

EGT1
ENGINEERING TRIPOS PART IB

Friday 7 June 2024 9.00 to 11.10

Paper 4

THERMOFLUID MECHANICS

*Answer not more than **four** questions.*

*Answer not more than **two** questions from each section.*

All questions carry the same number of marks.

*The **approximate** number of marks allocated to each part of a question is indicated in the right margin.*

Answers to questions in each section should be tied together and handed in separately.

*Write your candidate number **not** your name on the cover sheet.*

STATIONERY REQUIREMENTS

Single-sided script paper

SPECIAL REQUIREMENTS TO BE SUPPLIED FOR THIS EXAM

CUED approved calculator allowed

Engineering Data Book

10 minutes reading time is allowed for this paper at the start of the exam.

You may not start to read the questions printed on the subsequent pages of this question paper until instructed to do so.

You may not remove any stationery from the Examination Room.

SECTION A

Answer not more than **two** questions from this section

1 The cross-section of a spherical tank for cryogenic liquid is shown in Fig. 1. Both inner and outer shells have conductivity $140 \text{ W m}^{-1}\text{K}^{-1}$. The insulation has thermal conductivity $2 \times 10^{-4} \text{ W m}^{-1}\text{K}^{-1}$. The average heat transfer coefficients for the inner and outermost surfaces are $100 \text{ W m}^{-2}\text{K}^{-1}$. The liquid in the tank is kept saturated at 20 K by allowing vapour to escape through the constant pressure valve. The ambient environment is 300 K .

- (a) (i) Show that the thermal resistance of a thick spherical shell, with thermal conductivity λ , inner radius r_1 and outer radius r_2 , is given by

$$\frac{1}{4\pi\lambda} \left\{ \frac{1}{r_1} - \frac{1}{r_2} \right\}. \quad [3]$$

- (ii) Calculate the thermal resistance of each layer and that of the overall tank (ignore the valve). What error would arise from considering the resistance of the insulation alone? [5]
- (iii) Sketch the temperature variation with radius through the tank wall. [2]
- (iv) If the latent heat of evaporation of the liquid is 448.9 kJ kg^{-1} , what is the loss of liquid in 24 hours? [2]

(b) In practice, radiative heat transfer in the insulation must be considered. Considering the worst case, i.e. if the insulation is completely transparent to radiation:

- (i) determine the view factor between the outer and inner shells and vice versa; [2]
- (ii) using a network diagram for radiative heat exchange between the inner and outer shells (or otherwise), calculate and compare this worst-case radiative heat loss to that from conduction in part (a)(ii). Take the emissivity of the surfaces to be 0.1, and assume that the inner shell is at the liquid temperature and the outer shell is at the temperature of the surroundings. [6]
- (iii) Derive an expression for the increase in radiative resistance achieved by the addition of a thin spherical shell placed between the inner and outer shells in terms of its radius r and emissivity ε . [5]

