

ENGINEERING TRIPOS PART IB

Wednesday 6 June 2012

2 to 4

Paper 5 - SOLUTIONS

ELECTRICAL ENGINEERING

SECTION A

1 (a) For the circuit of Fig. 1, the gain is $-R_2/R_1$, the input impedance is R_1 and the output impedance is 0. [2]

(b) For an ideal op. amp., the voltage at the two input terminals is the same. Therefore, if the non-inverting terminal is at 0 V, then so is the inverting terminal. If we call V_x the voltage at the node between resistor R_4 and the two resistors marked R_3 then, performing a nodal voltage analysis at the inverting input, gives

$$\sum I_{out} = 0 = \frac{-V_{in}}{R_3} - \frac{V_x}{R_3} \Rightarrow V_x = -V_{in} \quad (1)$$

Performing a nodal voltage analysis at the V_x node, gives

$$\begin{aligned} \sum I_{out} = 0 &= \frac{V_x}{R_3} + \frac{V_x}{R_4} + \frac{V_x - V_{out}}{R_3} \\ \frac{V_{out}}{R_3} &= V_x \left(\frac{2}{R_3} + \frac{1}{R_4} \right) \end{aligned}$$

Substituting for (1) gives

$$\frac{V_{out}}{V_{in}} = - \left(2 + \frac{R_3}{R_4} \right) \quad [5]$$

(c) (i) For the circuit of Fig. 1, the gain, G (from 1(a)) is $-R_2/R_1$ and R_1 is the input impedance, which are required to be 240 k Ω and -50 respectively, so

$$R_2 = -GR_1 = -(-50) \cdot 240 \times 10^3 = \underline{12 \text{ M}\Omega} \quad [2]$$

(ii) For the circuit of Fig. 2, the input impedance is R_3 , and the gain, G , is the expression in part (b), so given the same requirements for these values as is part (i)

$$R_4 = \frac{-R_3}{G+2} = \frac{-240 \times 10^3}{-50+2} = \underline{5 \text{ k}\Omega} \quad [2]$$

(iii) The circuit of Fig. 2 is preferable, as it will be easier to source a 5 k Ω resistor with a precise value than a 12 M Ω resistor of similar precision to give an accurate gain of -50 . [2]

(d) WE need an op. amp. With an input impedance at least 100 times greater than the circuit input impedance of $240 \text{ k}\Omega$ (otherwise our ideal op. amp. Assumption will be incorrect), so at least $10 \text{ M}\Omega$ is required. The output impedance of the op. amp. should be at least 100 times smaller than the circuit output impedance (again so we can assume ideality), so less than $\sim 1 \text{ k}\Omega$. Therefore Type C is the appropriate choice. [3]

(e) The gain of an amplifier in terms of its open loop gain, A , and feedback fraction, B , is $G = A/(1 + AB)$. For this op. amp., A is very large, and tending to infinity to a good approximation, so

$$G = \frac{1}{\frac{1}{A+B}} \xrightarrow{as A \rightarrow \infty} \frac{1}{B}$$

Hence, for this circuit,

$$B = \frac{1}{G} = \frac{1}{50}$$

The cut-off frequency is increased by a factor of $(1 + AB) = (1 + 10^6 \cdot [1/50]) = 20 \times 10^3$ for the circuit. If the cut-off frequency of the op. amp. is 10 Hz , then that of the circuit will be $20 \times 10^3 \cdot 10 = \underline{200 \text{ kHz}}$. [4]

2 (a) The signal into a differential amplifier applied at its two inputs may be deconstructed into a common mode and a differential mode component. A good differential amplifier will amplify the differential signal much more than the common mode signal (which may be due to effects such as low frequency pick up). The ratio of the differential to common mode gain is known as the common mode rejection ratio (CMRR) and this should ideally be infinite. [4]

(b) If h_{FE} is infinite, then the current into the base, I_B , is zero. Therefore, the voltage at the base, V_B , may be determined by potential division of the resistors R_1 and R_2 ,

