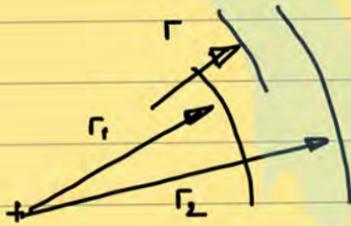


Q1)



$$Q = -\lambda 4\pi r^2 \frac{dT}{dr}$$

a) i)

$$\frac{1}{r^2} dr = \frac{-\lambda 4\pi}{Q} dT$$

$$\int_{r_1}^{r_2} \frac{1}{r^2} dr = \frac{-\lambda 4\pi}{Q} \int_{T_1}^{T_2} dT$$

$$+\left[\frac{1}{r}\right]_{r_1}^{r_2} = +\frac{\lambda 4\pi}{Q} [T]_{T_1}^{T_2}$$

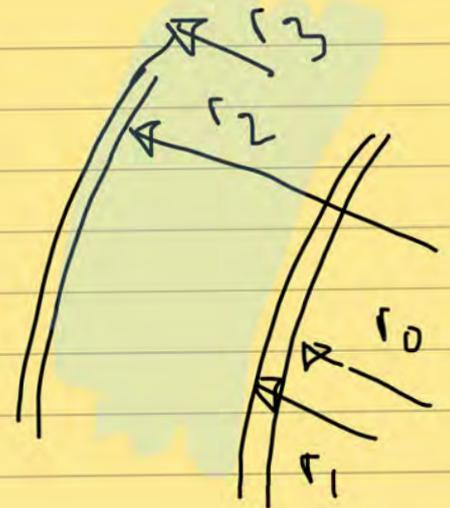
$$\left\{\frac{1}{r_2} - \frac{1}{r_1}\right\} = \frac{4\pi\lambda}{Q} (T_2 - T_1)$$

$$Q = \frac{T_1 - T_2}{R} \Rightarrow Q = \frac{-(T_2 - T_1)}{\frac{1}{4\pi\lambda} \left\{\frac{1}{r_2} - \frac{1}{r_1}\right\}}, R = \frac{1}{4\pi\lambda} \left\{\frac{1}{r_1} - \frac{1}{r_2}\right\}$$

ii)

$$300K \rightarrow \frac{1}{\lambda 4\pi r_3^2} \rightarrow \frac{r_3 - r_2}{4\pi\lambda r_3^2} \rightarrow \frac{1}{4\pi\lambda \left(\frac{1}{r_2} - \frac{1}{r_1}\right)} \rightarrow \frac{r_1 - r_0}{4\pi\lambda r_0^2} \rightarrow \frac{1}{\lambda 4\pi r_0^2} \rightarrow 20K$$

0.002 227.36 0.003

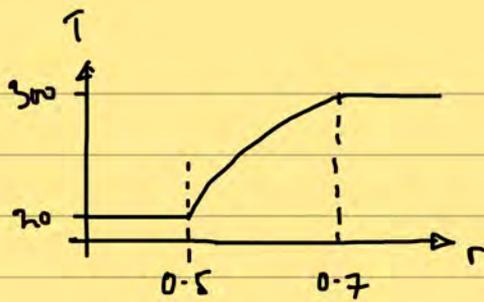


$$R_{\text{overall}} = 227.37 \text{ W}^{-1}\text{K}$$

$$\frac{R_{\text{overall}} - R_{\text{insulation}}}{R_{\text{overall}}} \times 100 = 0.002\% \text{ ERROR}$$

i.e. no error!

iii)



$$T = +Q \frac{1}{4\pi\lambda} \left\{ \frac{1}{r} - \frac{1}{r_1} \right\} + T_1$$

$$T = \underbrace{\frac{Q}{4\pi\lambda}}_{-c_2} \frac{1}{r} - \underbrace{\frac{Q}{4\pi\lambda} \frac{1}{r_1}}_{+c_1} + T_1$$

Note Q is negative

$$T = c_1 - c_2/r$$

iv)

$$Q = \frac{(300 - 20)}{227.4} = 1.23 \text{ W}$$

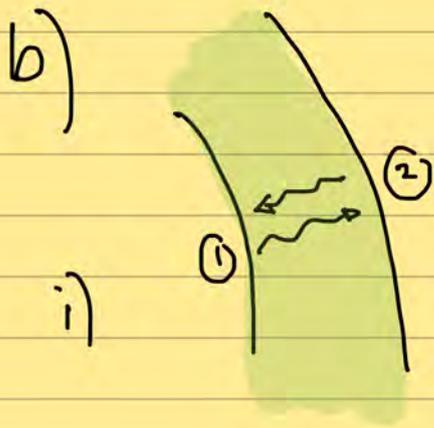
latent heat of evaporation 448.9 kJ kg^{-1}