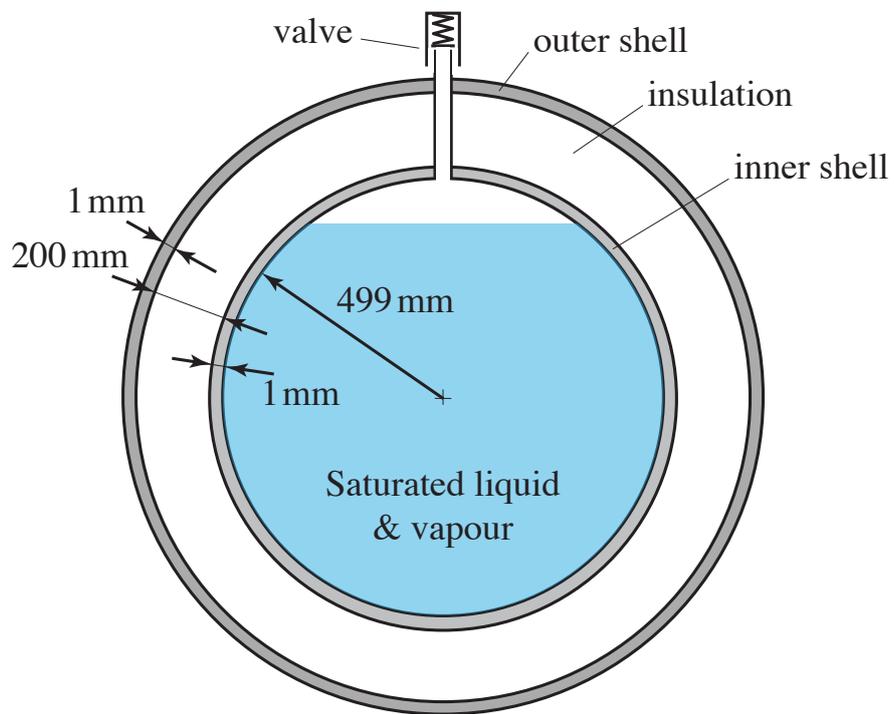


Fig. 1

2 (a) A real heat pump operates on a vapour compression cycle with a temperature of $T_1 = -15^\circ\text{C}$ in the evaporator. The refrigerant is R-134A, which enters the compressor as saturated vapour and leaves as a vapour superheated by 20°C . On exit from the condenser, the refrigerant is a saturated liquid at $T_3 = 50^\circ\text{C}$. It then passes through a throttle valve back into the evaporator. The properties of R-134a can be found in the Thermofluids Databook.

(i) Draw a T - s diagram for the real heat pump cycle and for a reversed Carnot heat pump operating between T_1 and T_3 . [4]

(ii) Show that the coefficient of performance of the reversed Carnot heat pump is given by

$$COP_{RC} = \frac{1}{1 - T_1/T_3},$$

and calculate the value. [4]

(iii) For the real heat pump, determine the pressure ratio of the compressor. [4]

(iv) Calculate the coefficient of performance, COP_{HP} , of the real heat pump. [4]

(b) To increase the temperature of the supplied heat, two heat pumps are used in series such that the heat output of the first heat pump is the heat intake of the second.

(i) Show that the ratio of input power between the two heat pumps

$$\dot{W}_1/\dot{W}_2 = (COP_2 - 1)/COP_1,$$

where COP_1 and COP_2 are the coefficients of performance of the first and second heat pumps. [4]

(ii) Determine a relationship for the total coefficient of performance COP_T in terms of COP_1 and COP_2 . Plot COP_T for varying COP_1 when $COP_2 = 4$ and explain its bounds for high and low COP_1 . [5]

3 A gas turbine for a power generation plant consists of a compressor, a combustor and a turbine. The compressor has a pressure ratio p_r .

(a) Assuming an isentropic compressor and turbine, and no pressure drop in the combustor, show that the plant thermal efficiency is

$$\eta = 1 - p_r^{(1-\gamma)/\gamma}.$$

The working fluid should be treated as a perfect gas with constant composition with specific heat ratio γ and the mass flow of fuel can be ignored. [7]

(b) The plant operates at ambient temperature $T_1 = 300\text{ K}$ with pressure ratio $p_r = 20$ and combustor temperature $T_3 = 1500\text{ K}$. The powers for the compressor and turbine are $\dot{W}_c = -15\text{ MW}$ and $\dot{W}_t = 24\text{ MW}$, respectively. Treat the working fluid as air throughout, with perfect gas properties of $c_p = 1.1\text{ kJ kg}^{-1}\text{ K}^{-1}$ and $\gamma = 1.4$. The mass flow of air is 30 kg s^{-1} and the mass flow of fuel can be ignored.

(i) Calculate the isentropic efficiencies of the compressor and turbine. [5]

(ii) Calculate the change in available power across the compressor $\Delta\dot{B}_{12}$. Plot the compression process from initial state (1) to final state (2) and identify the state representing isentropic compression (2s) on a T - s diagram. Mark the area on the diagram that represents the loss of available power due to irreversibilities. [6]

(iii) A carbon capture and storage system removes and compresses the CO_2 from the turbine exhaust after it has cooled to ambient temperature T_1 . The CO_2 results from burning CH_4 with air at stoichiometric conditions, i.e. $\lambda = 1$. Balance the stoichiometric combustion equation to determine the mass fraction of CO_2 in the exhaust. The CO_2 is compressed from a pressure of 1 bar to 100 bar. Assuming the compression process is isentropic, calculate the increase in available power of the CO_2 across the compressor. *NB:* Air has a molar ratio of nitrogen to oxygen of $n_{\text{N}_2}/n_{\text{O}_2} = 3.76$. Molar masses and CO_2 properties can be found in the Thermofluids Databook. You should assume that CO_2 behaves as a perfect gas. [7]

SECTION B

Answer not more than *two* questions from this section

4 Fluid with density ρ enters a pipe of diameter D from a large reservoir as illustrated in Fig. 2. The pipe wall is thin and has a sharp inlet. The liquid separates from the inlet edge and forms a jet in the centre of the pipe. The fluid surrounding the jet is stagnant and the streamlines at position 2 are straight and parallel. Friction on the walls should be neglected and there is no stagnation pressure loss between the reservoir and the jet.