$$V_B = -15 \cdot \frac{9.1}{2.7+9.1} = -11.57 \text{ V}$$

We are told that V_{BE} is 0.6 V, so the voltage at the emitter, V_E , is

$$V_E = -11.57 - 0.6 = -12.17 \text{ V}$$

Hence, the current at the emitter, I_E , is that through R_3 , which is given by

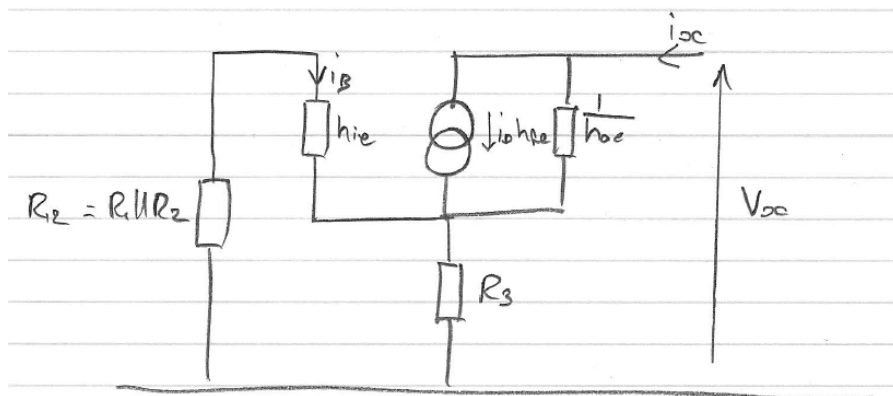
$$I_E = \frac{-12.17 - (-15)}{2.7 \times 10^3} = 1.05 \text{ mA}$$

Finally, if I_B is zero, then the collector and emitter currents must be the same, so

$$\underline{I_C = 1.05 \text{ mA}}$$

[4]

(c) (i) The small signal circuit is:



(ii) We know that $(1 + R_3 h_{oe})$ is negligible compared with the rest of the contents of the brackets in the expression for Z_c . Also, $h_{fe} R_3 \gg h_{oe} R_3^2$, so the expression for Z_c simplifies to

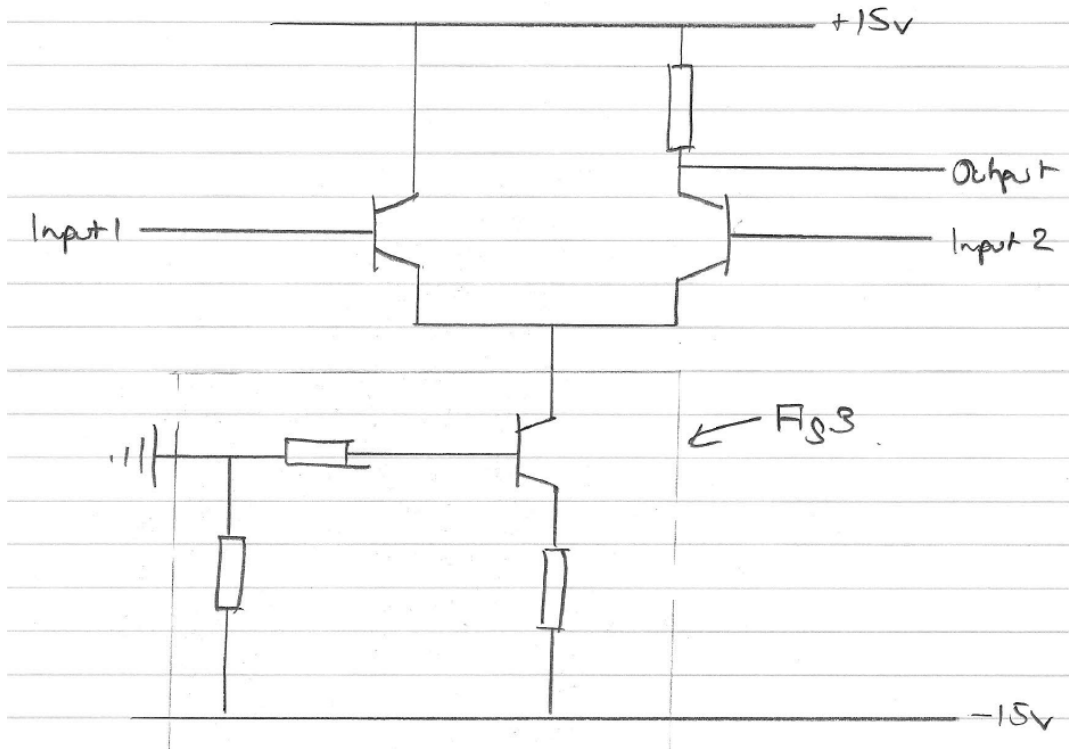
$$Z_c \approx \frac{1}{h_{oe}} \left(\frac{h_{fe} R_3}{R_3 + h_{ie} + R_{12}} \right) \quad (1)$$

