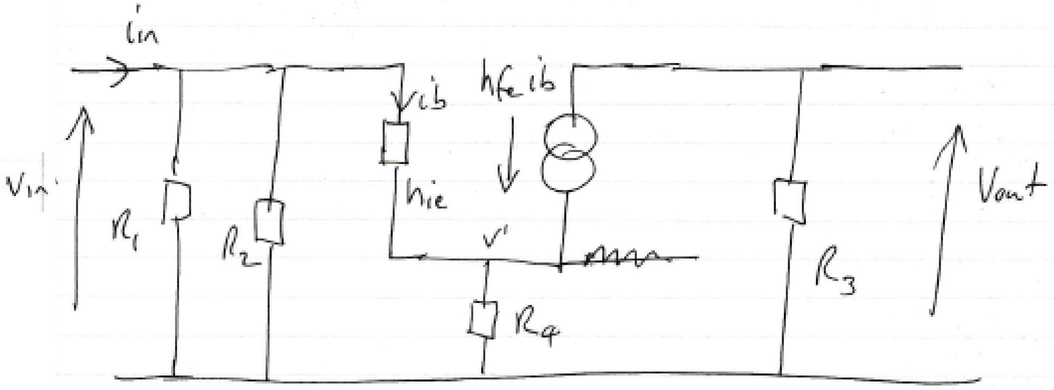
SECTION A

1

a) SSM mid-band, ignore C_1 + C_2, no h_{re}, h_{oe}



Voltage at R_4 node = V'

$\Rightarrow V_{in} = h_{ie} i_b + V' \qquad V' = (1+h_{fe}) i_b R_4$

at output $\dfrac{0 - V_{out}}{R_3} = h_{fe} i_b \Rightarrow V_{out} = -h_{fe} i_b R_3$

$$V_{in} = i_b[h_{ie} + (1+h_{fe})R_4] = \frac{-V_{out}}{h_{fe} R_3}[h_{ie} + (h_{fe}+1)R_4]$$

$$V_{out}/V_{in} = \frac{-h_{fe} R_3}{h_{ie} + (1+h_{fe})R_4}$$

$R_{in} = V_{in}/i_{in} \qquad i_{in} = i_b + V_{in}/R_1 /\!/ R_2$

from above

$$V_{in} = i_b(h_{ie} + (1+h_{fe})R_4)$$

$$\Rightarrow i_{in} = \frac{V_{in}}{R_1/\!/R_2} + \frac{V_{in}}{h_{ie} + (1+h_{fe})R_4}$$

$$\Rightarrow \frac{V_{in}}{i_{in}} = R_{in} = \frac{1}{1/R_1/\!/R_2 + 1/(h_{ie} + (1+h_{fe})R_4)}$$

$$= (R_1/\!/R_2)/\!/(h_{ie} + (1+h_{fe})R_4) = \frac{R_1/\!/R_2(h_{ie} + (1+h_{fe})R_4)}{R_1/\!/R_2 + h_{ie} + (1+h_{fe})R_4}$$

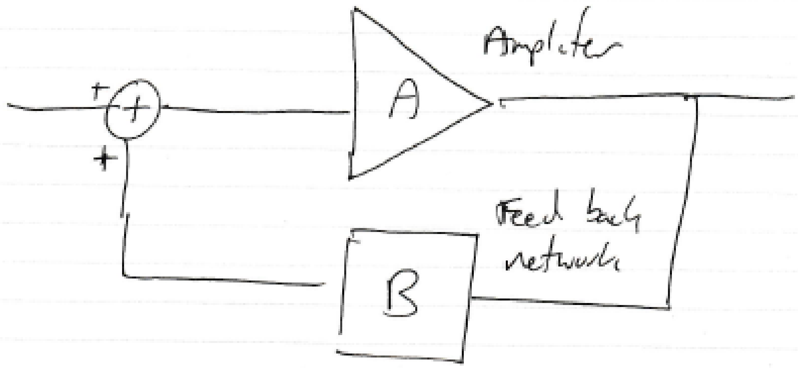
Calculating Rout is difficult as we have neglected the effect of hoe which make the SSM impossible to solve as the currentsource is directly connected to R_3 with no shunt ($1/h_{oe}$)

We either say $R_{out} \approx R_3$

or we have to include hoe in the SSM

b)

Oscillation requires positive feedback



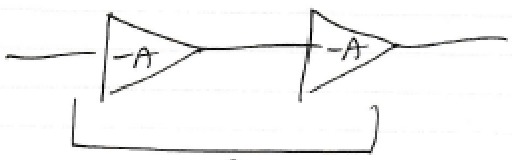
For oscillation to occur we need:

loop gain $|AB| \geq 1$

loop phase $\angle AB = 0$ or 2π (+ve F/B)

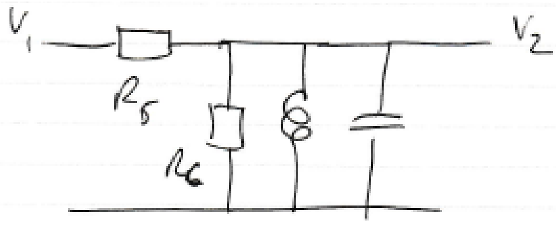
The amplifier in part (a) would not work as the oscillator as it is an inverting amplifier and has a phase of π (180°) which would give -ve feed back.

