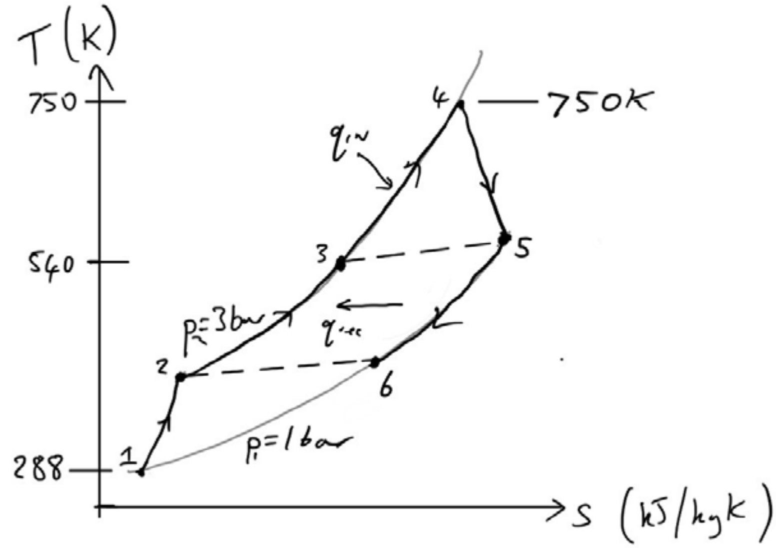
Q1

(a) $$\eta_c = \frac{T_{2s}-T_1}{T_2-T_1} = \frac{r_p^{\frac{\gamma-1}{\gamma}}-1}{\frac{T_2}{T_1}-1} \quad \therefore T_2 = T_1\left[\frac{r_p^{\frac{\gamma-1}{\gamma}}-1}{\eta_c}+1\right] = 288\times\left[\frac{3^{\frac{2}{7}}-1}{0.9}+1\right] = \underline{406.0\,\mathrm{K}}$$

$$\eta_t = \frac{T_4-T_5}{T_4-T_{5s}} = \frac{1-\frac{T_5}{T_4}}{1-\left(\frac{1}{r_p}\right)^{\frac{\gamma-1}{\gamma}}} \quad \therefore T_5 = T_4\left[1-\eta_t\left(1-\left(\frac{1}{r_p}\right)^{\frac{\gamma-1}{\gamma}}\right)\right] = 750\times\left[1-0.9\left(1-\left(\frac{1}{3}\right)^{\frac{2}{7}}\right)\right] = \underline{568.2\,\mathrm{K}}$$



[6]

(b) $$w_{net} = c_p\left(T_4-T_5\right)-c_p\left(T_2-T_1\right) = 1.01\times\left[\left(750-568.2\right)-\left(406-288\right)\right] = 64.44 \text{ kJ/kg}$$

$$\dot{m} = \frac{P}{w_{net}} = 2000/64.44 = \underline{31.0 \text{ kg/s}}$$

$$\eta_{th} = \frac{w_{net}}{q_{in}} = \frac{w_{net}}{c_p\left(T_4-T_3\right)} = \frac{64.44}{1.01\times\left(750-540\right))} = \underline{0.304}$$

Availability added in combustor,

$$b_{34} = \left(h_4-h_3\right)-T_0\left(s_4-s_3\right) = c_p\times\left(T_4-T_3\right)-288\times c_p\ln\left(\frac{T_4}{T_3}\right)$$

$$= 1.01\times\left(750-540\right)-288\times1.01\times\ln\left(\frac{750}{540}\right) = 116.5 \text{ kJ/kg}$$

$$\eta_{rat} = \frac{w_{net}}{b_{34}} = \frac{64.44}{116.5} = \underline{0.553}$$

[5]

(c) For the recuperator, the mass flows are equal:

$$q_{rec} = c_p\left(T_5 - T_6\right) = c_p\left(T_3 - T_2\right)$$

$$\therefore \quad T_6 = T_5 - \left(T_3 - T_2\right) = 568.2 - \left(540 - 406\right) = 434.2\ K$$

Rate of entropy generation in the recuperator,

$$\dot{S}_{gen} = \dot{m}\left(s_3 - s_2\right) + \dot{m}\left(s_6 - s_5\right) = \dot{m}\left[c_p \ln(T_3 / T_2) + c_p \ln(T_6 / T_5)\right]$$

$$\dot{S}_{gen} = 31.0 \times 1.01 \times \left[\ln(540/406) + \ln(434.2/568.2)\right] = \underline{0.509\text{ kJ/K}}\text{s}$$

This entropy is generated by the heat transfer within the recuperator across a finite temperature difference. It occurs across the tubes within the recuperator where there is a temperature gradient from the hot to the cold side. There is no irreversible entropy generation within the flow on either side.