Now, R_{12} is the resistance of R_1 and R_2 in parallel, which is 2.80 k Ω , so

$$Z_c = \frac{1}{200 \times 10^6} \left(\frac{250 \times 2.7 \times 10^3}{2.7 \times 10^3 + 1.5 \times 10^3 + 2.08 \times 10^3} \right) = \underline{537 \text{ k}\Omega} \quad [2]$$

(iii) Z_c scales with h_{fe} from the reduced equation (1) in part (ii). Hence, the tolerance on h_{fe} is also $\pm 10\%$. Therefore, the range of h_{fe} is between 225 and 275. [3]

(d) The circuit for the differential amplifier incorporating the circuit of Fig. 3 is



We want to sink a constant current through the tail of the long-tailed pair circuit. Hence, the output will not be affected by a common mode signal, but only the differential, resulting in a high CMRR.

SECTION B

3 (a) The advantages of using a per-unit system for analysing power supply networks are that all per-unit quantities are of a similar value and transformers are eliminated, factors of 3 and $\sqrt{3}$ are eliminated. [2]

(b) We can calculate the line current by simply considering the real power consumed by the town as

$$I_l = \frac{P}{\sqrt{3}V_l \cos \phi} = \frac{70 \times 10^6}{\sqrt{3} \times 9.9 \times 10^3 \times 0.85} = \underline{4800 \text{ A}} \quad [3]$$

(c) (i) The approach should be to use a per-unit calculation. First, we need to choose a base apparent power for the whole system, and 500 MVA is appropriate. We then need to scale the per-unit reactances for the transformers that are not rated at the base apparent power. As the step-up transformer is rated at 250 MVA, we need to multiply its per-unit reactance by two for a 500 MVA base, so its per-unit reactance becomes 0.2 pu. Similarly, for the step-down transformer, we must multiply by four to give a reactance of 0.4 pu.

Next, we need to define base voltages. It is normal to choose the rated voltage for the system, as this should scale appropriately with turns ratio of the transformers. Hence, the base voltage at the town is 11 kV. NOTE: many students incorrectly chose the actual voltage at the town as the base value at this stage. You can make this work by using the turns ratio to calculate the base voltage everywhere else. You would also need to recalculate the reactances of the two transformers as well. This is significantly more (and rather needless!) work.

Taking V_b at the town to be the rated voltage of 11 kV, we can calculate the base current at the town to be

$$I_b = \frac{VA_b}{\sqrt{3}V_b} = \frac{500 \times 10^6}{\sqrt{3} \cdot 11 \times 10^3} = 26240 \text{ A}$$

We can now calculate the per-unit line current as

$$I_{lpu} = \frac{I_l}{I_b} = \frac{4800}{26240} = 0.183 \text{ pu}$$

For the transmission line, the base voltage is 132 kV (using the rated values as discussed), so the base impedance over the transmission line is

$$Z_b = \frac{V_b^2}{VA_b} = \frac{(132 \times 10^3)^2}{500 \times 10^6} = 34.85 \Omega$$

Hence, the per-unit impedance of the transmission line is

$$Z_{lpu} = \frac{10 + 15j}{34.85} = (0.287 + 0.430j) \text{ pu}$$

Hence the per-unit power dissipated in the line is

$$\begin{aligned} P_{pu} &= I_{lpu}^2 R_{lpu} = 0.183^2 \times 0.287 = 9.61 \times 10^{-3} \text{ pu} \\ P &= P_{pu} VA_b = 9.61 \times 10^{-3} \cdot 500 \times 10^6 = \underline{4.8 \text{ MW}} \end{aligned} \quad [4]$$

(ii) To calculate the generator excitation voltage, we need to know the total real and reactive power consumed by the system. We will start with reactive power. For the town, we know that the power factor $\cos \phi = 0.85$, so $\phi = 31.79^\circ$. Therefore,

$$Q_{town} = P_{town} \tan \phi = 70 \times 10^6 \tan 31.79^\circ = 43.4 \text{ MVAR}$$

To calculate the reactive power dissipated in the rest of the system, we simply add up all of the reactances of the step-down transformer (0.4 pu), the transmission line (0.430 pu), the step-up transformer (0.2 pu) and the generator (0.2 pu) to give a total reactance of 1.230 pu. The total reactive power is then

$$\begin{aligned} Q_{suppu} &= I_{lpu}^2 X_{sup} = 0.183^2 \times 1.230 = 4.12 \times 10^{-2} \text{ pu} \\ Q_{sup} &= Q_{suppu} VA_b = 4.12 \times 10^{-2} \times 500 \times 10^6 = 20.6 \text{ MVAR} \end{aligned}$$