Either B must also have a phase of π or 180° which is not easy to achieve or we could cascade two amplifier in series



$+A^2$ phase of 0 or 2π.

c)



$$V_2 = V_1 \frac{Z}{Z + R_S}$$

where $Z = R_G // L // C$

$$L//C = \frac{j\omega L \cdot 1/j\omega C}{j\omega L + 1/j\omega C} = \frac{j\omega L}{1-\omega^2 LC}$$

$$Z = \frac{\frac{j\omega L}{(1-\omega^2 LC)} \times R_G}{R_G + \frac{j\omega L}{(1-\omega^2 LC)}} = \frac{j\omega L R_G}{j\omega L + R_G(1-\omega^2 LC)}$$

$$\frac{V_2}{V_1} = \frac{\frac{j\omega L R_G}{j\omega L + R_G(1-\omega^2 LC)}}{R_S + \frac{j\omega L R_G}{j\omega L + R_G(1-\omega^2 LC)}} = \frac{j\omega L R_G}{j\omega L R_G + R_S(j\omega L + R_G(1-\omega^2 LC))}$$

$$= \frac{j\omega L R_G}{j\omega L(R_S + R_G) + R_S R_G(1-\omega^2 LC)}$$

at resonance $1-\omega_0^2 LC = 0 \Rightarrow \omega_0 = \frac{1}{\sqrt{LC}}$

and Gain $\frac{V_2}{V_1} = \frac{R_G}{R_S + R_G}$

d)

$f_0 = 200\,kHz$ $\quad \omega_0 = 400\pi\,kHz$ $\quad R_S = R_G = 200k\Omega$

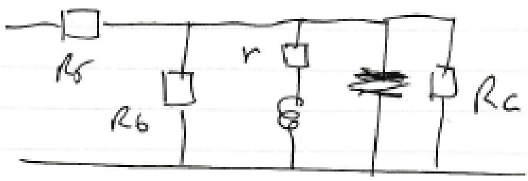
$L = 1\times 10^{-3} H$

$\Rightarrow C = \frac{1}{L\omega_0^2} = 6.33\times 10^{-10} F$

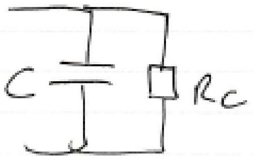
$(633\,pF)$

Gain $= 1/2$

New cct



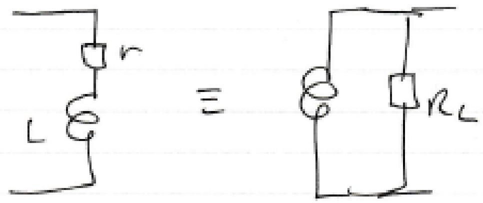
Need to combine effects of r & Rc with R_G



$$Q = 2000 = \omega_0 R_C C$$

$$\Rightarrow R_C = 2.5 \ M\Omega$$

For r need to convert to parallel resistance R_L



Both have same Q

$$\Rightarrow \frac{R_L}{\omega_0 L} = \frac{\omega_0 L}{r}$$

$$\Rightarrow R_L = \frac{(\omega_0 L)^2}{r} = 1.58 \times 10^5 \ \Omega$$

$$= 158 \ k\Omega$$

$\Rightarrow$ Total imperfections $= R_L // R_C$

$$= 1.49 \times 10^5 \ \Omega = 149 \ k\Omega$$

This is in parallel with $R_6 \Rightarrow$ new $R_6' = 200k // 149k$

$$= 85.3 \ k\Omega$$

$\Rightarrow$ New gain ~~output~~ would be $\frac{85k}{200k + 85k} = 0.3$.

The resistors R_5 + R_6 (or R_6') are also in series and parallel with R_{in} and R_{out} of the amplifier.

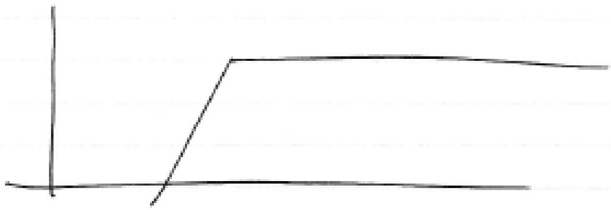
Hence the gain required for oscillation must be further adapted to take into account

$R_{in} + R_5$ and $R_6 // R_{out}$.