[6]

(d) Available power transferred to compressor exit flow is

$$\dot{m}b_{23} = \dot{m}\left\{\left(h_3 - h_2\right) - T_0\left(s_3 - s_2\right)\right\} = \dot{m}\left\{c_p \times \left(T_3 - T_2\right) - 288 \times c_p \ln\left(\frac{T_3}{T_2}\right)\right\}$$

$$= 31.0 \times 1.01 \times \left\{\left(540 - 406\right) - 288 \times \ln\left(\frac{540}{406}\right)\right\} = \underline{1.62\text{ MW}}$$

Available power lost in the exhaust flow (using conditions in the environment)

$$\dot{m}b_{60} = \dot{m}\left\{\left(h_6 - h_0\right) - T_0\left(s_6 - s_0\right)\right\} = \dot{m}\left\{c_p \times \left(T_6 - T_0\right) - 288 \times c_p \ln\left(\frac{T_6}{T_0}\right)\right\}$$

$$= 31.0 \times 1.01 \times \left\{\left(434.2 - 288\right) - 288 \times \ln\left(\frac{434.2}{288}\right)\right\} = \underline{875.5\text{ kW}}$$

Note that despite the entropy generation due to heat transfer, a large amount of available power is recovered from the turbine exit flow in the recuperator.

[4]

(e) The recuperator reduces the available power lost in the exhaust flow. Although there are losses within the recuperator (indicated by the entropy generation in part c), the large temperature difference between turbine exit and compressor exit enables a large transfer of availabile power to the flow entering the combustor (part d), which would otherwise be lost with the exhaust flow. In terms of an equivalent Carnot engine, the recuperator increases the mean temperature of heat reception (from 3 – 4) and reduces the mean temperature of heat rejection (from 6 – 1).

A low pressure ratio is required so that the turbine exit temperature T_5 is high and the compressor exit temperature T_2 is relatively low. This means that a large amount of heat (and available power) can be transferred between turbine exit and compressor exit.

[4]

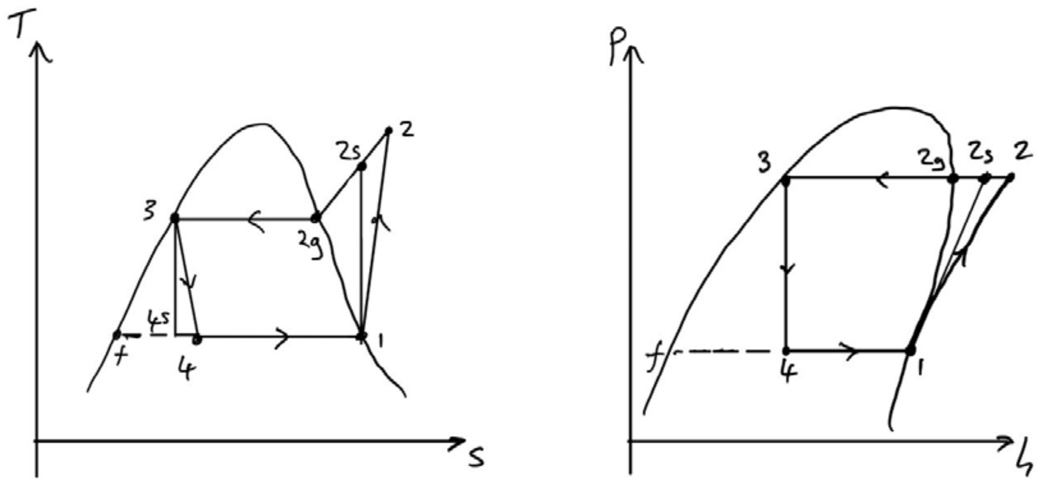
Q2

(a) (i) From R-143a table, page 37 of the databook, saturation pressure at $T_1 = -25^0 C$ is 1.06 bar

Saturation pressure at $T_3 = 40^0 C$ is 10.17 bar

[2]

(ii) T-s diagram and p-h diagram



[4]

(iii) From table, $h_1 = 383.4$ kJ/kg $h_2 = 441.2$ kJ/kg (20K superheat) $h_3 = h_4 = 256.4$ kJ/kg

$$COP = \frac{q_{evap}}{w_{in}} = \frac{h_1 - h_4}{h_2 - h_1} = \frac{h_1 - h_3}{h_2 - h_1} = \frac{383.4 - 256.4}{441.2 - 383.4} = \underline{2.197}$$

[3]

(iv) Entropy values from the table, $s_1 = 1.7458$ kJ/kgK $s_2 = 1.7786$ kJ/kgK

On the saturation line at $T_{2g} = 40^0 C$, $h_{2g} = 419.4$ kJ/kg and $s_{2g} = 1.7112$ kJ/kgK