The total reactive power is that of the supply and that of the load, which is

$$Q_{tot} = 20.6 + 43.4 = 64.0 \text{ MVAR}$$

Similarly, the total real power is that of the supply calculated in part (i) and that of the town, which is

$$P_{tot} = 70 + 4.8 = 74.8 \text{ MW}$$

The total apparent power is therefore

$$S_{tot} = \sqrt{(P_{tot}^2 + Q_{tot}^2)} = \sqrt{74.8^2 + 64.0^2} = 98.4 \text{ MVA}$$

We can re-express this as a per-unit apparent power as

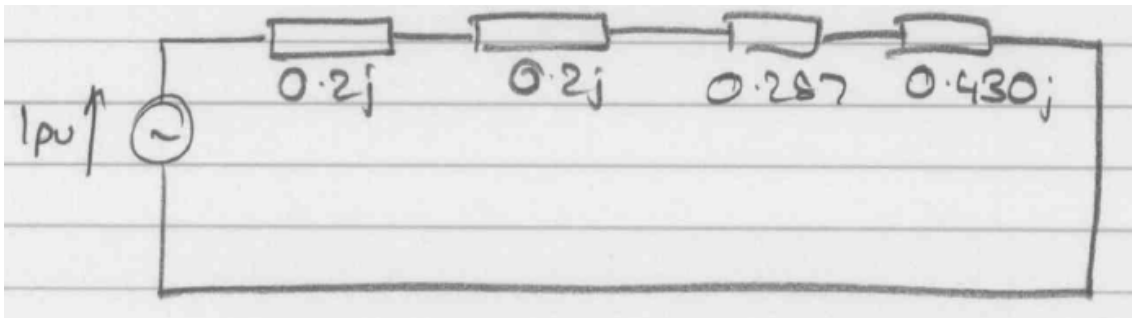
$$S_{totpu} = \frac{S_{tot}}{VA_b} = \frac{98.4}{500} = 0.197 \text{ pu}$$

Hence, the line voltage at the generator is

$$V_{lpu} = \frac{S_{totpu}}{I_{lpu}} = \frac{0.197}{0.183} = 1.077 \text{ pu}$$

$$V_l = V_{lpu} V_b = 1.077 \times 22 \times 10^3 = \underline{23.68 \text{ kV}} \quad [7]$$

(iii) Under the fault condition at point F, the per-unit circuit becomes



The total per-unit impedance of the circuit is then $(0.287 + 0.830j)$ pu. Therefore, the per-unit fault current is

$$I_{Fpu} = \frac{1}{\sqrt{0.287^2 + 0.830^2}} = 1.139 \text{ pu}$$

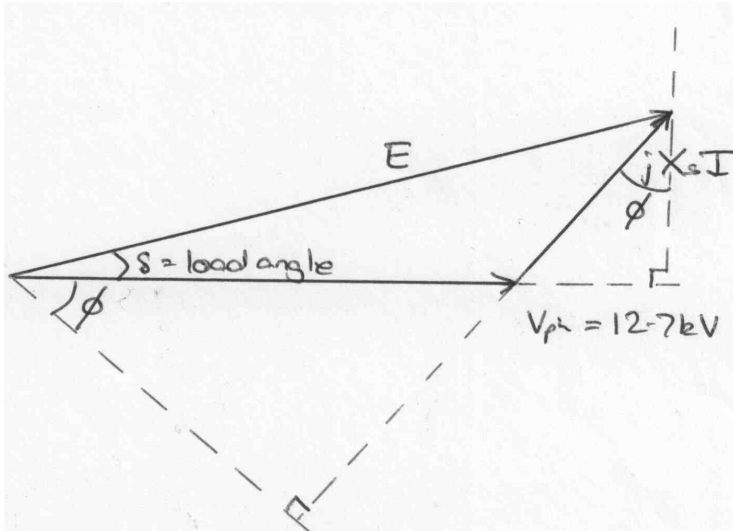
For the transmission line, the base current is

$$I_b = \frac{VA_b}{\sqrt{3}V_b} = \frac{500 \times 10^6}{\sqrt{3} \cdot 132 \times 10^3} = 2187 \text{ A}$$

