Version CAH/6

EGT1
ENGINEERING TRIPOS PART IB

Friday 13 June 2025 9.00 to 11.10

Paper 4

THERMOFLUID MECHANICS

Answer not more than ***four*** *questions.*

Answer not more than **two** *questions from each section.*

All questions carry the same number of marks.

The ***approximate*** *number of marks allocated to each part of a question is indicated in the right margin.*

Answers to questions in each section should be tied together and handed in separately.

Write your candidate number ***not*** *your name on the cover sheet.*

STATIONERY REQUIREMENTS
Single-sided script paper

SPECIAL REQUIREMENTS TO BE SUPPLIED FOR THIS EXAM
CUED approved calculator allowed
Engineering Data Book

10 minutes reading time is allowed for this paper at the start of the exam.

You may not start to read the questions printed on the subsequent pages of this question paper until instructed to do so.

You may not remove any stationery from the Examination Room.

SECTION A

*Answer not more than **two** questions from this section.*

1 A gas turbine has a power output of $2\,\text{MW}$. It uses a heat exchanger, known as a *recuperator*, between the turbine exit and the compressor exit to pre-heat the air before it enters the combustor, as shown in Fig. 1. The compressor has a pressure ratio of 3.0 and the inlet takes air from the environment at $p_1 = 1.0\,\text{bar}$ and $T_1 = 288\,\text{K}$. The combustor inlet temperature is $T_3 = 540\,\text{K}$ and the turbine entry temperature is $T_4 = 750\,\text{K}$. The turbine expands the air back to $p_5 = 1.0\,\text{bar}$ and there are no pressure drops in the recuperator or in the combustor. The turbine and compressor both have isentropic efficiencies of 0.90. Treat the working fluid as air throughout, with perfect gas properties of $c_p = 1.01\,\text{kJ}\,\text{kg}^{-1}\text{K}^{-1}$ and $\gamma = 1.4$.

(a) Calculate the temperatures at compressor exit and turbine exit and sketch the T-s diagram for the gas turbine, marking on the stations 1 – 6 shown in Fig. 1. [6]

(b) Calculate the mass flow rate of air through the gas turbine, the 1st Law thermal efficiency and the 2nd Law rational efficiency. [5]

(c) Determine the net rate of entropy generation within the recuperator. Describe the source of this entropy rise and where in the recuperator it occurs. [6]

(d) Calculate the available power transferred by the recuperator to the compressor exit flow and the available power lost in the exhaust flow (station 6). [4]

(e) Explain how the recuperator improves the 2nd Law rational efficiency of the gas turbine. Why is a low compressor pressure ratio required? [4]

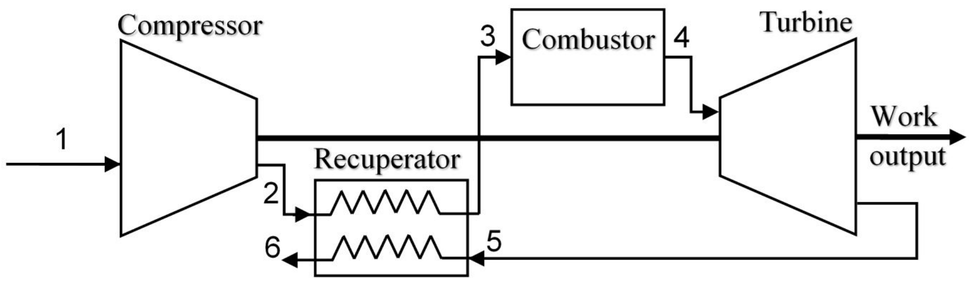

Fig. 1

2 (a) A refrigerator uses refrigerant R-134a as the working fluid. Saturated vapour enters the compressor at a temperature of $-25\,^{\circ}\mathrm{C}$ and leaves as superheated vapour at $60\,^{\circ}\mathrm{C}$. The vapour is cooled at constant pressure in the condenser until it reaches the saturated liquid state at a temperature of $40\,^{\circ}\mathrm{C}$. It then passes through a throttle valve to the evaporator inlet. In the evaporator the fluid is heated at constant pressure until it returns to the saturated vapour state at inlet to the compressor. Note that the properties of R-134a can be found in the Thermofluids Databook.