For isentropic compression, $s_{2s} = s_1 = 1.7458$ kJ/kgK

Find enthalpy at exit of isentropic compression, h_{2s}

Linear interpolation:

$$h_{2s} = h_{2g} + \left(h_2 - h_{2g}\right)\frac{s_{2s} - s_{2g}}{s_2 - s_{2g}} = 419.4 + (441.2 - 419.4)\frac{1.7458 - 1.7112}{1.7786 - 1.7112} = 430.6 \text{ kJ/kg}$$

$$\eta_c = \frac{h_{2s} - h_1}{h_2 - h_1} = \frac{430.6 - 383.4}{441.2 - 383.4} = \underline{0.817}$$

[4]

(v) From the table, $s_3 = 1.1906$ kJ/kgK

$$x_4 = \frac{h_4 - h_f}{h_g - h_f} = \frac{256.4 - 167.2}{383.4 - 167.2} = 0.4126$$

$$s_4 = s_f + x_4(s_g - s_f) = 0.8743 + 0.4126 \times \left(1.7458 - 0.8743\right) = 1.2339 \text{ kJ/kgK}$$

Throttle specific entropy increase = $s_4 - s_3 = 1.2339 - 1.1906 = \underline{0.0433 \text{ kJ/kgK}}$

[3]

(b) For an isentropic compressor, $h_2 = h_{2s} = 430.6$ kJ/kg

For isentropic turbine, $s_{4s} = s_3 = 1.1906$ kJ/kgK

$$x_{4s} = \frac{s_{4s} - s_f}{s_g - s_f} = \frac{1.1906 - 0.8743}{1.7458 - 0.8743} = 0.3629$$

$$h_{4s} = h_f + x_{4s}(h_g - h_f) = 167.2 + 0.3629 \times (383.4 - 167.2) = 245.7 \text{ kJ/kg}$$

Net work input to compressor for isentropic components.

$$w_{in,s} = (h_{2s} - h_1) - (h_3 - h_{4s}) = (430.6 - 383.4) - (256.4 - 245.7) = 36.5 \text{ kJ/kg}$$

$$COP_s = \frac{q_{evap,s}}{w_{in,s}} = \frac{h_1 - h_{4s}}{36.5} = \frac{383.4 - 245.7}{36.5} = \underline{3.77}$$

[6]

(c) COP for Carnot refrigerator

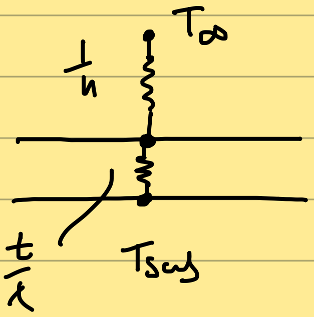
$$COP_{rev} = \frac{q_{in,rev}}{w_{in,rev}} = \frac{q_{in,rev}}{q_{out,rev} - q_{in,rev}} = \frac{T_{evap}}{T_{cond} - T_{evap}} = \frac{273 - 25}{40 + 25} = \underline{3.82}$$

This COP is slightly higher than the value in part (b) because in the idealised cycle in part (b) some of the heat is rejected at a temperature higher than the condenser saturation temperature (in the superheated region at compressor exit).

[3]

Q3.

a) i)



$$\frac{\dot{Q}}{A} = \frac{T_\infty - T_{surf}}{\frac{1}{h} + \frac{t}{\lambda}}$$

$$\Rightarrow R = \frac{1}{h} + \frac{t}{\lambda}$$

[3]

a) ii) $T_{film} = \frac{140+20}{2} = 80°C$

Page 29 of Thermofluids Databook:

$$Pr_f = 0.72 + (0.7 - 0.72)\,(80-0)/(100-0) = 0.704 \qquad \frac{\mu c_p}{\lambda}$$

$$\mu_f = \left\{17 + (22-17) \times 80/100\right\} \times 10^{-6} = 2.1 \times 10^{-5} \text{ kg/s m}$$

$$\lambda_f = 0.024 + (0.032 - 0.024)\,80/100 = 0.0304 \text{ W/m K}$$

$$\rho_f = \frac{p}{RT_f} = \frac{1.013 \times 10^5}{287.15 \times (80 + 273.15)} = 1.0 \text{ kg/m}^3$$

$$Re_L = \frac{\rho_f U_\infty L}{\mu_f} = \frac{1.0 \times 10 \times 1}{2.1 \times 10^{-5}} = 4.76 \times 10^5$$

$$Nu_{lam} = \frac{hL}{\lambda_f} = 0.332 \times Re_L^{1/2}\, Pr^{1/3} = 203.7$$

$$\Rightarrow h = \frac{203.7 \times 0.0304}{1.0} = 6.2$$

$$\dot{q} = \frac{\dot{Q}}{A} = \frac{140 - 20}{\frac{1}{6.2} + \frac{0.001}{1}} = \frac{120}{0.1715} = 700.0 \ W/m^2$$

$$T_{surface,lam} = \dot{q} \times \frac{t}{\lambda} + T_{sub} = \frac{700 \times 0.001}{1} + 20 = 27^\circ C$$