IN 1 SECOND LOSS IS

$$\frac{\dot{Q}}{\Delta h} = \frac{1.22}{448.9 \times 10^3} \text{ [kg]}$$
$$= 2.72 \times 10^{-6} \text{ kg}$$

$$24 \text{ hours} = 24 \times 60 \times 60 \text{ seconds}$$
$$= 86400$$

$$\Rightarrow \text{loss per hour} = 0.235 \text{ kg/day}$$



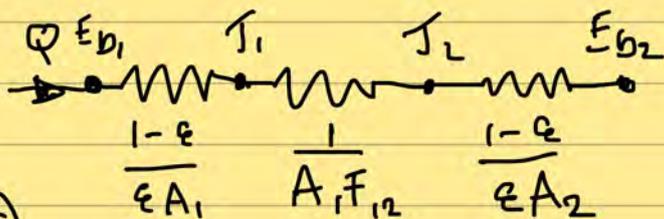
$$i) \sum_j F_{ij} = 1 \quad F_{11} + F_{12} = 1 \Rightarrow F_{12} = 1$$

$$F_{ij} A_i = F_{ji} A_j$$

$$F_{12} \pi r_1^2 = F_{21} \pi r_2^2$$

$$F_{21} = \left(\frac{r_1}{r_2} \right)^2 = \left(\frac{0.5}{0.7} \right)^2 = 0.51$$

what are T_1 & T_2 \rightarrow neglect thermal resistances except insulation.



$$Q = \frac{E_{b2} - E_{b1}}{\frac{1-\epsilon}{\epsilon A_1} + \frac{1}{A_1 F_{12}} + \frac{1-\epsilon}{\epsilon A_2}}$$

$$= \sigma (T_2^4 - T_1^4)$$

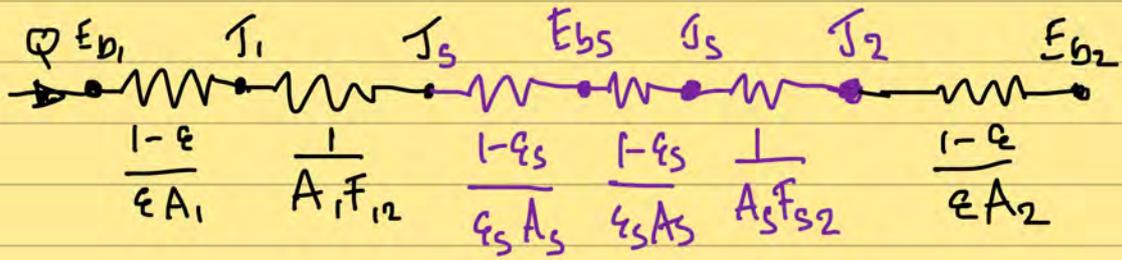
$$\frac{1-\epsilon}{\epsilon A_1} + \frac{1}{A_1} + \frac{1-\epsilon}{\epsilon A_2}$$

$$= \frac{5.67 \times 10^{-8} (300^4 - 20^4)}{\frac{1-0.1}{0.1 \times 4\pi \times 0.5^2} + \frac{1}{4\pi \times 0.5^2} + \frac{1-0.1}{0.1 \times 4\pi \times 0.7^2}}$$

$$\frac{1-0.1}{0.1 \times 4\pi \times 0.5^2} + \frac{1}{4\pi \times 0.5^2} + \frac{1-0.1}{0.1 \times 4\pi \times 0.7^2}$$

$$\frac{\dot{Q}_{radiation}}{\dot{Q}_{conduction}} = \frac{98.9}{1.2} = 80.3 = 98.9 \text{ W}$$

iii)