(i) Determine the pressures in the evaporator and the condenser. [2]

(ii) Sketch the refrigeration cycle on T-s and p-h diagrams. [4]

(iii) Determine the refrigerator Coefficient of Performance (COP). [3]

(iv) Calculate the compressor isentropic efficiency. [4]

(v) Calculate the specific entropy increase through the throttle valve. [3]

(b) Consider a refrigeration cycle with the same pressures in the evaporator and condenser as found in Part (a)(i), but with an isentropic compressor, and an isentropic turbine in place of the throttle valve. Assuming the power output from the turbine is used to reduce the compressor power input, determine the refrigerator COP for this cycle. [6]

(c) Calculate the refrigerator COP for a Carnot cycle operating between the condenser and evaporator saturation temperatures. Explain why there is a difference between this COP and the value obtained in Part (b). [3]

3 Air flows above a flat plate, as shown in Fig. 2. The free stream velocity remains constant whilst a boundary layer grows on the plate. The free stream temperature is $T_\infty = 140\,°\mathrm{C}$, the pressure is $p_\infty = 1.013\,\mathrm{bar}$ and the velocity is $U_\infty = 10\,\mathrm{m\,s^{-1}}$. At a position $L = 1\,\mathrm{m}$, the plate has an insert of thickness $t = 1\,\mathrm{mm}$ and thermal conductivity $\lambda = 0.1\,\mathrm{W\,m^{-1}\,K^{-1}}$. The remainder of the plate is made from a material with high conductivity so that it can be considered isothermal and is maintained at a temperature $T_{\mathrm{sub}} = 20\,°\mathrm{C}$. The properties of air should be evaluated at a temperature mid way between that of the free stream and the plate.

(a) (i) Derive an expression for the thermal resistance per unit area for the heat transfer from the free stream, through the insert, to the plate below. Give your answer in terms of the heat transfer coefficient above the insert h, the insert thickness t and conductivity λ. [3]

(ii) If the the flow remains laminar over the insert, compute the heat transfer per unit surface area through the insert and find its surface temperature, T. [6]

(iii) A disturbance is added upstream such that the flow is turbulent from the leading edge of the plate. What is the heat transfer per unit area through the insert and its surface temperature in this case? [3]

(iv) Sketch the velocity and approximate near wall temperature profiles for the two cases (i.e. Parts (a)(ii) and (a)(iii)). Explain the physical mechanism driving the difference in the shape of the profiles in each case. [5]

(b) Considering the thermal resistance of the insert derived in Part (a)(i), find the optimal thickness, t, that would maximise the surface temperature difference *between* the laminar case (Part (a)(ii)) and the turbulent case (Part (a)(iii)) for the given free stream conditions. [8]

You should consider the following Nusselt number correlations:

$$\mathrm{Nu}_{x,\mathrm{laminar}} = 0.332\,\mathrm{Re}_x^{1/2}\,\mathrm{Pr}^{1/3}$$

$$\mathrm{Nu}_{x,\mathrm{turbulent}} = 0.0296\,\mathrm{Re}_x^{4/5}\,\mathrm{Pr}^{1/3}$$