[6]

a) iii)

$$Nu_{turb} = \frac{h L}{\lambda_f} = 0.0296 \ Re^{0.8} \ Pr^{1/3} = 916.9$$

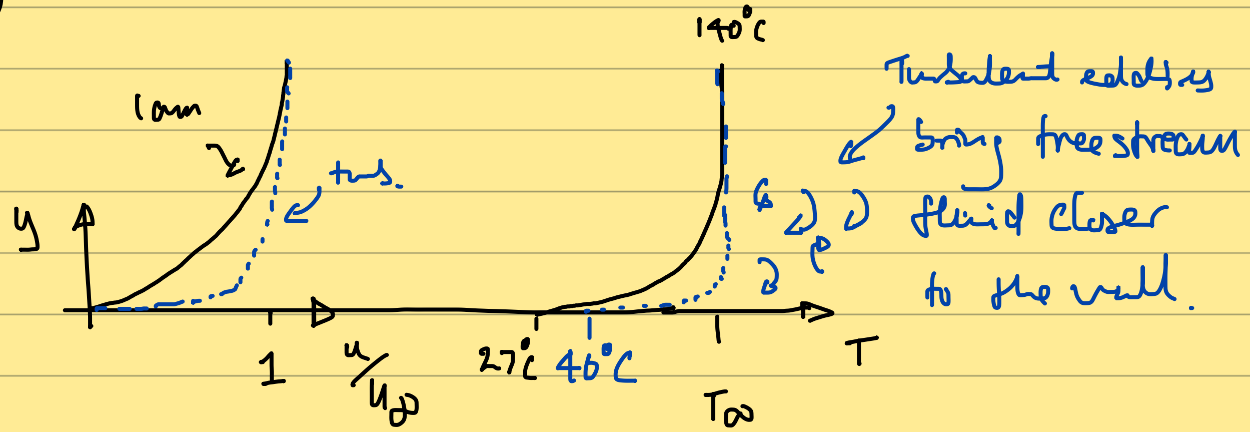
$$\Rightarrow h_{turb} = 27.9$$

$$\dot{q} = \frac{\dot{Q}}{A} = \frac{140 - 20}{\frac{1}{27.9} + \frac{0.001}{1}} = \frac{120}{0.0459} = 2.616 \ kW/m^2$$

$$T_{surface,turb} = \dot{q}_{turb} \times \frac{t}{\lambda} + T_{sub} = \frac{2.616 \times 10^3}{1} \times 0.001 + 20 = 46^\circ C$$

[3]

a(iv)



[5]

b) $$T_{turb} - T_{lam} = \dot{q}_{turb}\frac{t}{\lambda} + \cancel{T_{sub}} - \dot{q}_{lam}\frac{t}{\lambda} - \cancel{T_{sub}}$$

$$= \left(T_\infty - T_{sub}\right)\frac{t}{\lambda}\left\{\frac{1}{\frac{1}{h_{turb}} + \frac{t}{\lambda}} - \frac{1}{\frac{1}{h_{lam}} + \frac{t}{\lambda}}\right\}$$

$$\frac{T_{turb} - T_{lam}}{T_\infty - T_{sub}} = \frac{t}{\lambda/h_{turb} + t} - \frac{t}{\lambda/h_{lam} + t}$$

Let $\alpha = \lambda/h_{turb}$ & $\beta = \lambda/h_{lam}$ & $\theta = \dfrac{T_{turb} - T_{lam}}{T_\infty - T_{sub}}$

$$\theta = \frac{t}{\alpha + t} - \frac{t}{\beta + t}$$

$$\frac{\partial \theta}{\partial t} = \frac{\partial}{\partial t}\left\{\frac{t}{\alpha + t} - \frac{t}{\beta + t}\right\} = 0$$

$$\frac{\alpha}{(\alpha + t)^2} - \frac{\beta}{(\beta + t)^2} = 0$$

$$\alpha(\beta + t)^2 - \beta(\alpha + t)^2 = 0$$

$$\alpha\beta^2 + \alpha t^2 + \cancel{2\alpha\beta t} - \beta\alpha^2 - \cancel{2\beta\alpha t} - \beta t^2 = 0$$

$$t^2\cancel{(\alpha - \beta)} - \beta\alpha\cancel{(\alpha - \beta)} = 0$$

$$t = \sqrt{\beta\alpha} = \lambda / \sqrt{h_{lam} h_{turb}}$$

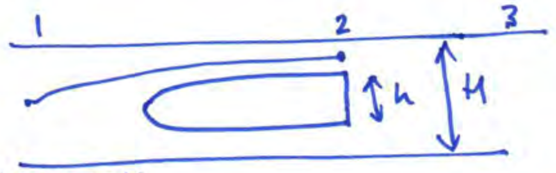
$$t = 0.1 / \sqrt{\{6.2 \times 27.9\}} = 7.6\,\text{mm}$$