(a) Explain why the pressure close to the inlet at position 1 is not uniform and why the pressure in the jet at position 2 is uniform. [2]

(b) Explain why the fluid does not apply a resultant force on the pipe if there is flow separation at position 1. [3]

(c) Using an appropriate control volume enclosing the inlet flow and the jet, determine the jet diameter at position 2, as a proportion of the pipe diameter D . [5]

(d) The jet eventually mixes to a uniform velocity V far downstream of the inlet at position 3. Determine the total pressure loss coefficient,

$$K = \frac{-\Delta p_0}{\frac{1}{2}\rho V^2},$$

between the reservoir and position 3. [5]

(e) The inlet to the pipe is now modified to have a smooth profile, as illustrated in Fig. 3. The inlet profile has a semi-circular shape of diameter d .

(i) Sketch the streamline pattern if the flow now remains attached to the inlet. Hence find the force F on the pipe exerted by the fluid. Explain where this force is exerted and the direction it acts in. Express your answer as a non-dimensional force coefficient,

$$C_F = \frac{F}{\rho V^2 \pi D^2}.$$

[6]

(ii) Explain why the flow separates when there is a sharp inlet. Describe the variation in K and C_F with d/D . [4]

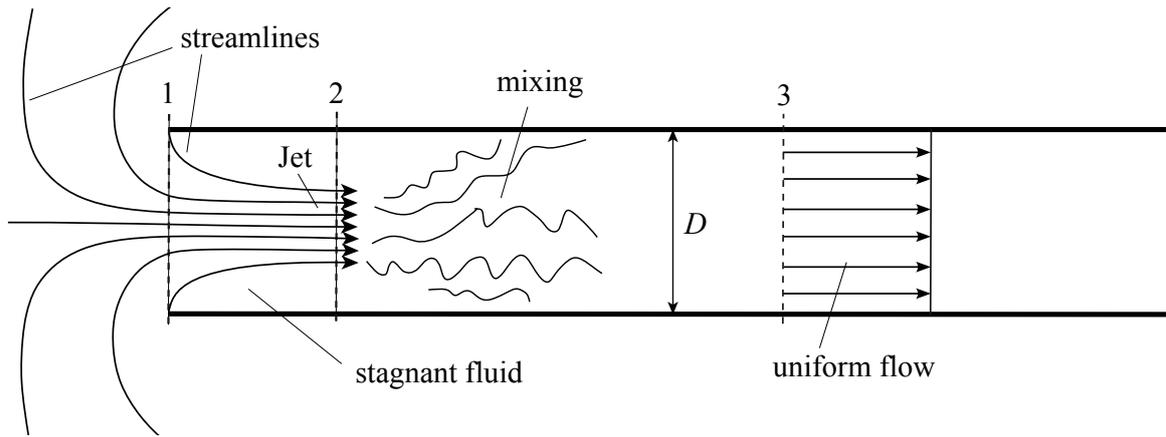


Fig. 2

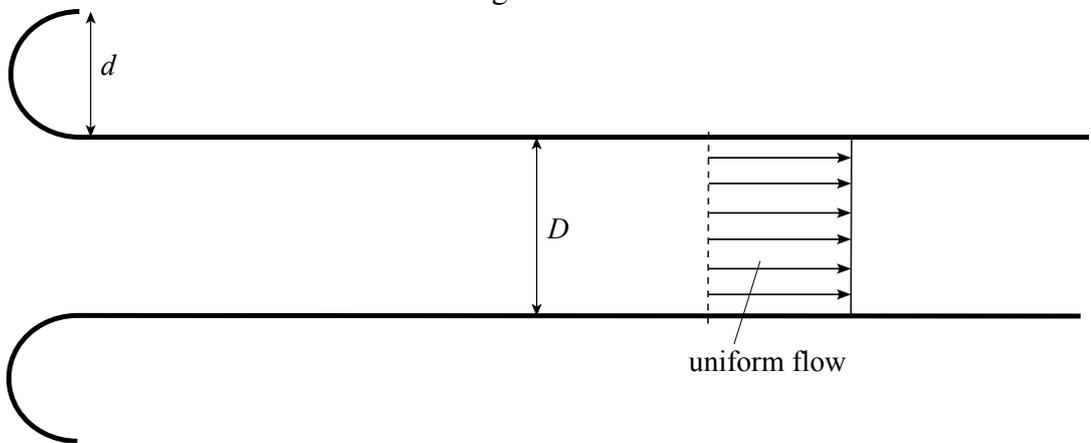


Fig. 3

5 A two-dimensional flow control valve is illustrated in Fig. 4. The valve is controlled by a spring with stiffness per unit width (into the page) k connected to a piston of length L which can move vertically, thus varying the height g of the channel AB. The total pressure on the high-pressure side of the valve is p_0 and varies sufficiently slowly that the flow can be considered as quasi-steady. The flow leaves the valve at B at atmospheric pressure p_{atm} . The flow is incompressible with uniform density of ρ and dynamic viscosity μ . When the spring extension $e = 0$, the channel height $g = 0.1L$.