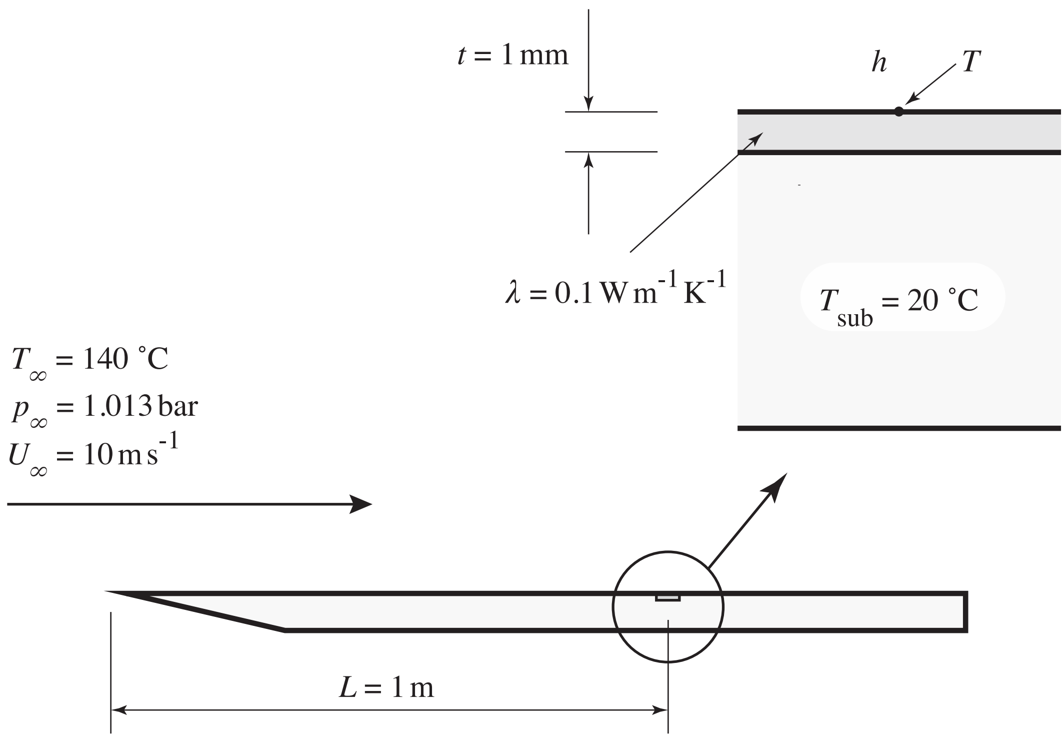

Fig. 2

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SECTION B

*Answer not more than **two** questions from this section.*

4 Consider the flow past a two-dimensional object of height h placed centrally between two parallel walls a distance H apart as illustrated in Fig. 3. The object has a streamlined nose but the rear is cut off square, so that the flow leaves the object parallel to the walls at position 2. At positions 1 and 3, the velocities V_1 and V_3 are uniform. The pressure is uniform at positions 1, 2 and 3. Gravity should be neglected. The density ρ is constant and friction on all the surfaces should be neglected.

(a) Identify any of the following criteria which must be satisfied for the pressure to be uniform in y. For each case state true or false.

(i) If the pressure is uniform, the flow must be irrotational. [1]

(ii) If the pressure is uniform, the flow must be incompressible. [1]

(iii) If the pressure is uniform, the streamlines are straight and parallel. [1]

(iv) If the pressure is uniform, there can be no viscosity. [1]

(b) Assuming frictionless flow from 1 to 2, show that the pressure difference between 1 and 2 is given by

$$p_2 - p_1 = -\frac{1}{2}\rho {V_1}^2 \left(\phi^2 - 1\right)$$

where $\phi = H/(H - h)$. [4]

(c) By considering a control volume encompassing the region between positions 1 and 2, find an expression for the force F per unit depth into the page exerted by the flow on the object. Indicate the direction in which this acts. Express your answer as a non-dimensional force coefficient $C_F = F/(\frac{1}{2}\rho {V_1}^2 H)$ in terms of ϕ only. [6]

(d) Sketch the streamlines around the object and indicate regions of high and low pressure. Verify that your sketch is consistent with the force of the flow on the object. [3]

(e) Explain why frictional effects are important in the region between 2 and 3. [2]

(f) Determine the change in total pressure from position 1 to 3. Express your answer as a loss coefficient $K = (p_{01} - p_{03})/(\frac{1}{2}\rho {V_1}^2)$ in terms of C_F only. Determine the rate of loss of mechanical energy in terms of F and V_1 . [6]

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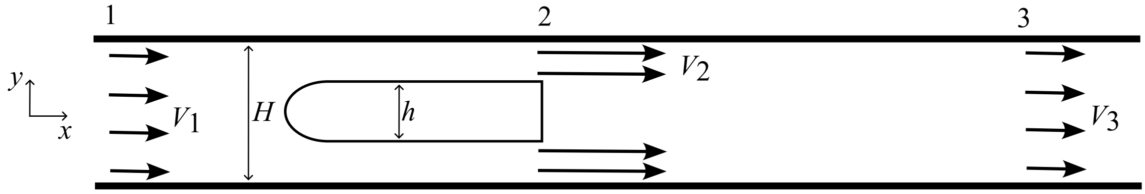

Fig. 3

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5 For this question, assume that the flow is incompressible with a constant density ρ.

(a) Air passes through a pump as illustrated in Fig. 4. The duct cross-sectional area A_d upstream and downstream of the pump is constant. Losses in the pump and in the inlet and exit duct should be neglected. The pressure rise across the pump is

$$p_2 - p_1 = \alpha\,(1 - Q/\beta)$$

where Q is the volumetric flow rate and α and β are positive constants.

(i) Determine the SI units of the constants α and β. Sketch the pump pressure rise against Q. Comment on the physical meaning of α and β. [4]

(ii) Determine (in terms of α and β) the value of Q at maximum pump power. Hence find the maximum pump power in terms of α and β. [5]

(b) The pump from Part (a) is used to draw air into a box as illustrated in Fig. 5. Air is drawn from atmosphere and passes through a filter of cross-sectional area A_f before entering the box. The filter total pressure loss coefficient is $-\Delta p_0/(\frac{1}{2}\rho {V_f}^2) = 0.5$. The flow leaving the pump leaves the exit duct at a speed V_p and at atmposheric pressure p_a.