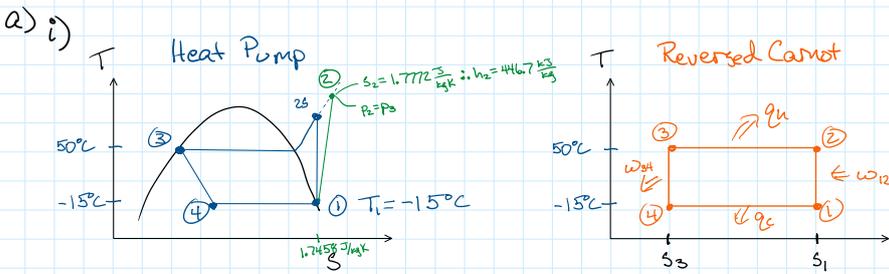


Extra terms.
from shield.

increasing
view factor always
↓

$$R_{\text{EXTRA}} = 2 \left\{ \frac{1-\epsilon}{\epsilon 4\pi r^2} \right\} + \frac{1}{4\pi r^2 \cdot 1}$$

$$= \left[2 \frac{(1-\epsilon)}{\epsilon} + 1 \right] \frac{1}{4\pi r^2}$$



ii) $COP_{HP} = \frac{q_h}{q_h - q_c} = \frac{1}{1 - q_c/q_h} = \frac{1}{1 - T_1/T_3}$ $COP_{HP} = \frac{1}{1 - \frac{273-15}{273+50}} = 4.97$

$q_h = \int_2^3 T ds = T_3 \Delta s_{2-3}$ $\Delta s_{2-3} = \Delta s_{4-1}$
 $q_c = \int_4^1 T ds = T_1 \Delta s_{4-1}$

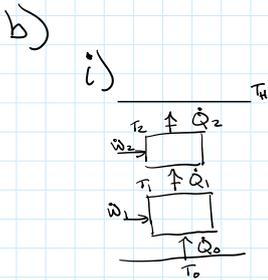
iii) ① $T_1 = -15^\circ C$ Sat Vap $p_1 = 1.64 \text{ bar}$ $s_1 = 1.7458 \text{ J/kg K}$ $h_1 = 389.6 \text{ kJ/kg}$

③ sat liquid at $T_3 = 50^\circ C$ $p_2 = p_3 = 13.18 \text{ bar}$

② → ③ Isobaric $p_3 = p_2$ $r_c = \frac{p_3}{p_1} = \frac{13.18}{1.64} = 8.0 = r_c$

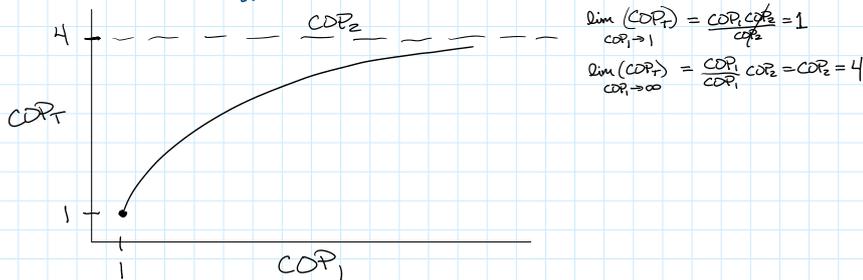
iv) $T_3 = 50^\circ C + 20^\circ C$ From Table $h_2 = 446.7 \text{ kJ/kg}$ $s_2 = 1.7772 \text{ kJ/kgK}$ superheat ③ sat liq @ $50^\circ C$ $h_3 = 271.6 \text{ kJ/kg}$

$COP = \frac{Q_H}{W_{in}} = \frac{h_2 - h_3}{h_2 - h_1} = \frac{446.7 \frac{\text{kJ}}{\text{kg}} - 271.6 \frac{\text{kJ}}{\text{kg}}}{446.7 \frac{\text{kJ}}{\text{kg}} - 389.6 \frac{\text{kJ}}{\text{kg}}} = 3.06$



$COP_1 = \frac{\dot{Q}_1}{\dot{W}_1} \Rightarrow \dot{W}_1 = \frac{\dot{Q}_1}{COP_1}$
 $COP_2 = \frac{\dot{Q}_2}{\dot{W}_2} = \frac{\dot{Q}_1 + \dot{W}_2}{\dot{W}_2} \Rightarrow \dot{W}_2 = \frac{\dot{Q}_1}{COP_2 - 1}$
 $\frac{\dot{W}_1}{\dot{W}_2} = \frac{COP_2 - 1}{COP_1}$

ii) $COP_T = \frac{\dot{Q}_2}{\dot{W}_2 + \dot{W}_1} = \frac{\dot{Q}_2}{\dot{W}_2 (1 + \frac{COP_2 - 1}{COP_1})} = \frac{COP_1 COP_2}{COP_1 + COP_2 - 1}$



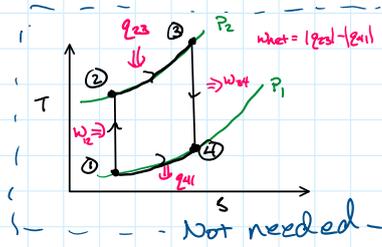
At low COP_1 the total COP_T is no better than 1 despite having $COP_2 = 4$.