Friction on the vertical surfaces of the piston can be neglected. Neglect any corner effects at the entry to the valve and assume the piston mass is negligible. The fluid on the high-pressure side of the valve is able to pass freely through the spring.

(a) Show that if frictional effects are negligible within the valve, the mass flow per unit width is

$$\dot{m} = C\sqrt{e}(0.1L - e),$$

where e is the spring extension and C is a constant which should be derived in terms of ρ , k and L . [6]

(b) Using the expression determined in (a) find the pressure difference $(p_0 - p_{\text{atm}})$ at which the maximum mass flow passes through the valve in terms of k . [3]

(c) In reality, frictional effects do occur within channel AB.

(i) Assuming fully developed flow in channel AB, find the streamwise pressure gradient dp/dx in terms of the wall shear stress τ_w and the channel height g . [4]

(ii) If the flow in the channel is laminar, explain why the velocity profile in the channel has a parabolic form

$$u(y) = \alpha y^2 + \beta y + \zeta.$$

Determine the constants α , β and ζ in terms of the bulk velocity V_b and channel height g . [6]

(iii) Hence, find a relationship between the skin friction coefficient $c_f = \tau_w / \left(\frac{1}{2} \rho V_b^2 \right)$ and a Reynolds number based on the channel height g . Give a physical explanation for your result. [3]

(iv) Without further calculation, describe the range of possible flow regimes that might occur as $(p_0 - p_{\text{atm}})$ is varied and the subsequent effects on c_f . [3]

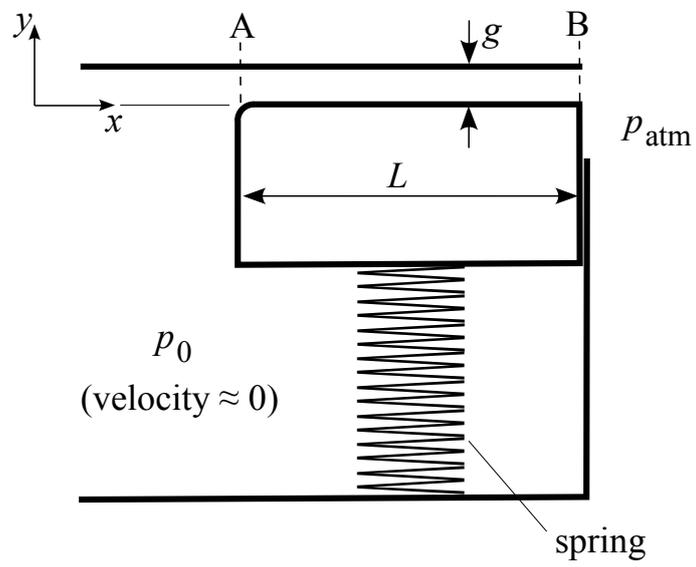


Fig. 4

6 A water tank delivers water to a nozzle via a hose connected to a valve, as illustrated in Fig. 5. The surface of the water in the tank is maintained at 4 m above the centerline of the nozzle. The hose diameter is 25 mm and the nozzle exit diameter is 10 mm. The pressure loss coefficient of the valve K_v , based on inlet dynamic pressure, is