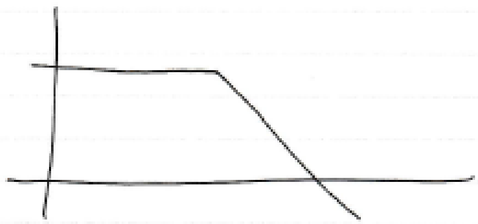
2

a)

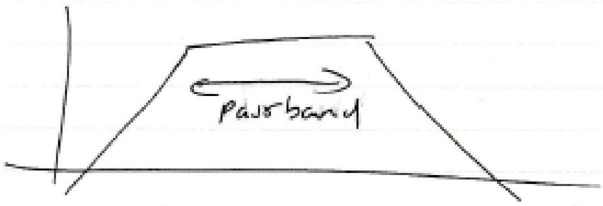
High pass – passes high freq



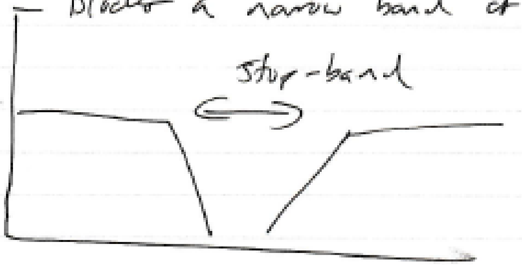
Low pass – passes low freq



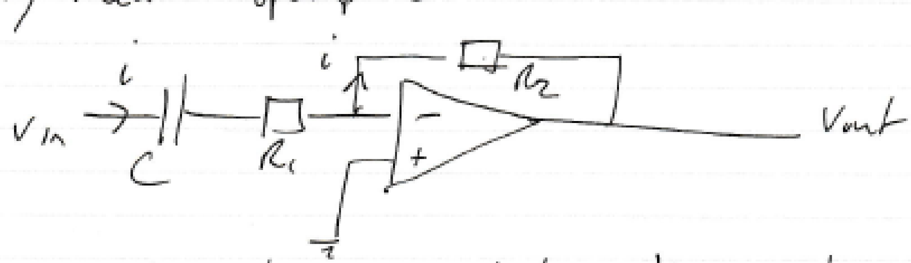
Band pass – passes a narrow band of freq (pass band)



Band Stop – Blocks a narrow band of freq (also a notch filter)



b) Ideal opamp ⇒ $A = \infty$ $R_i = \infty$ $R_o = 0$



(–) input is a virtual earth ⇒ at 0V

no current into (–) or (+) inputs

$$\Rightarrow \quad i = \frac{V_{in} - 0}{1/j\omega C + R_1} = \frac{0 - V_{out}}{R_2}$$

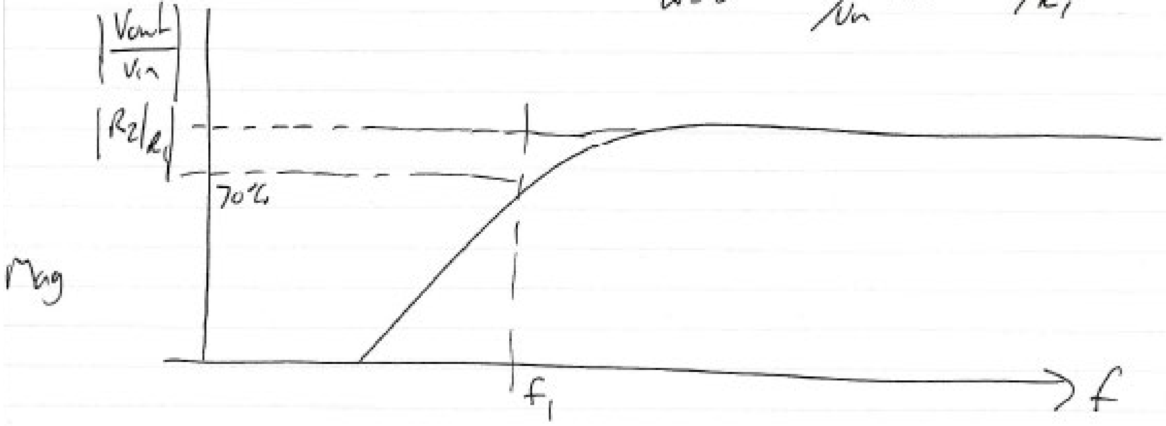
$$\frac{V_{out}}{V_{in}} = \frac{-R_2}{R_1 + 1/j\omega C} = \frac{-j\omega R_2 C}{1 + j\omega C R_2}$$