(i) Consider the flows into and out of the box. Identify, using sketches of the flow, where mechanical energy is dissipated. [2]

(ii) Considering the losses in the system, find an expression for the total pressure rise across the pump. Express your answer as a total pressure rise coefficient $C_p = \Delta p_0/(\frac{1}{2}\rho {V_p}^2)$ in terms of the area ratio (A_d/A_f) only. [5]

(iii) For the case when $A_f >> A_d$, determine an expression for the volumetric flow rate Q in terms of ρ, α, β and A_d only. [4]

(c) The internal rotor of the pump from Part (a) has a rotational speed of Ω and a diameter of D.

(i) Suggest non-dimensional parameters to characterise the pump pressure rise and flow rate. Hence state how both α and β depend on Ω and D. [3]

(ii) In practise the pump is not loss free. Suggest a third non-dimensional parameter which would also be required to characterise the pump performance. [2]

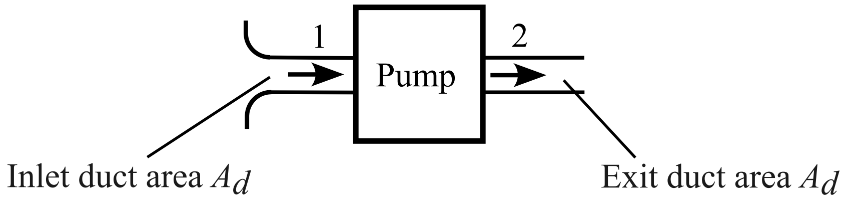
1
2
Pump
Inlet duct area A_d
Exit duct area A_d

Fig. 4

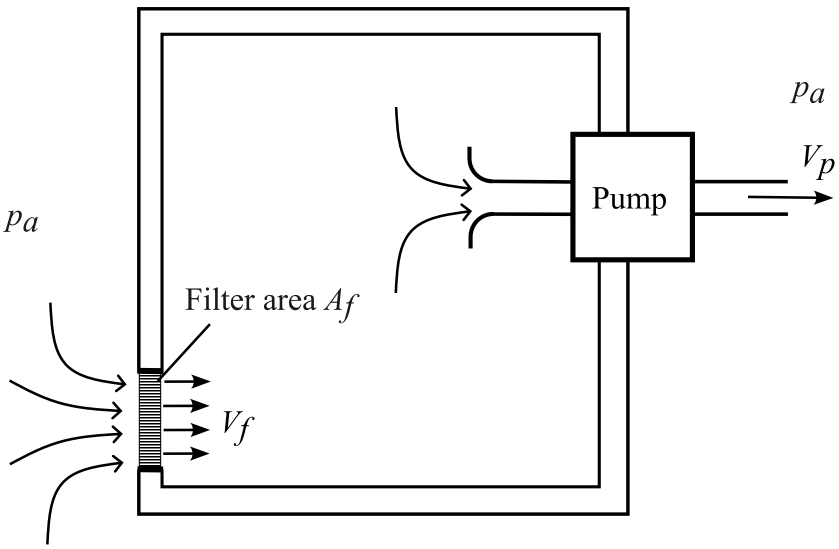
p_a
V_p
Pump
p_a
Filter area A_f
V_f

Fig. 5

6 (a) Fig. 6a shows a plate sliding beneath a block at speed V. The block has length L and the gap between the plate and the block has height h. The gap height h is very much smaller than the length L. The gap is filled with fluid of viscosity μ and the pressure is greater at $x = L$ than at $x = 0$ by ΔP.

(i) Show that the flow in the gap is governed by the equation

$$\frac{\mathrm{d}p}{\mathrm{d}x} = \mu \frac{\mathrm{d}^2 u}{\mathrm{d}y^2}$$

where u is the velocity in the x direction. Explain why the velocity is not a function of x and the pressure gradient is not a function of y. [5]

(ii) Explain why the pressure gradient must be constant and show that the volumetric flow rate through the gap, per unit depth into the page, is given by

$$Q = \frac{Vh}{2} - \frac{h^3}{12\mu}\frac{\mathrm{d}p}{\mathrm{d}x}$$

where

$$\frac{\mathrm{d}p}{\mathrm{d}x} = \frac{\Delta P}{L}.$$ [5]

(b) In Fig. 6b, the lower surface of the block is modified to include a step at $x = L_1$. The gap height from $0 < x < L_1$ is h_1 and from $L_1 < x < (L_1 + L_2)$ is h_2. In this configuration, both ends of the block are at the same pressure, p.