Hence,

$$I_F = I_{Fpu} I_b = 1.139 \times 2187 = \underline{2491 \text{ A}} \quad [4]$$

4 (a) To draw the phasor diagram for one phase, we need to know that the phase voltage $V_{ph} = V_l/\sqrt{3} = 22/\sqrt{3} = 12.7$ kV, and that the phase angle between the voltage and current is $\phi = \cos^{-1} 0.75 = 41.1^\circ$. The diagram, therefore, is



[4]

(b) As state in the question, the generator is operating at rated MVA (*not* at rated MW as used by many candidates who attempted this question) and so we can simply calculate the phase current as

$$I_{ph} = \frac{S}{3V_{ps}} = \frac{600 \times 10^6}{3 \times 12.7 \times 10^3} = \underline{15748 \text{ A}}$$

The excitation can then be calculated using the cosine rule applied to the phasor diagram as

$$\begin{aligned} E^2 &= V_{ph}^2 + (X_s I)^2 - 2V_{ph} X_s I \cos(90^\circ + \phi) \\ &= (12.7 \times 10^3)^2 + (1.5 \times 15748)^2 - 2 \cdot (12.7 \times 10^3) \cdot (1.5) \cdot 15748 \cos(90^\circ + 41.1^\circ) \\ E^2 &= 1.116 \times 10^9 \end{aligned}$$

$$\underline{E = 33.4 \text{ kV}}$$

Finally, we can calculate the load angle by equating the vertical components of E and $X_s I$ as

$$E \sin \delta = X_s I \cos \phi$$

$$\therefore \delta = \sin^{-1} \left(\frac{X_s I \cos \phi}{E} \right) = \sin^{-1} \left(\frac{1.5 \times 15748 \times 0.75}{33.4 \times 10^3} \right) = \underline{32.0^\circ} \quad [5]$$

(c) The excitation has been increased to E' . The system is operating at rated MVA still, and therefore there will be a change in power (many candidates incorrectly assumed constant power output). We can calculate the new load angle δ' using the cosine rule again, as

$$\begin{aligned} \cos(\delta') &= \frac{E^2 + V_{ph}^2 - (X_s I)^2}{2E'V_{ph}} \\ \cos(\delta') &= \frac{(1.05 \times 33.4 \times 10^3)^2 + (12.7 \times 10^3)^2 - (1.5 \times 15748)^2}{2 \times 1.05 \times 33.4 \times 10^3 \times 12.7 \times 10^3} = 0.935 \\ &\quad \underline{\delta' = 20.7^\circ} \end{aligned}$$

The power factor may then be found by comparing the vertical components of E' and $X_s I$ again as

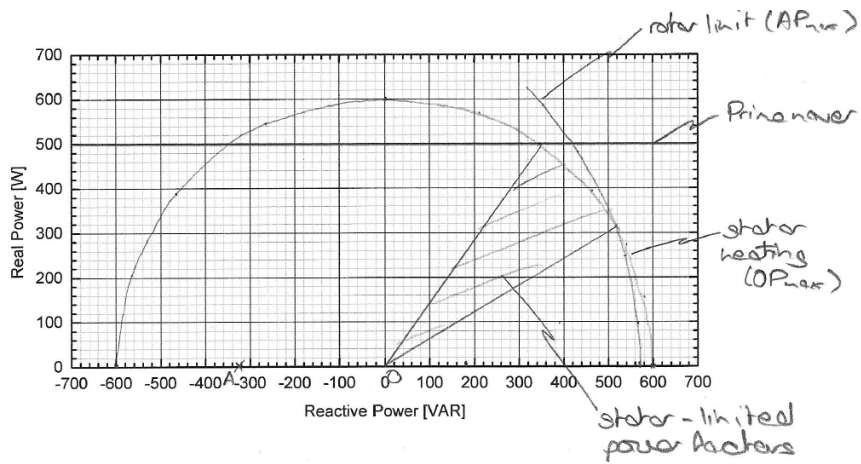
$$\cos \phi = \frac{E' \sin \delta'}{X_s I} = \frac{1.05 \times 33.4 \times 10^3 \sin 20.7^\circ}{1.5 \times 15748} = \underline{0.52} \quad [5]$$

(d) To draw the operating chart, we need to determine the length AO, which is

$$AO = \frac{3V_{ph}^2}{X_s} = \frac{3 \cdot (12.7 \times 10^3)^2}{1.5} = 322.6 \text{ MVAR}$$

The maximum length of OP is the stator heating limit of 600 MVA, which is a circle of this length around O. The maximum length of AP is

$$AP_{max} = \frac{3V_{ph}E_{ph(max)}}{X_s} = \frac{3 \times 12.7 \times 10^3 \times 60.6 \times 10^3}{\sqrt{3} \times 1.5} = 890 \text{ MVA}$$



From the figure, the range of ϕ is 31.2° to 55.0° for which stator heating is the limit, leading to a range of power factors of 0.518 to 0.819.