3dB freq when $1 = \omega C R_2 \Rightarrow f_1 = \frac{1}{2\pi R_2 C}$

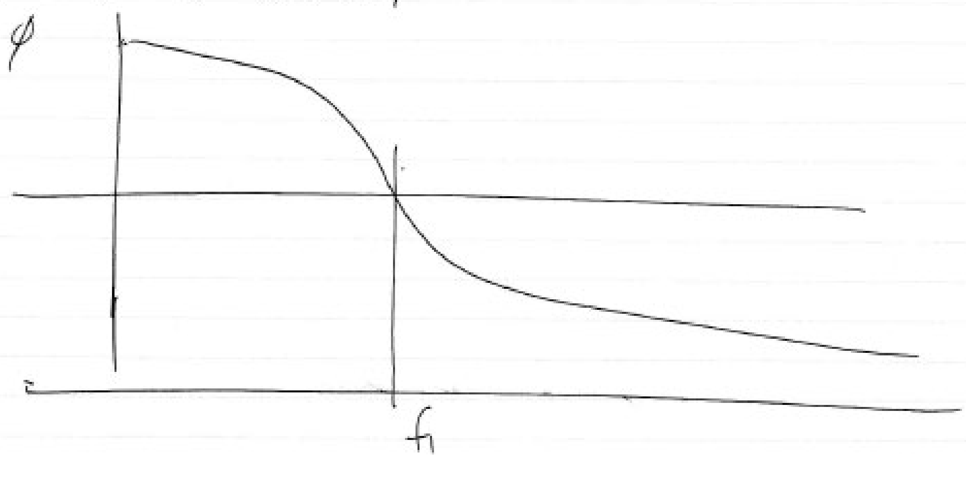
Sketch Bode plot

when $\omega = 0$ $\frac{V_{out}}{V_{in}} = 0 \Rightarrow$ high pass filter

$\omega = \infty$ $V_{out}/V_{in} = -R_2/R_1$

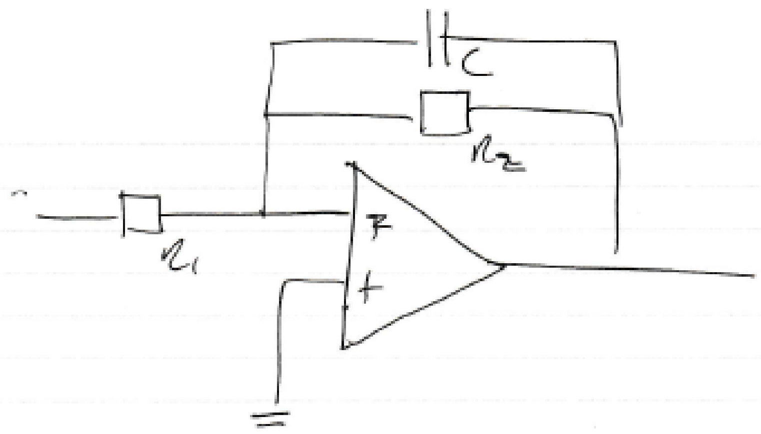


phase = 45° at 3dB freq



c)

Low pass characteristic can be made by moving C to feed back network.

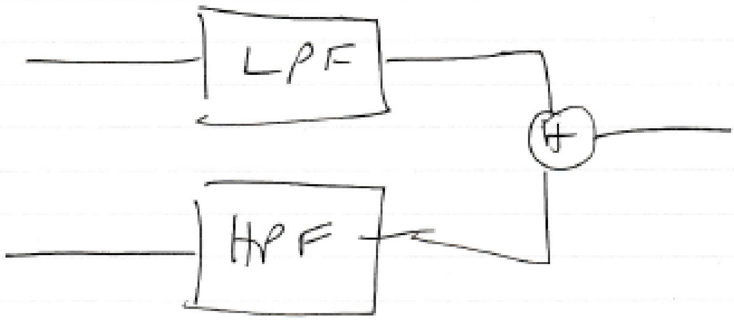


$$\frac{V_o}{V_i} = \frac{-R_2 // C}{R_1}$$

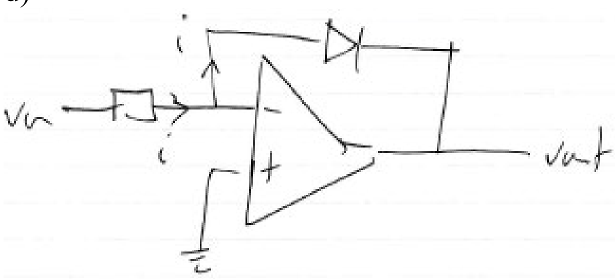
$$= \frac{-R_2}{1 + j\omega C R_2}$$

Band stop

To create a bandstop the two filter must be in parallel



d)



$V_D = -V_{out}$

$$\frac{V_{in} - 0}{R} = i_{diode} = I_S e^{(V_D/V_T)}$$

if $I_D = I_S e^{(V_D/V_T)}$

then $V_D = V_T \ln\left(\frac{I_D}{I_S}\right) = V_T \ln\left(\frac{i}{I_S}\right)$

$$= V_T \ln\left(\frac{V_{in}}{R I_o}\right)$$

Given $V_D = -V_{out}$

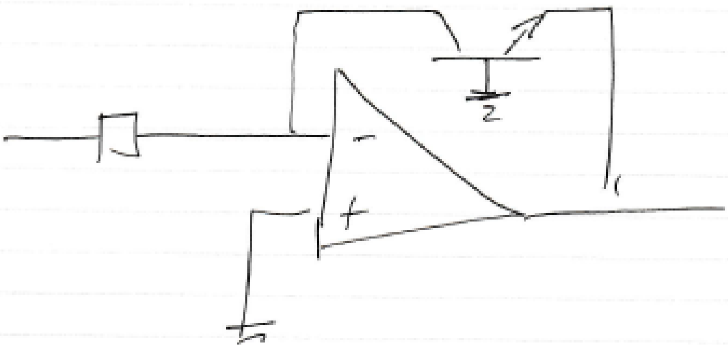
$$\Rightarrow V_{out} = -V_T \ln\left(\frac{V_{in}}{R I_o}\right)$$

This is a logarithmic amplifier.