For $COP_1 \rightarrow \infty$ the best the system can be is $COP_T = COP_2 = 4$

Conclusion: Heat pumps in series give poor performance.

$$2a) \quad \eta = \frac{\dot{q}_{23} + \dot{q}_{41}}{\dot{q}_{23}} = \frac{c_p(T_3 - T_2) - (T_4 - T_1)}{c_p(T_3 - T_2)}$$

$$= 1 - \frac{T_4 - T_1}{T_3 - T_2} = 1 - \frac{T_1(T_4/T_1 - 1)}{T_2(T_3/T_2 - 1)}$$



Iisentropic Relations $P_4/P_3 = (T_4/T_3)^{\gamma/\gamma-1} = (T_1/T_2)^{\gamma/\gamma-1} = P_1^{-1}$

$\therefore T_4/T_3 = T_1/T_2$

thus $T_2/T_1 = T_3/T_2$

$\therefore \eta = 1 - T_1/T_2 = 1 - P_1^{(1-\gamma)/\gamma}$

b)

i)

$$\eta_c = \frac{\dot{W}_{cs}}{\dot{W}_c} = \frac{\dot{m} c_p T_1 (1 - P_1^{-\gamma/\gamma})}{\dot{W}_c} = \frac{30 \text{ kg/s} \cdot 1.1 \frac{\text{kJ}}{\text{kgK}} \cdot 300 \text{ K} (1 - 20^{-0.4/1.4})}{-15 \text{ kW}} = 0.89$$

$$\eta_T = \frac{\dot{W}_T}{\dot{W}_{T3}} = \frac{\dot{W}_T}{\dot{m} c_p T_3 (1 - P_1^{-\gamma/\gamma})} = \frac{24 \text{ kW}}{30 \frac{\text{kg}}{\text{s}} \cdot 1.1 \frac{\text{kJ}}{\text{kgK}} \cdot 1500 \text{ K} (1 - 20^{-0.4/1.4})} = 0.84$$

ii)

$$\Delta \dot{B}_{12} = \dot{m} c_p (T_2 - T_1) - \dot{m} T_0 \left(c_p \ln \left(\frac{T_2}{T_1} \right) - R \ln \left(\frac{P_2}{P_1} \right) \right)$$

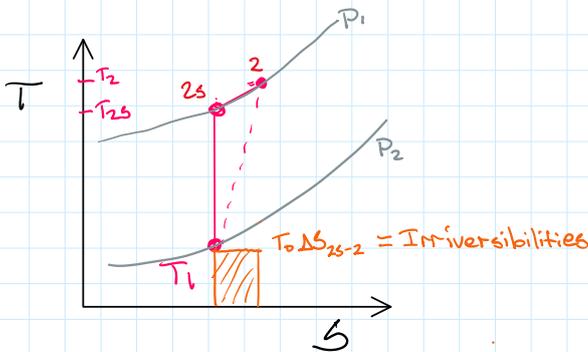
$$\dot{B}_{12} = 15,000 \text{ kW} - 30 \frac{\text{kg}}{\text{s}} \cdot 300 \text{ K} \left(1.1 \frac{\text{kJ}}{\text{kgK}} \ln \left[\frac{15,000}{30 \frac{\text{kg}}{\text{s}} \cdot 1.1 \frac{\text{kJ}}{\text{kgK}} + 300 \text{ K}} / 300 \text{ K} \right] - 0.314 \frac{\text{kJ}}{\text{kgK}} \ln [20] \right)$$

$\Delta \dot{B}_{12} = 14,335 \text{ kW}$

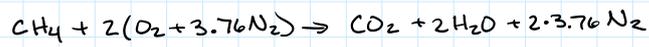
$\dot{W}_c = \dot{m} c_p (T_2 - T_1)$
 $T_2 = \dot{W}_c / (\dot{m} c_p) + T_1$