[6]

5 (a) The physical significance of each of the six components in the equivalent circuit is

R_1	Stator winding resistance
X_1	Stator leakage reactance
R_2'	Rotor winding resistance referred to the stator
X_2'	Rotor leakage reactance referred to the stator
X_m	Magnetising reactance
R_i	Iron loss resistance

The slip, s , is the difference (fractionally) between the stator field speed and the physical rotor speed. The excitation of the rotor coil is dependent on this slip being non-zero. R_2' must be scaled by $1/s$ to allow this effect to be accounted for in the equivalent circuit. [4]

(b) (i) Let the phase current (which is also equal to the line current as this is star-connected) be I . We are told that we can ignore the parallel loss components, so the total power input is

$$P_{in} = 3I^2 \left(R_1 + \frac{R_2'}{s} \right)$$

The component R_2'/s is made up from the actual referred rotor resistance R_2' , which is a loss term, and the remainder in which useful power is output. Hence, power losses are given by

$$P_{loss} = 3I^2(R_1 + R_2')$$

By conservation of power, the output power is therefore

$$P_{out} = P_{in} - P_{loss} = 3I^2 \left(R_1 + \frac{R_2'}{s} \right) - 3I^2(R_1 + R_2') = \frac{3I^2 R_2'}{s} (1 - s)$$

This can then be related to mechanical torque by

$$P_{out} = T\omega_r = T\omega_s(1 - s) = \frac{3I^2 R_2'}{s} (1 - s)$$

where ω_r and ω_s are the rotor and stator rotation speeds respectively. By rearranging this, we have

$$T = \frac{3I^2 R'_2}{s\omega_s} \quad (1)$$

which is the equation in the Data Book. However,

$$\begin{aligned} |V_{ph}| &= |I_{ph}| |Z| = I \sqrt{\left(R_1 + \frac{R'_2}{s}\right)^2 + (X_1 + X'_2)^2} \\ \therefore I^2 &= \frac{V_{ph}^2}{\left(R_1 + \frac{R'_2}{s}\right)^2 + (X_1 + X'_2)^2} \end{aligned}$$

Hence, by substituting into equation (1), we gain the required result that

$$T = \frac{3V_{ph}^2 R'_2}{\left[\left(R_1 + \frac{R'_2}{s}\right)^2 + (X_1 + X'_2)^2\right] s\omega_s} \quad [7]$$

(ii) Maximum torque occurs when we dissipate maximum power in R'_2/s , which according to the maximum power transfer theorem occurs when

$$\begin{aligned} R'_2/s &= |Z_{in}| = \sqrt{R_1^2 + (X_1 + X'_2)^2} \\ \therefore s &= \frac{R'_2}{\sqrt{R_1^2 + (X_1 + X'_2)^2}} \quad [5] \end{aligned}$$

(iii) For maximum power output, we divide R'_2/s into the loss term R_2 and the output term, as

$$\frac{R'_2}{s} = R_2 + R'_2 \frac{(1-s)}{s}$$

From the maximum power transfer theorem,

$$\begin{aligned} R'_2 \frac{(1-s)}{s} &= |Z_{in}| = \sqrt{(R_1 + R'_2)^2 + (X_1 + X'_2)^2} \\ \therefore s &= \frac{R'_2}{R_2 + \sqrt{(R_1 + R'_2)^2 + (X_1 + X'_2)^2}} \quad [4] \end{aligned}$$

SECTION C

- 6 (a) We first determine the power in the laser pulse from

$$\text{Pulse power} = \frac{\text{Energy}}{\text{Time}} = \frac{25}{5} = 5 \text{ W}$$

We now want the power per unit area, \bar{N} , which is

$$\bar{N} = \frac{\text{Power}}{\pi r^2} = \frac{5}{\pi(0.5 \times 10^{-3})^2} = 6.37 \times 10^6 \text{ W m}^{-2}$$

From the Poynting Vector, $\bar{N} = |E||H|/2$ and the fact that the characteristic impedance of free space, $\eta = E/H = 377 \Omega$, we have that

$$\begin{aligned} |H| &= \sqrt{\frac{2\bar{N}}{\eta}} = \sqrt{\frac{2 \times 6.37 \times 10^6}{377}} = \underline{183.8 \text{ A m}^{-1}} \\ |E| &= \eta |H| = 377 \times 183.8 = \underline{69.3 \text{ kV m}^{-1}} \end{aligned} \quad [5]$$