[8]

Q4. 2P4 JAN 25

a) (i) FALSE (ii) FALSE (iii) TRUE (iv) FALSE [4]

b)



CONTINUITY:

$$V_2 (H-h) = V_1 H$$

$$\rightarrow \quad V_2/V_1 = \frac{H}{H-h} = \phi$$

BERNOULLI:

$$P_2 + \tfrac{1}{2}\rho V_2^2 = P_1 + \tfrac{1}{2}\rho V_1^2$$

$$P_2 - P_1 = \tfrac{1}{2}\rho\left[V_1^2 - V_2^2\right]$$

$$= \tfrac{1}{2}\rho V_1^2\left[1 - \left(\frac{V_2}{V_1}\right)^2\right]$$

$$P_2 - P_1 = -\tfrac{1}{2}\rho V_1^2\left[\phi^2 - 1\right]$$

c)

F is force on fluid
= force needed to support object

Here 'F' is the force on the flow which is in the opposite direction to the force on the object required in the question

$$F + P_1 H - P_2 H = \rho V_2^2 (H-h) - \rho V_1^2 H$$

$$F = \rho V_2^2 (H-h) - \rho V_1^2 H + (P_2 - P_1) H$$

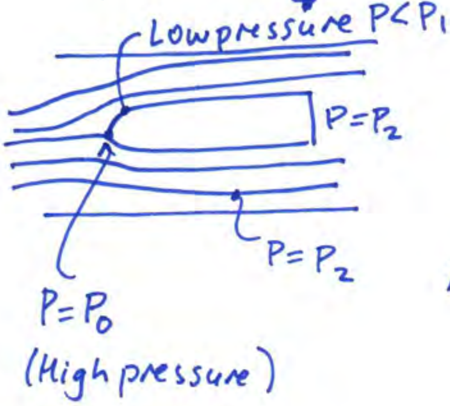
$$F = \tfrac{1}{2}\rho V_1^2 H \left[2\left(\frac{H-h}{H}\right)\left(\frac{V_2}{V_1}\right)^2 - 2 - \left[\phi^2 - 1\right]\right]$$

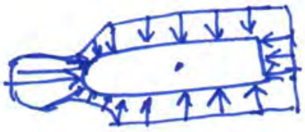
$$C_F = -F \Big/ \tfrac{1}{2}\rho V_1^2 H$$

CF in terms of the force on the object is
CF = (1 - phi)^2 which is always positive
i.e. the force on the object from the flow is to the right

[6]

d)





Pressure force on object →
force on flow ←
force needed to hold object in equilibrium ← [3]

e) viscous dissipation due to mixing of flows at V_2 and 0 speed [2]

f)

$$P_{01} = P_1 + \tfrac{1}{2}\rho V_1^2$$

$$P_{03} = P_3 + \tfrac{1}{2}\rho V_3^2$$

$$V_1 = V_3 \rightarrow P_{01} - P_{03} = P_1 - P_3$$

$$F + (P_1 + \cancel{\rho V_1^2})H = (P_3 + \cancel{\rho V_3^2})H$$

$$F = -(P_1 - P_3)H \quad \rightarrow \quad F = -(P_{01} - P_{03})H$$

$$C_F = \frac{-F}{\tfrac{1}{2}\rho V_1^2} = \frac{(P_{01} - P_{03})H}{\tfrac{1}{2}\rho V_1^2}$$

$$\rightarrow \quad K = C_F$$

(2)

$$\text{loss} \sim Q\Delta P_0 = V_1 H \times F/H$$

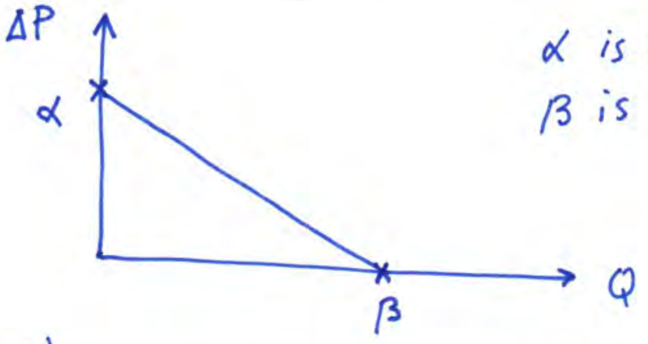
$$= F V_1$$

[6]

Q5. ③

a) $P_2 - P_1 = \alpha\,(1 - Q/\beta)$

i) $\alpha \rightarrow$ units of pressure, e.g. Pa, $\beta \rightarrow$ units of volumetric flow rate e.g. m^3/s



α is the maximum pressure rise
β is the maximum flow rate.