Given $c_p = 1.1 \frac{\text{kJ}}{\text{kgK}}$ $R = c_p - c_v$
 $\gamma = 1.4$ $\gamma = c_p / c_v$
 $R = c_p (1 - 1/\gamma)$
 $= 1.1 \frac{\text{kJ}}{\text{kgK}} (1 - 1/1.4) = 0.314 \frac{\text{kJ}}{\text{kgK}}$

$\dot{W}_{12} > \Delta \dot{B}_{12} > \dot{W}_{12s}$
 $15 \text{ MW} > 13.6 \text{ MW} > 13.4 \text{ MW}$



iii)



$$f_{\text{CO}_2} = \frac{M_{\text{CO}_2} n_{\text{CO}_2}}{M_{\text{CO}_2} n_{\text{CO}_2} + M_{\text{H}_2\text{O}} n_{\text{H}_2\text{O}} + M_{\text{N}_2} n_{\text{N}_2}} = \frac{44}{44 + 2 \cdot 18 + 2 \cdot 3.76 \cdot 28} = 0.15$$

$$\dot{m}_{\text{CO}_2} = \dot{m} f_{\text{CO}_2} = 30 \cdot 0.15 = 4.5 \text{ kg/s}$$

$$\Delta B = W_{\text{ccs}} = c_p \dot{m}_{\text{CO}_2} T_1 (1 - p_r^{\gamma-1/\gamma})$$

$$0.83 \text{ kJ/kgK} \cdot 4.5 \text{ kg/s} \cdot 300\text{K} (1 - 100^{0.31/1.31})$$

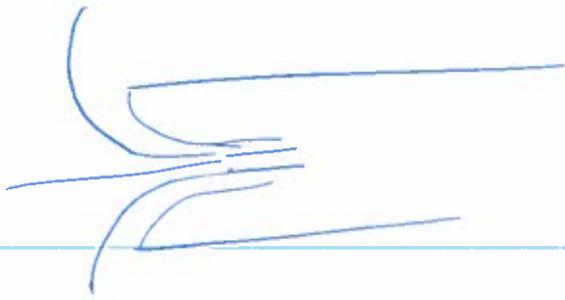
CO₂ @ 300K

$$c_p = 0.83$$

$$\gamma = 1.31$$

$$W_{\text{ccs}} = -2211.4 \text{ kW}$$

1.



a) Streamline curvature close to inlet
Zero streamline curvature in jet. [2]

b) No wall friction
Thin wall [no pressure loading on pipe] [3]

$$\text{SFME: } [P_0 - P_j] A = \rho [V_j^2 - 0] A_j$$



BERNOULLI:

$$P_0 = P_j + \frac{1}{2} \rho V_j^2$$

$$\rightarrow \frac{1}{2} \rho V_j^2 = \rho V_j^2 \times A_j / A$$

$$A_j = \frac{1}{2} A$$

$$d_j^2 = \frac{1}{2} D^2$$

$$d_j / D = \frac{1}{\sqrt{2}} \approx 0.71$$

[5]

d)

$$K = \frac{-\Delta P_0}{\frac{1}{2} \rho V^2}$$

$$P_{\text{mix}} = P_{\text{mix}} + \frac{1}{2} \rho V^2$$

$$\text{SFME} \rightarrow [P_0 - P_{\text{mix}}] A = \rho V^2 A$$

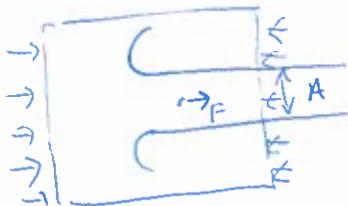
$$P_0 - (P_{\text{mix}} + \frac{1}{2} \rho V^2) = \rho V^2$$

$$P_0 - P_{\text{mix}} = \frac{1}{2} \rho V^2$$

$$K = 1.0$$

[5]

e)



$$F + [P_0 - P] A = \rho V^2 A$$

$$P_0 - P = \frac{1}{2} \rho V^2$$

$$F + [\frac{1}{2} \rho V^2] A = \rho V^2 A$$

$$\dot{m} = \rho V A$$

$$V = \dot{m} / \rho A$$

$$F = 2 \frac{\dot{m}^2}{\pi \rho D^2}$$

Applied on inlet

$$F = \frac{1}{2} \rho V^2 A = \frac{1}{2} \rho \left[\frac{\dot{m}}{\rho A} \right]^2 A$$