This type of amplifier is useful in circuits with high dynamic range. It allows ~~direct~~ measurements ~~of~~ of signals ~~on~~ in decibels ~~and~~. They are also used in compression circuits like those used with analogue to digital converters or when multiplying analogue signals.

e)

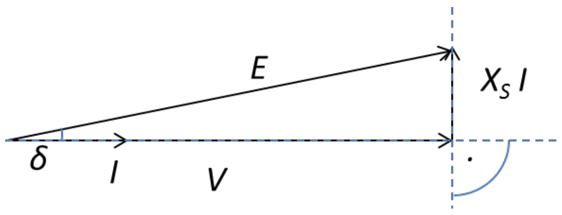
The diode is a basic pn junction * is not as stable as the combination of a BJT. A BJT can be used as a diode to give the same log response but over a more stable range



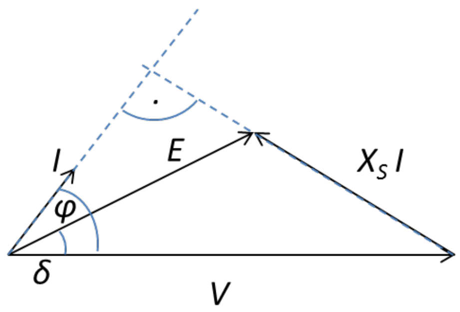
3

a)

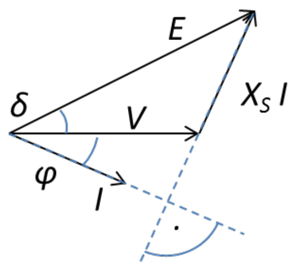
[5]



power factor of one



leading



lagging

b)

$V_{ph} = 22\ \text{kV}/\sqrt{3} = 12.7\ \text{kV}$

$P = 3\ V\ I \cos\varphi$ $\quad\Rightarrow I_{ph} = 250\ \text{MW} / (3\cdot 12.7\ \text{kV} \cdot 0.8) = 8.2\ \text{kA}$

$\sin^2\varphi + \cos^2\varphi = 1$

$\cos\varphi = 0.8 \quad \Rightarrow \sin\varphi = 0.6$

$X_sI = 8.2\ \text{kA} \cdot 1\ \text{V/A} = 8.2\ \text{kV}$

$X_sI \cos\varphi = 6562\ \text{V}$

$X_sI \sin\varphi = 4921\ \text{V}$

$\Rightarrow E^2 = (V_{ph} + X_sI \sin\varphi)^2 + (X_sI \cos\varphi)^2 = 3.54\cdot 10^8\ \text{V}^2$

$\Rightarrow E = 18.8\ \text{kV}$

$\tan\delta = X_sI \cos\varphi / (V_{ph} + X_sI \sin\varphi) \quad \Rightarrow \quad \delta = 18.1°$

c)

Change in power factor achieved through change in rotor excitation. To have a leading power factor, the excitation has to be increased (so that the line practically does not "see" the winding reactance any more but the back-emf voltage over-compensates it and pushes reactive current into the line).

Size of increase (now leading current):

Voltage unchanged $V_{ph} = 22\ kV/\sqrt{3} = 12.7\ kV$

$I_{ph} = 250\ MW / (3 \cdot 12.7\ kV \cdot 0.9) = 7.29\ kA$

$\cos\varphi = 0.9 \quad \Rightarrow \sin\varphi = 0.4359$

$X_s I = 7.29\ kA \cdot 1\ V/A = 7.29\ kV$

$V_{ph} \sin\varphi = 5.536\ kV > X_s I$ => triangle as follows:

$X_s I - x = V_{ph} \sin\varphi = 5.536\ kV$

$V_{ph} \cos\varphi = y = 11.43\ kV$

$x^2 + y^2 = E^2$

$\Rightarrow E^2 = (X_s I - V_{ph} \sin\varphi)^2 + (V_{ph} \cos\varphi)^2 = 1.34 \cdot 10^8\ V^2$

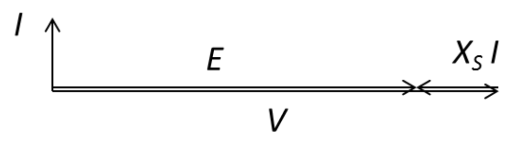
$\Rightarrow E = 11.56\ kV$

d)

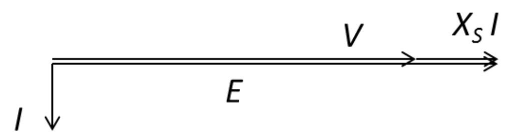
50 MVar / (3·12.7 kV) = 1312 A (fully reactive)

$X_s I = 1312\ V$

Acting as a capacitor (compensating inductive load): $E = V_{ph} + X_s I = 14\ kV$



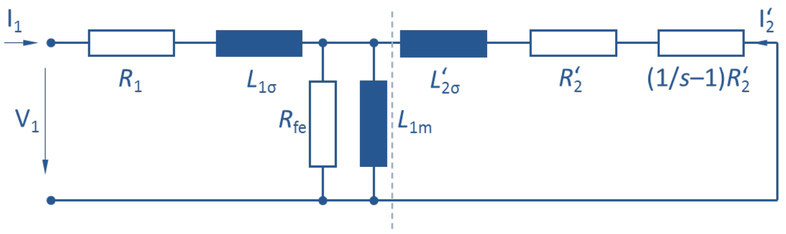
Acting as an inductor (compensating capacitive load): $E = V_{ph} - X_s I = 11.4\ kV$



SECTION B
4
a)

Both the stator and rotor are wound with 3-phase windings. The current applied to the stator produces a rotating magnetic field, which induces currents in the rotor if the rotor is not at the same speed as the rotating stator field due to the changing flux linkage (similar to a transformer). The rotor currents then produce their own magnetic field. The rotor and stator magnetic fields interact to produce a torque. At synchronous speed, the rotor is spinning at the same frequency as the stator field, so the flux linkage remains constant, and there is no induced emf in the rotor coils, so there will be no rotor field, and no torque on the rotor.