[4]

ii) Pump power $Q\Delta P_0$ $\quad \Delta P_0 = \Delta P$ (duct area constant)

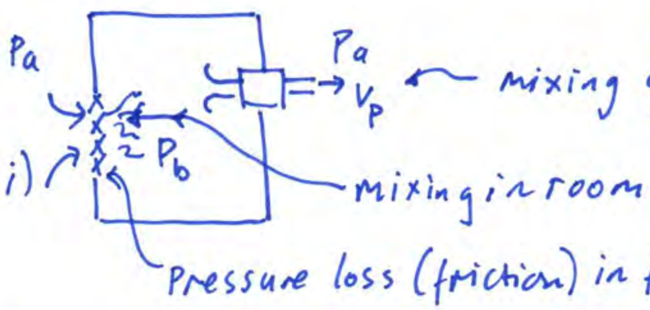
$$\rightarrow \text{Power} = \alpha Q - \alpha Q^2/\beta$$

$$\partial \text{Power}/\partial Q = \alpha - 2Q\alpha/\beta$$

Max power $\alpha = 2Q\alpha/\beta \qquad Q = \beta/2 \qquad$ MAX POWER $= \beta/2 \times \alpha/2$

$= \alpha\beta/4$. [5]

b) i)



[2]

ii)

$$P_b + \tfrac{1}{2}\rho V_f^2 = P_a - 0.5 \times \tfrac{1}{2}\rho V_f^2 \rightarrow P_b = P_a - (1.5)\tfrac{1}{2}\rho V_f^2$$

$$P_a + \tfrac{1}{2}\rho V_p^2 = (\Delta P_0)_{pump} + P_b$$

$$\cancel{P_a} + \tfrac{1}{2}\rho V_p^2 = (\Delta P_0)_{pump} + \cancel{P_a} - (1.5)\tfrac{1}{2}\rho V_f^2$$

$$(\Delta P_0)_{pump} = \tfrac{1}{2}\rho V_p^2 + 1.5\,\tfrac{1}{2}\rho V_f^2, \quad V_p A_d = V_f A_f \quad V_f/V_p = A_d/A_f$$

$$C_p = \frac{\Delta P_0}{\tfrac{1}{2}\rho V_p^2} = 1 + 1.5\left(\frac{V_f}{V_p}\right)^2 = 1 + 1.5\,(A_d/A_f)^2$$ [5]

iii) if $A_f >> A_d$ $\quad C_p = 1.0 \rightarrow \alpha(1 - Q/\beta) = \tfrac{1}{2}\rho V_p^2$

$$\alpha(1 - Q/\beta) = \tfrac{1}{2}\rho (Q/A_d)^2$$

$$\rightarrow \quad \frac{\rho}{2}\frac{1}{A_d^2}Q^2 + \frac{\alpha}{\beta}Q - \alpha = 0$$

$$\rightarrow \quad Q = \frac{-\alpha/\beta + \sqrt{(\alpha/\beta)^2 + 4\alpha\,\rho/2A_d^2}}{2\,\rho/2\,\frac{1}{A_d{}^2}} \qquad \text{(reject -ve)}$$

$$Q = \frac{-\alpha/\beta + \sqrt{(\alpha/\beta)^2 + \frac{2\rho\alpha}{A_d{}^2}}}{\rho/A_d^2}$$

[4]

c) i) $\Pi_1 = \dfrac{\Delta P}{\rho\,\Omega^2 D^2}$ $\quad \Pi_2 = \dfrac{Q}{\Omega D^3}$

if Π_1, Π_2 fixed

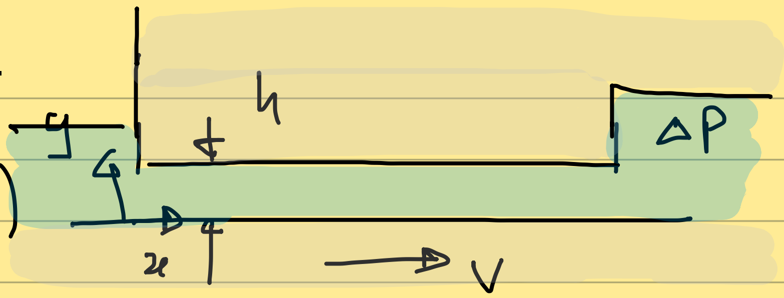
$\alpha \propto \rho\,\Omega^2 D^2$ (if non-dimensional performance matched) [3]