~~160~~

force $F = 2m^2 / \pi D^2$
on water

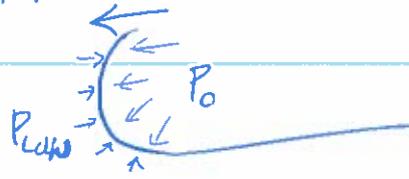
$$C_F = \frac{-F}{\pi D^2 \rho V^2} =$$

$$\frac{\frac{1}{2} \rho V^2 \pi D^2 / 4}{\pi D^2 \rho V^2} =$$

force on pipe $F_{\text{pipe}} = -F = -2m^2 / \pi D^2$

$$C_F = 1/8$$

(i.e. water
tight to left)



[6]



Shape parameter = d/D

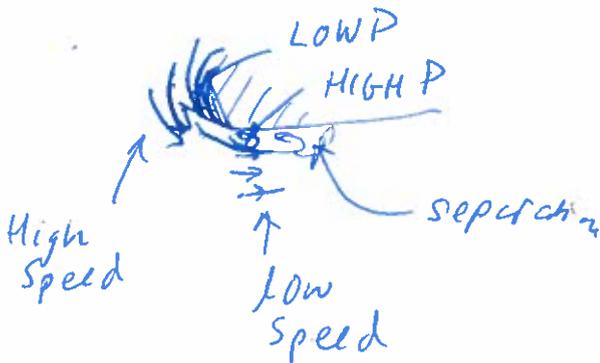
as $d/D \rightarrow 0$

streamline curvature increases
 $dp/dr \uparrow$

P_{min} reduces

→ Adverse pressure gradient rises

→ flow separates



Attached [4]



d/D large

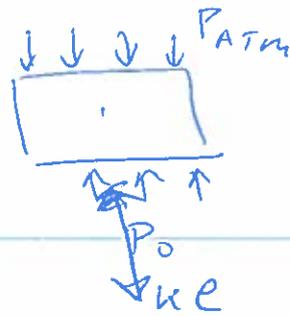
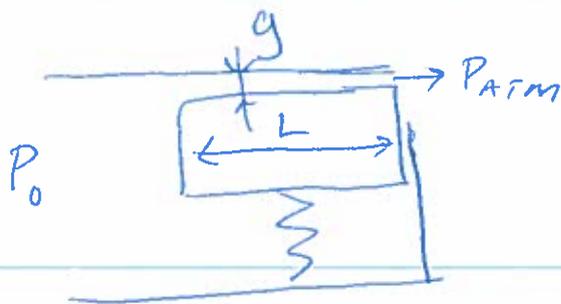
$$C_F = 1/8, K = 0.$$

$d/D \rightarrow 0$

$$C_F = 0, K = 1$$

2.

a)



$$[P_0 - P_{atm}] L = ke$$

Bernoulli
(loss free)

$$P_0 = P_{atm} + \frac{1}{2} \rho V^2$$

$$P_0 - P_{atm} = \frac{1}{2} \rho V^2$$

When $ke = 0$

$$P_0 = P_{atm}$$

$$g = 0.1 L$$

answer: $g = 0.1 L - e$

$$\dot{m} = \rho V g$$

$$= \sqrt{[P_0 - P_{atm}] \frac{2}{\rho}} \times \rho g$$

$$\dot{m} = \sqrt{2\rho(ke/L)} g$$

$$\dot{m} = \underbrace{\sqrt{\frac{2\rho k}{L}}}_C \times \sqrt{e} \times [0.1 L - e] \quad [6]$$

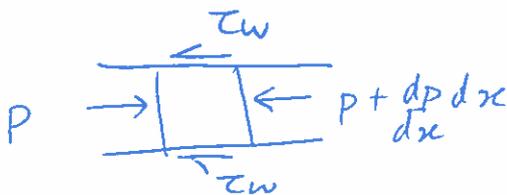
$$b) \frac{d\dot{m}}{de} = \frac{1}{2} C e^{-1/2} [0.1 L - e] - C \sqrt{e} = 0$$

$$2e = 0.1 L - e$$

$$e = 0.1 L / 3 \text{ for } \dot{m}_{max}$$

$$(\Delta P)_{\dot{m}_{max}} = \frac{ke}{L} = 0.1 L / 3 \times k/L = \frac{0.1}{3} k \quad [3]$$

c)