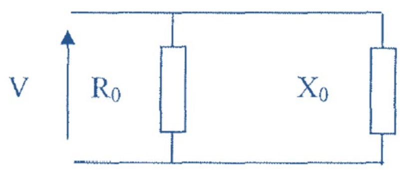
(b)



R_1: copper resistance in stator
L_1: stator stray inductance/reactance
R_{fe}: equivalent iron-loss resistance
L_{1m}: magn. reactance
L'_2: rotor stray inductance/reactance (referred to stator)
R': rotor resistance (split into loss and the equivalent part converted into mechanical power, both referred to the stator)

Or any somehow similarly detailed equivalent representation of an induction machine.

c)



$s = 0$

$P = 3 \cdot V^2/R_0$; $V = 1/\sqrt{3}\ 450\ \text{V} = 259.8076 \ldots\ \text{V}$

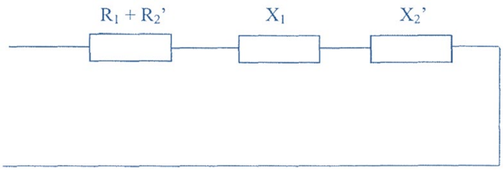
$R_0 = 3\ V^2/P = 40.5\ \Omega$

$S^2 = P^2 + Q^2 = (3\ VI)^2$

$Q = (3VI)^2 - P^2 = 3\ V^2/X_0 = 10.568\ \text{kVAr}$

$\Rightarrow X_0 = 3\ V^2/Q = (450\ \text{V})^2/Q = 19.16\ \text{V/A}$

d)



Slip $s = 1$ per definition, i.e., rotor standing still, i.e., $n = 0$ rpm; induced rotor current is at full frequency fed from the outside, i.e., $f = 50$ Hz

$R_{1\mathrm{cu}} = 0.1\ \Omega$

$P_{\mathrm{in}} = RI^2 \Rightarrow R = 30\ \mathrm{kW}/(300\ \mathrm{A})^2 = 1/3\ \Omega == 0.1\ \Omega + \mathrm{R'}_2$

$\Rightarrow R'_2 = 0.2333\ \Omega$

$Q^2 = S^2 - P^2 = (V_{\mathrm{line}}\ I_{\mathrm{line}}/\sqrt{3})^2 - P^2 = 9.75 \cdot 10^8\ \mathrm{V^2A^2} = (31224.99\ \mathrm{VA})^2 == XI^2$

$\Rightarrow X = X_1 + X'_2 = Q/I^2 = 0.3469\ \mathrm{V/A}$

Ratio 2:3

$X_1 = 2/5\ X = 0.1388\ \mathrm{V/A}$

$X_2 = 3/5\ X = 0.2082\ \mathrm{V/A}$

e)

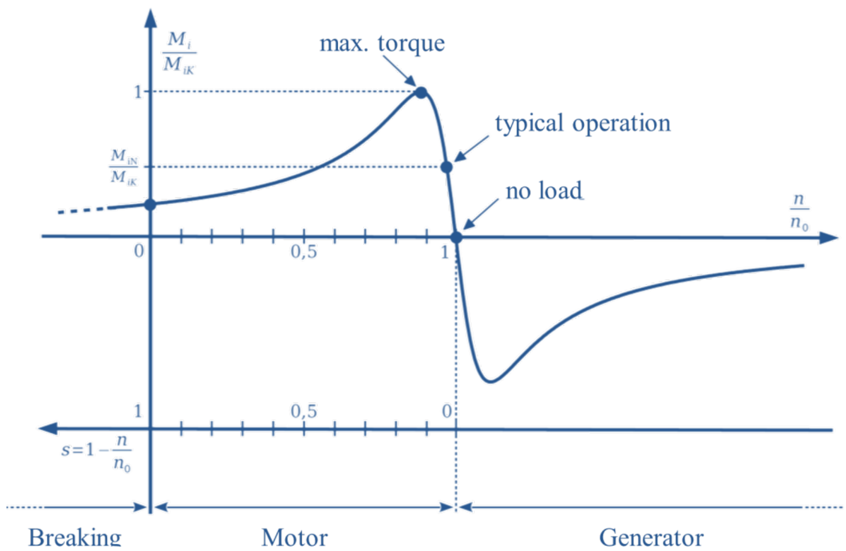
$n_{\mathrm{tyres}} = 190 \cdot 10^3\ \mathrm{m/h} / 1.8\ \mathrm{m} = 1.0556 \cdot 10^5\ \mathrm{1/h} = 1759\ \mathrm{1/min}$

$n_{\mathrm{motor}} = n_{\mathrm{tyres}} \cdot 17/20 = 1495.4\ \mathrm{1/min} = 24.92\ \mathrm{1/s} \Rightarrow n_{\mathrm{motor,sync}} = 25\ \mathrm{1/s}$

$f = n_{\mathrm{motor,sync}}\ 2p = 50\ \mathrm{Hz} \Rightarrow p = 4$ poles or 2 pole pairs

$s = 1 - n_{\mathrm{motor}} / n_{\mathrm{motor,sync}} = 0.0031$

f)



Example for operation as a motor. For generative operation, equivalent point with negative torque.