- (b) (i) We need to calculate the angular frequency of the light, which is

$$\omega = \frac{2\pi c}{\lambda} = \frac{2\pi \times 2.998 \times 10^8}{300 \times 10^9} = 6.28 \times 10^{15} \text{ rad s}^{-1}$$

In a conductor where $\sigma \gg \omega\epsilon$, the equation for the characteristic impedance of a conductive medium from the Data Book becomes

$$\eta = \sqrt{\frac{j\omega\mu}{\sigma}} = \sqrt{\frac{j6.28 \times 10^{15} \times 4\pi \times 10^{-7}}{10^6}} = (1 + j)62.8 \Omega$$

We now need the reflection coefficient for waves entering the conductor from free space, which is

$$\begin{aligned} \rho &= \frac{\eta_2 - \eta_1}{\eta_2 + \eta_1} = \frac{(1+j)62.8 - 377}{(1+j)62.8 + 377} = -0.68 + 0.24j \\ &\therefore |\rho| = 0.72 \end{aligned}$$

$|\rho|^2$ is the energy reflected at the interface, and so the energy which enters the film is $1 - |\rho|^2 = 0.481$, so the energy entering the film is

$$E_{in} = 25 \times 0.481 = \underline{12.0 \text{ mJ}} \quad [2]$$

(ii) For a conductor where $\sigma \gg \omega\epsilon$, the equation for the propagation constant of a conductive medium in the Data Book becomes

$$\begin{aligned} \gamma &= \sqrt{j\omega\mu\sigma} = \sqrt{j6.28 \times 10^{15} \times 4\pi \times 10^{-7} \times 10^6} \\ \gamma &= \alpha + j\beta = (1 + j)6.28 \times 10^7 \end{aligned}$$

Power decays with distance, d , according to the real part of the propagation constant as $\exp(-2\alpha d)$. We want power to decay by a factor of 10^{-2} , so

$$\begin{aligned} 10^{-2} &= \exp(-2\alpha d) \\ -4.60 &= -2\alpha d \\ \therefore d &= \frac{4.60}{2\alpha} = \frac{4.60}{2 \times 6.28 \times 10^7} = \underline{37 \text{ nm}} \end{aligned}$$

Therefore, this set up is well designed to absorb the majority of the power in a 50 nm thick film.

(c) The power density in the electromagnetic wave at the breakdown strength of air is

$$|N| = \frac{|E|^2}{2\eta} = \frac{(3 \times 10^6)^2}{2 \times 377} = 1.19 \times 10^{10} \text{ W m}^{-2}$$

The increase in power as a factor compared with part (a) is $1.19 \times 10^{10} / 6.37 \times 10^6 = 1874$. This is the area factor increase, so

$$\begin{aligned} \frac{r_{new}^2}{r_{old}^2} &= \frac{1}{1874} \\ \therefore r_{new} &= \frac{r_{old}}{\sqrt{1874}} = \frac{0.5 \times 10^{-3}}{\sqrt{1874}} = \underline{11.6 \text{ } \mu\text{m}} \quad [4] \end{aligned}$$

7 (a) To show that the velocity of the wave is $1/\sqrt{LC}$, we need to show that the Telegrapher's Equations can be converted into a wave equation. To do this, we differentiate the first of the two equations with respect to x ,

$$\frac{\partial^2 V}{\partial x^2} = -L \frac{\partial}{\partial x} \left(\frac{\partial I}{\partial t} \right) = -L \frac{\partial}{\partial t} \left(\frac{\partial I}{\partial x} \right)$$

We can now substitute from the other Telegrapher's Equation to give

$$\frac{\partial^2 V}{\partial x^2} = LC \frac{\partial^2 V}{\partial t^2}$$

This is a standard wave equation of the form $\nabla^2 \psi = \frac{1}{v^2} \frac{\partial^2 \psi}{\partial t^2}$ where v is the velocity of the wave. Hence by comparison with equation (1), we can see that

$$\underline{v = 1/\sqrt{LC}}$$

(b) The characteristic impedance, Z_0 , of a transmission line is the ratio of voltage to current at any point in the absence of reflections. Applying the first of the two Telegrapher's Equations to the equations given for V_F and I_F gives

$$\begin{aligned} \frac{\partial V_F}{\partial x} &= -j\beta V_F e^{j(\omega t - \beta x)} = -Lj\omega I_F e^{j(\omega t - \beta x)} = -L \frac{\partial I_F}{\partial t} \\ \therefore Z_0 &= \frac{V_F}{I_F} = \frac{L\omega}{\beta} \end{aligned}$$