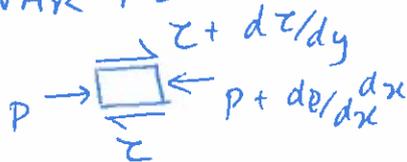
$$-\frac{dp}{dx} dx - 2\tau_w dx = 0$$

$$\rightarrow dp/dx = -2\tau_w/g \quad [4]$$

$$\frac{dp}{dx} = -\frac{dz}{dy} = -\mu \frac{d^2 u}{dy^2} = -\frac{2\tau_w}{g}$$

d)

LAMINAR FLOW



$$\frac{du}{dy} = -\frac{2\tau_w}{\mu g} y + B$$

$$u = -\frac{\tau_w}{\mu g} y^2 + By + C$$

$$u = 0$$

at $y=0$ and $y=g$

$$\rightarrow C = 0$$

$$u = \tau_w \frac{y}{\mu}$$

$$0 = \frac{-\tau_w}{\mu} g^2 + Bg$$

$$A = \frac{-\tau_w}{\mu g}$$

$$\rightarrow B = \frac{\tau_w}{\mu}$$

$$\rightarrow u = -\frac{\tau_w}{\mu g} y^2 + \frac{\tau_w}{\mu} y$$

$$u = \frac{\tau_w g}{\mu} \left[-\left(\frac{y}{g}\right)^2 + \left(\frac{y}{g}\right) \right]$$

$$V_b = \int u dy = \frac{\tau_w g}{\mu} \left[-\frac{y^3}{3g^2} + \frac{y^2}{2g} \right]_0^g$$

$$= \frac{\tau_w g}{\mu} \left[-\frac{1}{3} + \frac{1}{2} \right] = \frac{\tau_w g}{6\mu}$$

$$u = 6V_b \left[\left(\frac{y}{g}\right) - \left(\frac{y}{g}\right)^2 \right]$$

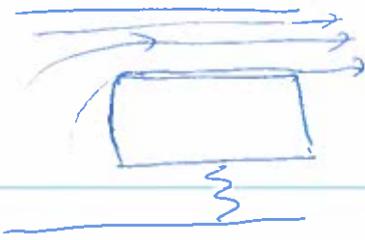
$$\alpha = -6V_b/g^2 \quad \beta = 6V_b/g \quad \tau_w = 0 \quad [7]$$

$$e) \quad C_f = \frac{\tau_w}{\frac{1}{2} \rho V_b^2} = \frac{\tau_w}{\frac{1}{2} \rho V_b^2} \quad \tau_w = \frac{V_b \cdot 6\mu}{g}$$

$$C_f = \frac{V_b \times 6\mu/g}{\frac{1}{2} \rho V_b^2} = \frac{12\mu}{\rho V_b g}$$

$$Re = \frac{\rho V_b g}{\mu} \rightarrow C_f = 12/Re$$

As Re rises frictional effects reduce.



$$P_0 \approx P_{atm}$$

~~low flow rate~~
low flow rate, laminar flow

$$g/L = 0.1$$



$$\hat{P}_i = P_{i_{max}}$$

high flow rate

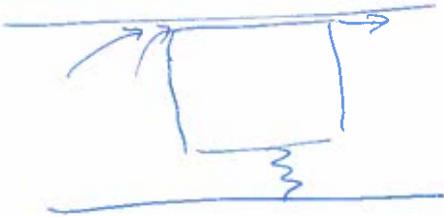
Re high

potential

~~boundary layer effects~~
~~separation~~

to transition

to turbulence



→ Re very small, low flow rate

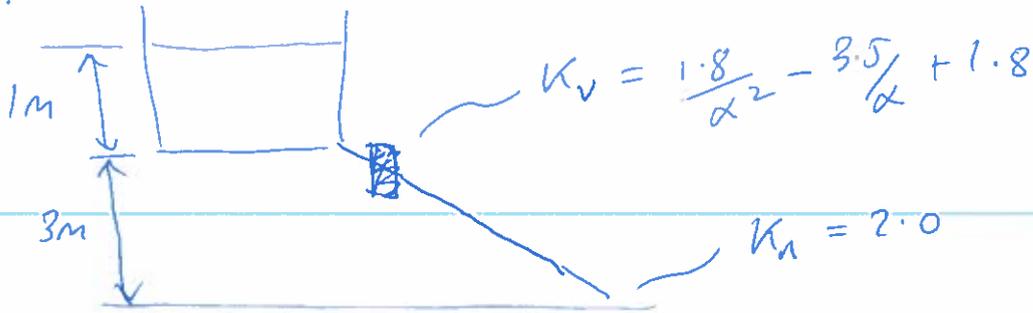
~~low flow rate~~

$g/L \rightarrow 0$, very viscous ~~low flow rate~~

laminar flow

[5]