(i) Explain why the pressure gradient must be a function of x in this case. [4]

(ii) Find the maximum pressure in the gap, p^*, in terms of $h_1, h_2, L_1, L_2, \mu, V$ and p. [5]

(iii) Find the load per unit depth into the page that can be supported by the block for the case when $h_1/h_2 = 2$ and $L_1 = L_2 = L/2$ in terms of h_1, L, μ, and V. [4]

(iv) Comment on possible applications for this stepped configuration. [2]

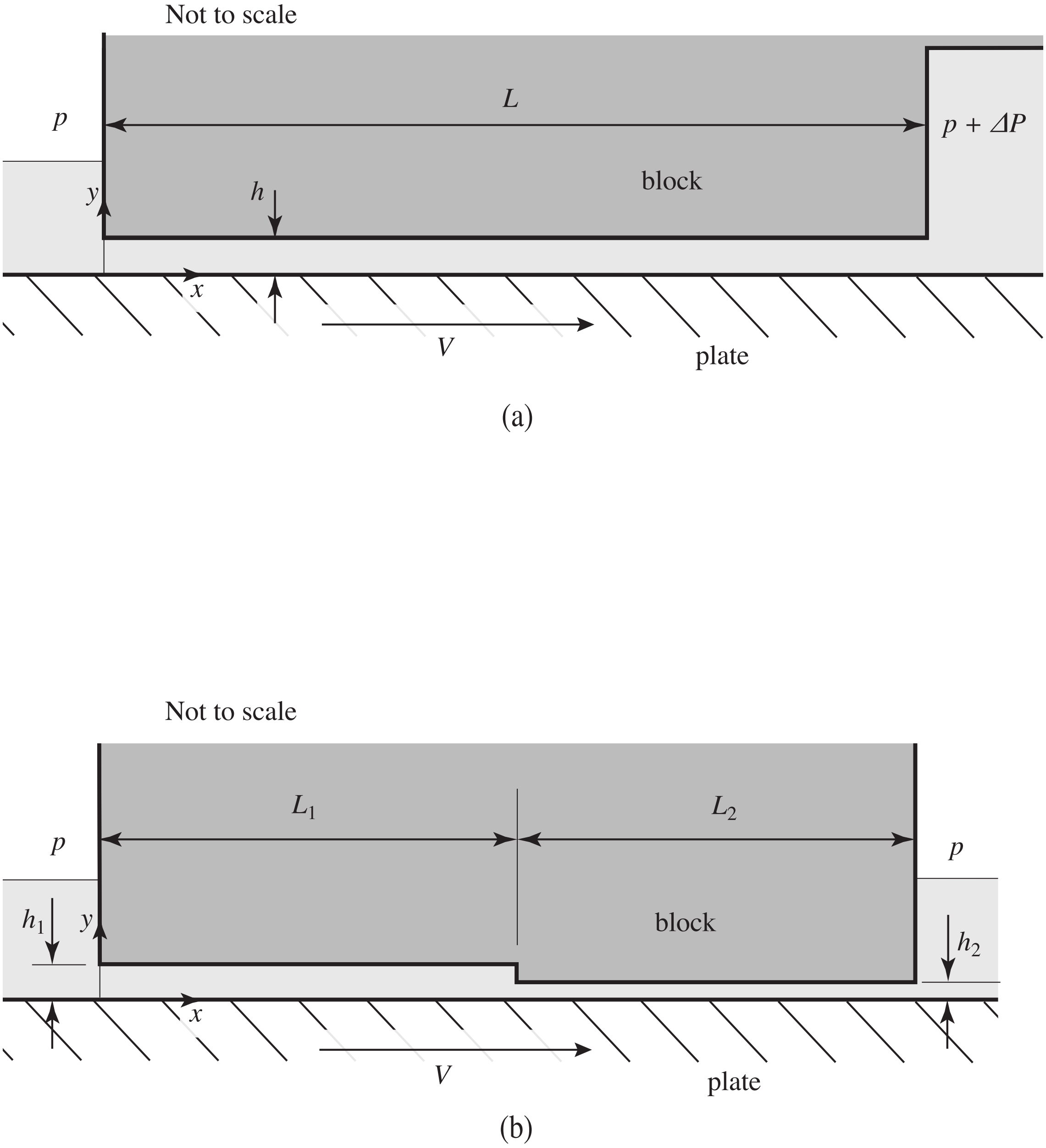

Fig. 6

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