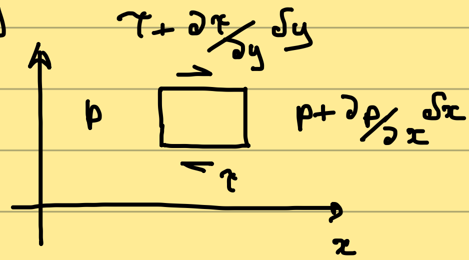
$\beta \propto \Omega D^3$

ii) Losses due to friction characterized by Reynolds number $Re = \dfrac{\rho\,\Omega D^2}{\mu}$ [2]

Q6.

a) i)





$$\cancel{p\,\delta y} - \left(\cancel{p} + \frac{\partial p}{\partial x}\delta x\right)\delta y - \cancel{\tau\,\delta x} + \left(\cancel{\tau} + \frac{\partial \tau}{\partial y}\delta y\right)\delta x = 0$$

$$-\frac{\partial p}{\partial x}\cancel{\delta x\,\delta y} + \frac{\partial \tau}{\partial y}\cancel{\delta y\,\delta x} = 0$$

$$\frac{\partial \tau}{\partial y} = \frac{\partial p}{\partial x}$$

$$\tau = \mu\frac{\partial u_x}{\partial y} \qquad \mu\frac{\partial^2 u_x}{\partial y^2} = \frac{\partial p}{\partial x}$$

Streamlines are straight and parallel so

$u_x \neq f(x)$ and $\frac{\partial p}{\partial y} = \frac{\rho u_x^2}{R} \to 0$ as $R \to \infty$

so $$\mu\frac{d^2 u_x}{dy^2} = \frac{dp}{dx}$$ [5]

Integrate: $$\mu \frac{du}{dy} = \left(\frac{dp}{dx}\right) y + a$$

$$\mu u = \left(\frac{dp}{dx}\right)\frac{y^2}{2} + ay + b$$

BCs: @ $y=0,\ u=V$, $y=h,\ u=0$ $\Rightarrow$ $\mu V = b$

$$\left(\frac{dp}{dx}\right)\frac{h^2}{2} + ah + \mu V = 0$$

$$\Rightarrow a = -\left(\frac{dp}{dx}\right)\frac{h}{2} - \frac{\mu V}{h}$$

So: $$\cancel{\mu} u = \frac{1}{\mu}\left(\frac{dp}{dx}\right)\frac{y^2}{2} - \left(\left(\frac{dp}{dx}\right)\frac{h}{\cancel{\mu}2} + \frac{\cancel{\mu}V}{h}\right) y + \cancel{\mu}V$$

$$u = \frac{1}{\mu}\left(\frac{dp}{dx}\right)\left(\frac{y^2}{2} - \frac{yh}{2}\right) + V\left(1 - \frac{y}{h}\right)$$

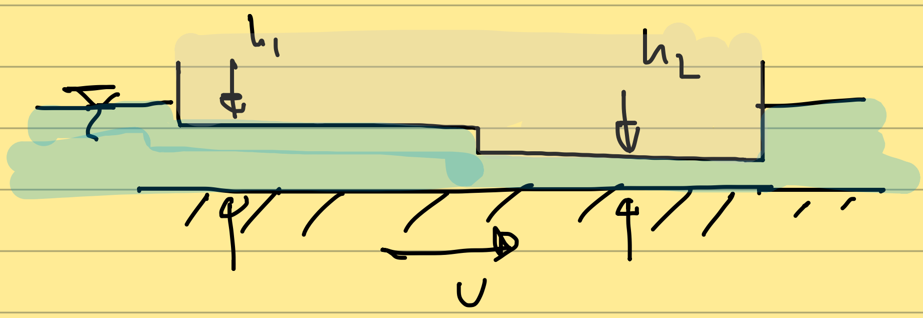
$$Q = \int_0^h u\,dy = \frac{1}{2\mu}\left(\frac{dp}{dx}\right)\left(\frac{y^3}{3} - \frac{y^2 h}{2}\right) + V\left(y - \frac{y^2}{2h}\right)\Bigg|_0^h$$

$$= \frac{1}{2\mu}\left(\frac{dp}{dx}\right)\left(\frac{h^3}{3} - \frac{h^3}{2}\right) + V\left(h - \frac{h}{2}\right)$$