5)

a)

Let us differentiate both equations with respect to x

$$\frac{\partial^2 V}{\partial x^2} = -L\frac{\partial}{\partial t}\left(\frac{\partial I}{\partial x}\right) = LC\frac{\partial^2 V}{\partial t^2} \qquad (5a1)$$

$$\frac{\partial^2 I}{\partial x^2} = -C\frac{\partial}{\partial t}\left(\frac{\partial V}{\partial x}\right) = LC\frac{\partial^2 I}{\partial t^2} \qquad (5a2)$$

Then in we substitute in 5a1 and 5a2 $\frac{\partial I}{\partial x}$ and $\frac{\partial V}{\partial x}$ from the Telegrapher's Equations, and get:

$$\frac{\partial^2 V}{\partial x^2} = LC\frac{\partial^2 V}{\partial t^2} \qquad (5a3)$$

$$\frac{\partial^2 I}{\partial x^2} = LC\frac{\partial^2 I}{\partial t^2} \qquad (5a4)$$

These have the same functional form as the wave equation:

$$\frac{\partial^2 A}{\partial x^2} = \frac{1}{v^2}\frac{\partial^2 A}{\partial t^2}$$

Hence:

$$v^2 = \frac{1}{LC}$$

b)

The characteristic impedance, Z_0 is defined as the ratio between the voltage and the current of a unidirectional forward wave on a transmission line at any point, with no reflection

Applying the first of the Telegrapher's Equations to the given equations for V_F and I_F gives:

$$\frac{\partial V_F}{\partial x} = -j\beta\overline{V_F}e^{j(\omega t-\beta x)} = -Lj\omega\overline{I_F}e^{j(\omega t-\beta x)} = -L\frac{\partial I_F}{\partial t}$$

Hence

$$Z_0 = \frac{\overline{V_F}}{\overline{I_F}} = \frac{L\omega}{\beta}$$

Since

$$\omega = 2\pi f \quad \text{and} \quad \beta = \frac{2\pi}{\lambda}$$

We get $Z_0 = Lf\lambda$

Considering that

$$v = f\lambda = \frac{1}{\sqrt{LC}}$$

We then get

$$Z_0 = \sqrt{\frac{L}{C}}$$

c) (i) We know the capacitance per unit length of the line and its characteristic impedance

The inductance per unit length is $L=CZ_0^2=375$ nH m^{-1}

Therefore the velocity is:

$$v = \frac{1}{\sqrt{LC}} = 1.33\text{x}10^8 \text{ m/s}$$

A different dielectric with lower relative permittivity would have to be used to increase the wave velocity

(ii) The VSWR is given by:

$$VSWR = \frac{\text{Maximum voltage}}{\text{Minimum voltage}} = \frac{|\overline{V_F}| + |\overline{V_B}|}{|\overline{V_F}| - |\overline{V_B}|}$$

This can be rewritten in terms of the reflection coefficient $\overline{\rho}_L$ as

$$VSWR = \frac{1+\frac{\left|\overline{V}_B\right|}{\left|\overline{V}_F\right|}}{1-\frac{\left|\overline{V}_B\right|}{\left|\overline{V}_F\right|}} = \frac{1+\left|\frac{\overline{V}_B}{\overline{V}_F}\right|}{1-\left|\frac{\overline{V}_B}{\overline{V}_F}\right|} = \frac{1+\left|\rho_L\right|}{1-\left|\rho_L\right|}$$

Or:

$$\left|\rho_L\right| = \frac{VSWR-1}{VSWR+1}$$

Hence, for VSWR=1.7 we get:

$$|\rho_L| = \frac{VSWR-1}{VSWR+1} = \frac{1.7-1}{1.7+1} = 0.26$$

We can relate the reflection coefficient to the impedance of the transmission line and its load according to:

$$\overline{\rho}_L = \frac{\overline{Z}_L - Z_0}{\overline{Z}_L + Z_0}$$

Hence

$$Z_L = Z_0 \frac{1+|\rho_L|}{1-|\rho_L|} = 85.14\Omega$$

(iii) From the Data Book

$$\overline{Z}_b = \overline{Z}(-b) = Z_0 \frac{\overline{Z}_L + jZ_0 \tan(\beta b)}{Z_0 + j\overline{Z}_L \tan(\beta b)}$$

We want $\bar{Z}_{in} = \bar{Z}_l = \bar{Z}(-l) = Z_0 \frac{\bar{Z}_L + jZ_0\tan(\beta l)}{Z_0 + j\bar{Z}_L\tan(\beta l)}$

