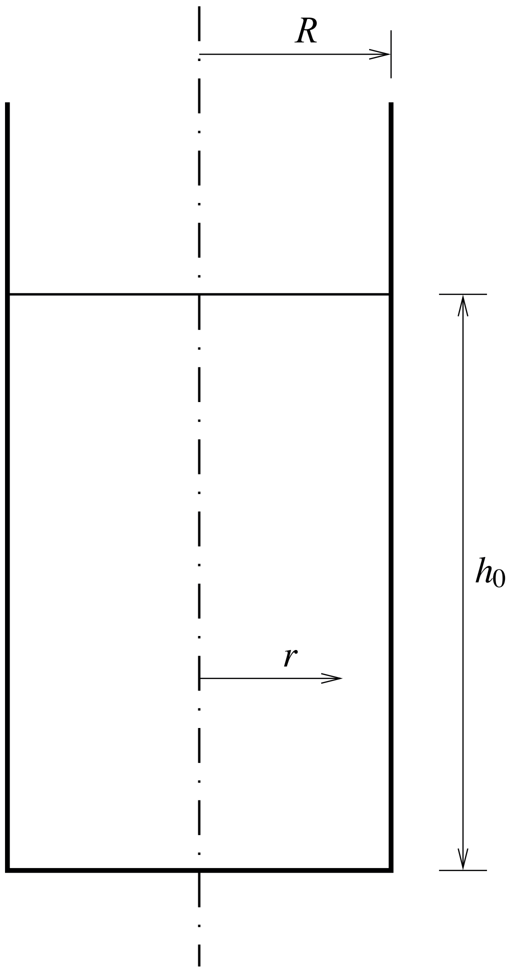

Fig. 4

END OF PAPER

1 (a) -

(i) 520 °C.

(ii) 1056 kg/s.

(b) (i) 150.3 kg/s, 122.4 °C

(ii) 219 MW

(c) (i) 59.2 %

(ii) -

2 (a) (i) -

(ii) 0.65

(iii) 0.267

(iv) 193.7 kJ/kg, 151 kJ/kg

(b) 4.54 (60%), 6.42 (85%)

(c) 15.5%

3 (a) (i) 0.875 kg/s

(ii) 413, 330 $\mathrm{Wm^2K^{-1}}$

(iii) 4.2 m

(b) (i) -

(ii) 3 m

(c) -