3.



$$A_c V_e = A V$$

$$2 \cdot 10^2 V_e = 25^2 V$$

$$V_e = 6.25 V$$

[2]



straight +
parallel streamlines

→ pressure uniform → $dp/dr = 0$

[2]

$$c) \quad 4\rho g = \frac{1}{2} \rho V^2 [K_v + K_n] = \frac{1}{2} \rho V_e^2$$

$$\rightarrow \frac{1}{2} \rho V^2 [6.25^2 + K_v + K_n] = 4\rho g$$

$$V = \frac{7.5}{6.25} = 1.2 \text{ m/s}$$

$$V^2 =$$

$$\frac{8g}{41.0625 + K_v}$$

$$\rightarrow K_v + 41.0625 = 54.5$$

$$\rightarrow K_v = 13.4375 = \frac{1.8}{d^2} - \frac{3.5}{d} + 1.8$$

$$\rightarrow \frac{1.8}{d^2} - \frac{3.5}{d} - 11.6375 = 0$$

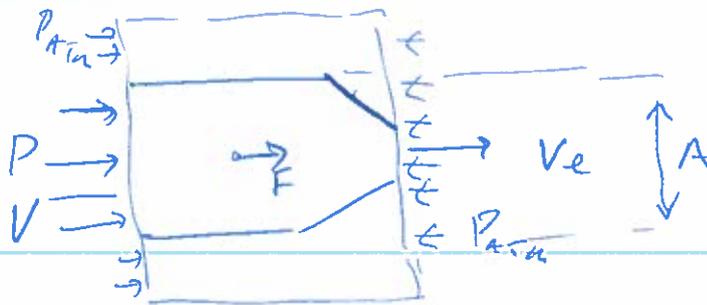
$$d^{-1} = \frac{3.5 \pm \sqrt{3.5^2 + 4 \times 1.8 \times 11.6375}}{3.6} = 3.6944$$

(reject -ve)

$$d = 0.2707$$

[6]

d)



$$A(P - P_{ATM}) + F = \cancel{\rho V^2 A} = \dot{m}(V_e - V)$$

$$= 1000 \times 1.2 \times \pi \left(\frac{0.025}{2}\right)^2 [7.5 - 1.2]$$

$$[P - P_{ATM}] + \frac{\rho}{2} [V^2 - V_e^2] = \frac{K_n \rho V^2}{2} = \rho V^2$$

$$P - P_{ATM} = -\rho/2 [1.2^2 - 7.5^2] + \rho(1.2^2)$$

$$= \rho \times 28.845 =$$

$$F = [7560 - 28845] \pi \left(\frac{0.025}{2}\right)^2 = -10.45 \text{ N}$$

force on fluid ←

force on nozzle →

e)

~~4pg + \Delta P_{pump} = \frac{1}{2} \rho V^2 [K_r + K_n] = \frac{1}{2} \rho V_e'^2~~

$$V' = 2V$$

$$\rightarrow 4pg + \Delta P_{pump} = 4 \left[\frac{1}{2} \rho V^2 [K_r + K_n] + \rho V_e'^2 \right]$$

$$\boxed{\Delta P_{pump} = 12pg}$$

Equivalent to increasing depth in tank

by ~~12~~ 12 metres!

i.e. total * 16pg $\left(v \propto \sqrt{h} \right)$
 $h \propto v^2$

[5]

$$f) \quad \text{Power} = Q \Delta P_0$$

Q_{max} when $\alpha = 1.0$ (hence max power)

$$\rightarrow \quad \cancel{V_0} = 1.8 - 3.5 + 1.8 = 0.1$$

$$V = 2 \sqrt{\frac{8g}{41.1625}} = \overset{2.746}{\cancel{1.777}} \text{ m/s}$$

$$\begin{aligned} \rightarrow \quad \text{Power} &= 12 \rho g \times \overset{2.746}{\cancel{1.777}} \times \pi \left(\frac{0.025}{2}\right)^2 \\ &= \frac{158.7}{\cancel{1.777}} \text{ Watts} \end{aligned} \quad (4)$$