However, $\omega = 2\pi f$ and $\beta = 2\pi/\lambda$, so

$$Z_0 = Lf\lambda$$

and $f\lambda = v = 1/\sqrt{LC}$, so

$$\underline{Z_0 = \sqrt{\frac{L}{C}}} \quad [4]$$

(c) (i) We know the capacitance per unit length of the line and its characteristic impedance, so from the result to part (b) we can calculate the inductance per unit length to be

$$L = CZ_0^2 = 100 \times 10^{-12} \cdot 50^2 = 250 \text{ nH m}^{-1}$$

Therefore, the velocity can be found as

$$v = \frac{1}{\sqrt{LC}} = \frac{1}{\sqrt{250 \times 10^{-9} \times 100 \times 10^{-12}}} = \underline{2.0 \times 10^8 \text{ m s}^{-1}}$$

(i.e. two-thirds of the velocity of an electromagnetic wave in free space). A different dielectric would have to be used with a lower relative permittivity if the velocity of the wave is to be increased. [2]

(ii) The VSWR is given by

$$\begin{aligned} \text{VSWR} &= \frac{\text{Maximum voltage}}{\text{Minimum voltage}} = \frac{1+\rho}{1-\rho} \\ \therefore \rho &= \frac{\text{VSWR}-1}{\text{VSWR}+1} = \frac{1.5-1}{1.5+1} = 0.2 \end{aligned}$$

We can relate the reflection coefficient to the impedance of the transmission line and its load according to

$$\begin{aligned} \rho &= \frac{Z_L - Z_0}{Z_L + Z_0} \\ Z_L &= Z_0 \frac{1+\rho}{1-\rho} = 50 \frac{1+0.2}{1-0.2} = \underline{75 \Omega} \end{aligned}$$

NOTE: it is also acceptable to use a reflection coefficient of -0.2 , in which case the load impedance is 33.3Ω . [5]

(iii) From the Data Book,

$$Z_{in} = Z_0 \frac{Z_L + Z_0 j \tan(\beta l)}{Z_0 + Z_L j \tan(\beta l)}$$

We want $Z_{in} = Z_L$, so $\tan(\beta l) = 0$, and therefore

$$\begin{aligned} \beta l &= \pi \\ \therefore l &= \frac{\pi}{\beta} = \frac{\pi}{2\pi/\lambda} = \frac{\lambda}{2} = \frac{v}{2f} = \frac{2 \times 10^8}{2 \times 10 \times 10^6} = \underline{1 \text{ m}} \end{aligned} \quad [4]$$

ENGINEERING TRIPOS PART IB

Wednesday 6 June 2012

2 to 4

Paper 5 – NUMERICAL SOLUTIONS

ELECTRICAL ENGINEERING

- 1 (c) (i) $R_1 = 240 \text{ k}\Omega$, $R_2 = 12 \text{ M}\Omega$
(ii) $R_3 = 240 \text{ k}\Omega$, $R_4 = 5 \text{ k}\Omega$
(ii) Circuit of Fig. 2
(d) Type C
(e) 200 kHz
- 2 (b) $I_C = 1.05 \text{ mA}$
(c) (ii) $Z_C = 537 \text{ k}\Omega$
(iii) h_{fe} should be in the range of 225 to 275
- 3 (b) $I_l = 4800 \text{ A}$
(c) (i) $P = 4.8 \text{ MW}$
(ii) $V_l = 23.68 \text{ kV}$
(iii) $I_F = 2491 \text{ A}$
- 4 (b) Stator current (phase) = 15750 A, Excitation = 33.4 kV, Load angle = 32.0°
(c) Load angle = 20.7° , Power factor = 0.52
(d) Power factor in the range of 0.518 to 0.819
- 6 (a) $|H| = 183.8 \text{ A m}^{-1}$, $|E| = 69.3 \text{ kV m}^{-1}$
(b) (i) 8.9 mJ
(ii) 55 nm
(c) 11.6 μm
- 7 (c) (i) $2.0 \times 10^8 \text{ m s}^{-1}$
(ii) 75 Ω (or 33.3 Ω)
(iii) 1m