$$K_v = \frac{-\Delta p_0}{\frac{1}{2}\rho V^2} = 1.8\alpha^{-2} - 3.5\alpha^{-1} + 1.8,$$

where α is the fraction of open area in the valve. The value of α ranges from $\alpha = 0$ when the valve is closed to $\alpha = 1$ when the valve is fully open. The pressure loss coefficient for the nozzle based on inlet dynamic pressure is $K_n = 2.0$. All other losses may be neglected.

In the following questions, where appropriate, give numerical answers in terms of the product ρg , where ρ is the density of water, and g is the acceleration due to gravity.

- (a) Find the nozzle exit velocity V_e in terms of the velocity in the hose V . [2]
- (b) The nozzle exit is at atmospheric pressure p_{atm} . Describe the shape of the streamlines leaving the nozzle and explain your reasoning. [2]
- (c) Determine the value of α when the flow velocity at nozzle exit is $V_e = 7.5 \text{ m s}^{-1}$. [6]
- (d) The hose is not load-bearing. Find the force necessary to support the nozzle when $V_e = 7.5 \text{ m s}^{-1}$. Indicate the direction this force acts. [6]
- (e) A pump is added downstream of the valve in order to double the flow rate through the nozzle. Determine the required pump pressure rise and show it is independent of α . Give a physical explanation for your result. [5]
- (f) Neglecting inefficiencies in the pump, find the power input to the pump when $\alpha = 1$. [4]

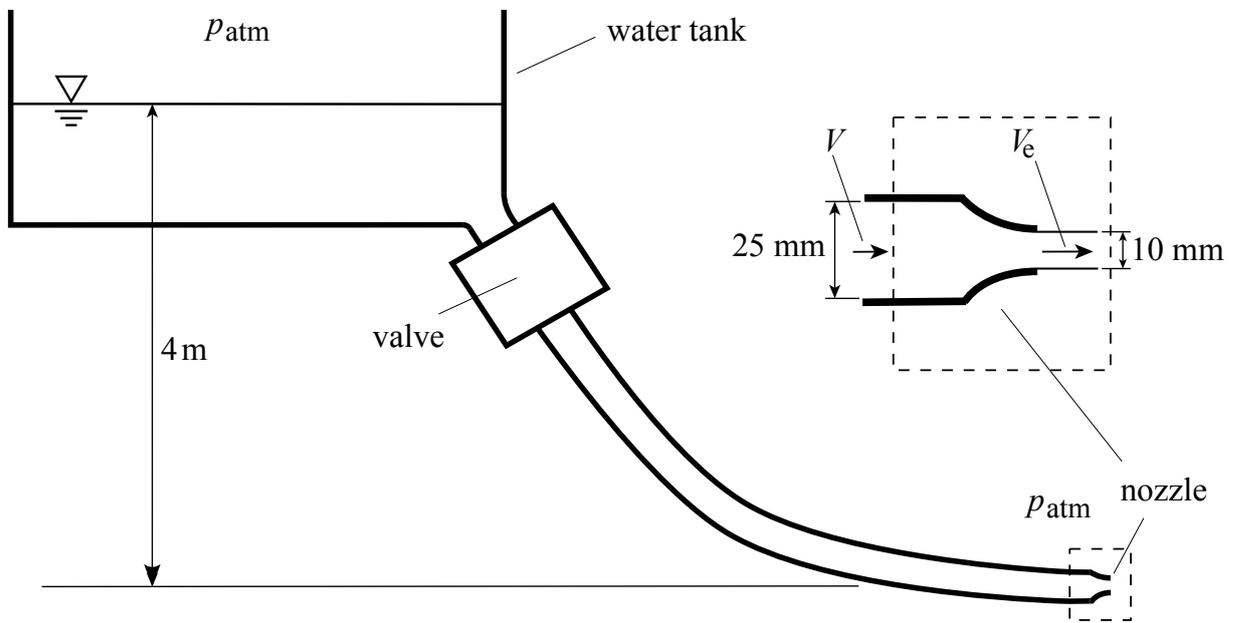


Fig. 5

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