Hence, $\tan(\beta l)=0$, thus: $\beta l=\pi$

Therefore

$$l = \frac{\pi}{\beta} = \frac{\pi}{2\pi/\lambda} = \frac{\lambda}{2} = \frac{v}{2f} = 0.44\text{m}$$

6)

(a)

Wave equation in one dimension $\left(\frac{\partial^2}{\partial t^2} - \varepsilon_0\mu_0\frac{\partial^2}{\partial z^2}\right)E_x = 0$; substitute $E_0\, e^{j(\omega t-\beta z)}$ into it

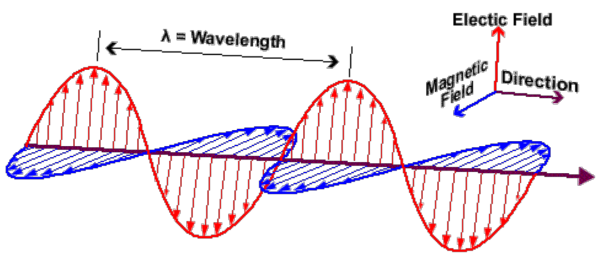
$$E_0 j^2\omega^2\, e^{j(\omega t-\beta z)} - \varepsilon_0\mu_0 E_0 j^2\beta^2 e^{j(\omega t-\beta z)} = 0$$

Thus: $\frac{\omega}{\beta} = \varepsilon_0\mu_0$

Since $\beta = \frac{2\pi}{\lambda}$ and $\omega=2\pi f$

We get $f\lambda = c = \frac{1}{\sqrt{\varepsilon_0\mu_0}}$

b)



Set $\mathbf{H} = \mathbf{u}_y\, H_0\, e^{j(\omega t-\beta z)}$

Impedance defined as $\eta_0 = \frac{E_0}{H_0}$

Putting E into Faraday-Maxwell:

$$\frac{\partial E_x}{\partial z} = -\mu\frac{\partial H_y}{\partial t}$$

$$\frac{E}{H} = \frac{\mu_0 \omega}{\beta} = \sqrt{\frac{\mu_0}{\epsilon_0}} \Rightarrow H = E_0/\eta_0 \;\; e^{j(\omega t - \beta z)}$$

Since $\underline{\overline{S}} = \frac{1}{2} \underline{\overline{E}} \times \underline{\overline{H}}^*$

We get

$$\underline{\overline{S}} = \frac{1}{2} E_x H^*_y \underline{a_z}$$

And

$$\text{Average Power} = \frac{|E_x|^2}{2\eta_0}$$

c)

100 kW = 100 kJ/s = 10^4 1/s· 10 J for the pulse energy

Average (and here also peak) power during a pulse: 10 J / 20 fs = ½ · 10^{15} W = 5 · 10^{14} W

Power density (Intensity) $= \frac{4P}{d^2\pi} = \frac{2 \cdot 10^{15}\ \mathrm{W}}{1\ \mathrm{mm}^2 \pi} = 6.37 \cdot 10^{20}\ \mathrm{W/m^2}$

$$\eta_0 = 377\ \Omega$$

$$I = \frac{1}{2} S = \frac{E_0^2}{2\eta_0}$$

$$\Rightarrow E_0^2 = 2\eta_0\, I = 6.37 \cdot 10^{20} \frac{\mathrm{VA}}{\mathrm{m}^2} \cdot 2 \cdot 377 \frac{\mathrm{V}}{\mathrm{A}} = 4.8 \cdot 10^{23} \frac{V^2}{m^2} = \left(6.93 \cdot 10^{11} \frac{\mathrm{V}}{\mathrm{m}}\right)^2$$

$$E_0 = 6.93 \cdot 10^{11} \frac{\mathrm{V}}{\mathrm{m}}$$

$$H_0 = \frac{E_0}{\eta_0} = 1.84 \cdot 10^9 \frac{\mathrm{A}}{\mathrm{m}}$$

d) New impedance $\eta_{\text{inside}} = \sqrt{\frac{\mu_0}{2\epsilon_0}} = \frac{\eta_0}{\sqrt{2}} = \eta_0 \cdot 0.7071$

Reflection $\frac{E_{0r}}{E_0} = \frac{\eta - \eta_0}{\eta + \eta_0} = 17.16\%$

Transmission $\frac{E_{0t}}{E_0} = 1 - \frac{\eta - \eta_0}{\eta + \eta_0} = \frac{2\eta_0}{\eta + \eta_0} = 82.84\%$ => $E_{ot} = 5.74 \cdot 10^{11}$ V/m

$$H_{0t} = \frac{E_{0t}}{\eta} = 2.15 \cdot 10^9 \, \frac{\text{A}}{\text{m}}$$

e)

$\mathbf{E} = \mathbf{u}_x \, E_0 \, e^{j(\omega t - \beta z) - \alpha z}$ with damping α

$\exp(-\alpha \cdot 1 \text{ m}) = 0.9$

$\alpha = -\ln(0.9) \ \text{m}^{-1} = 0.105361 \ \text{m}^{-1}$