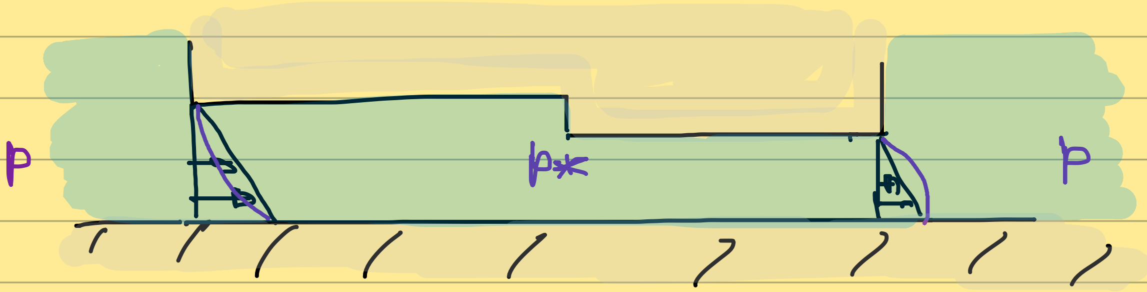
$$= \frac{Vh}{2} - \frac{h^3}{12\mu}\frac{dp}{dx} \qquad \frac{dp}{dx} = \frac{\Delta P}{L}$$

[5]

b)



Consider velocity profiles are always linear, plus a parabola from pressure term.



p^* must be higher to allow flow rates to match under each part of the bearing.

[4]

from a(ii)
$$Q = \frac{Vh_1}{2} - \frac{h_1^3}{12\mu}\frac{(p^* - p)}{L_1}$$

$$Q = \frac{Vh_2}{2} - \frac{h_2^3}{12\mu}\frac{(p - p^*)}{L_2}$$

$$\Rightarrow \frac{Vh_1}{2} - \frac{h_1^3}{12\mu}\frac{(p^* - p)}{L_1} = \frac{Vh_2}{2} - \frac{h_2^2}{12\mu}\frac{(p - p^*)}{L_2}$$

$$\frac{Vh_1}{2} - \frac{Vh_2}{2} = \frac{h_1^3}{12\mu} p^* - \frac{p}{L_1} \cdot \frac{h_1^3}{12\mu} - \frac{h_2^2}{12\mu} \frac{p}{L_2} + \frac{p^* h_2^2}{12\mu L_2}$$

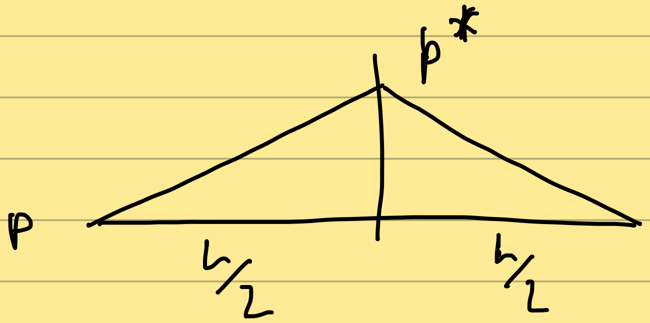
$$\frac{V}{2}\left(h_1 - h_2\right) + \frac{p}{12\mu}\left(\frac{h_1^3}{L_1} + \frac{h_2^2}{L_2}\right) = \frac{p^*}{12\mu}\left\{\frac{h_1^3}{L_1} + \frac{h_2^2}{L_2}\right\}$$

$$p^* = p + \frac{\overset{6}{\cancel{12}}\mu \frac{V}{\cancel{2}}\left(h_1 - h_2\right)}{\left\{\frac{h_1^3}{L_1} + \frac{h_2^2}{L_2}\right\}}$$

$$= p + \frac{6\mu V\left(h_1 - h_2\right)}{\left(\frac{h_1^3}{L_1} + \frac{h_2^2}{L_2}\right)} \quad //$$

[5]

iii) $h_1/h_2 = 2$ and $L_1 = L_2 = L/2$



$$F/_{\text{unit depth}} = \left(p^* - p\right)\frac{L}{2\cancel{4}} \times \cancel{2}$$

$$F/_{\text{unit depth}} = \frac{6\mu V \cancel{h_1}\left(1 - h_2/h_1\right)}{h_1^{\cancel{3}2}\left(\frac{2}{\cancel{2}L} + \frac{h_2^3}{h_1^3}\cdot\frac{2}{L}\right)}$$

$$= \frac{6\mu V\left(1 - \frac{1}{2}\right)}{h_1^2\,\frac{2}{L}\left(1 + \left(\frac{1}{2}\right)^3\right)}$$

$$= \frac{\cancel{12}\cancel{3}\mu V L}{h_1^2\,\cancel{2}^{1}\left(\frac{8+1}{8}\right)} = \frac{\cancel{12}^{4}}{3\cancel{9}}\frac{\mu V L}{h_1^2}$$

$$= \frac{4}{3}\frac{\mu V L}{h_1^2}$$

[4]

iv) viscous friction provides the pressure, so it can be a self supporting slider bearing. The plane parallel arrangement needs to be sealed to provide thrust, whereas the step doesn't. (Lord Rayleigh first to publish....)

[2]