## Version ACF/4

EGT1
ENGINEERING TRIPOS PART IB

14 June 202214.00 to 16.10

## Paper 5

## ELECTRICAL ENGINEERING

Answer not more than four questions.
Answer not more than two questions from each section.
All questions carry the same number of marks.
The approximate percentage of marks allocated to each part of a question is indicated in the right margin.

Answers to questions in each section should be tied together and handed in separately.

Write your candidate number not your name on the cover sheet.

## STATIONERY REQUIREMENTS

Single-sided script paper

SPECIAL REQUIREMENTS TO BE SUPPLIED FOR THIS EXAM
CUED approved calculator allowed
Engineering Data Book

10 minutes reading time is allowed for this paper at the start of the exam.

You may not start to read the questions printed on the subsequent pages of this question paper until instructed to do so.

You may not remove any stationery from the Examination Room.

## Version ACF/4

## SECTION A

Answer not more than two questions from this section
1 A BC182L silicon bipolar junction transistor (BJT) is to be used in the amplifier circuit shown in Fig. 1 with the following parameters:

$$
h_{i e}=4 \mathrm{k} \Omega, h_{f e}=250, V_{B E}=0.7 \mathrm{~V}, h_{o e} \text { and } h_{r e} \text { negligible, } V_{C C}=10 \mathrm{~V} .
$$

(a) Sketch the small signal equivalent model for the circuit in Fig. 1 at mid-band frequencies, stating any assumptions made. Derive expressions for the gain $\left(v_{o u t} / v_{\text {in }}\right)$ and input resistance $\left(R_{i n}\right)$ for the circuit.
(b) An oscillator can be created using an amplifier and a suitable feedback network. State, with the aid of a diagram, the conditions required for oscillation to occur. How could the amplifier in Fig. 1 be modified to be suitable for use in such an oscillator?
(c) The feedback network for an oscillator can be created using an inductor $(L)$ and a capacitor ( $C$ ) in parallel for a resonant circuit as shown in Fig. 2. The two resistances $\left(R_{5}\right.$ and $R_{6}$ ) are used to control the amplitude of the signal at resonance. Derive an expression for the frequency of oscillation and the gain of the network at this frequency.
(d) The network shown in Fig. 2, for a 200 kHz oscillator, has values of $R_{5}=R_{6}=200 \mathrm{k} \Omega$, $L=1 \mathrm{mH}$. Calculate the required capacitance for this oscillator and the overall gain at the oscillating frequency. Neither $L$ nor $C$ are ideal, with the inductor having a series resistance of $10 \Omega$ and the capacitor having a $Q$ factor of 2000. Calculate the new required gain given these imperfections.
(e) Explain how the values calculated in part (d) impact on the parameters of the amplifier chosen for an oscillator such as that in Fig. 1.


Fig. 1

Version ACF/4


Fig. 2

## Version ACF/4

2 (a) Explain, with the aid of a simple sketch, what is meant by the terms high pass filter, low pass filter, bandpass filter and bandstop filter in an analogue circuit.
(b) The circuit in Fig. 3(a) is a filter based around a single ideal operation amplifier. Derive an expression for the gain of the filter and its 3 dB cut off frequency. Sketch a Bode plot (magnitude and phase versus frequency) for the filter. What type of filter is it?
(c) Show how can the filter in Fig. 3(a) be modified to perform the opposite frequency response (i.e. high to low pass, or low to high pass). How can a high-pass and a low-pass filter be combined to make a bandstop filter? State any assumption made.

A forward biased silicon diode has a current versus voltage approximated by the relation:

$$
I_{D}=I_{S} e^{\left(\frac{V_{D}}{V_{T}}\right)}
$$

where $I_{D}$ is the current through the diode, $V_{D}$ is the voltage across the diode, $I_{S}$ is the diode saturation current and $V_{T}$ depends on the physical constants of the materials.
(d) For the circuit in Fig. 3(b), derive an equation for the output voltage $V_{\text {out }}$ as a function of the input voltage $V_{i n}$. What sort of circuit is this and what sort of application might it be used for?
(e) The diode characteristic above is very prone to variations due to temperature and the parameters of the diode. How could the diode be replaced by a transistor to make a more stable circuit with the same gain performance?


Fig. 3

## Version ACF/4

3 (a) Sketch a labelled phasor diagram for one phase of a synchronous generator when delivering power to an infinite bus for the following cases:
(i) Power factor of 1.
(ii) Leading power factor of 0.85 .
(iii) Lagging power factor of 0.85 .

Name all phasors in your diagram and highlight the characteristic angles between them.
(b) A $22 \mathrm{kV}, 1000 \mathrm{MVA}$ synchronous turbo generator in Y configuration (star) running at a speed of 3000 rpm (revolutions per minute) has a synchronous reactance of $1 \mathrm{~V} / \mathrm{A}$ per phase. It delivers 250 MW into an infinite bus at a lagging power factor of 0.8 . Calculate the stator line current, the excitation line voltage, and the load angle.
(c) The operator of the generator received a call to switch from the lagging 0.8 power factor to a leading 0.9 power factor to compensate reactive power without any change of the power of the prime mover. What would the operator need to do or change and why? By how much does the operator need to change it? What is the new value of the stator current?
(d) The primary mover is ramped down so that the machine can be used entirely for compensation of 10 MVar and no active power. The line voltage is unchanged. What is the current? What would be the respective excitation line voltage for the two cases of inductive and capacitive compensation? Draw the phasor diagram for one of these cases.

## SECTION B

## Answer not more than two questions from this section

4 (a) Briefly describe the structure and the principle of operation of induction motors, explaining why they produce no torque at synchronous speed.
(b) Sketch the full equivalent circuit of an induction machine, name all components, and describe what they represent. Split the stator-referred rotor resistance into a slip-independent part and the rest.
(c) A 50 Hz , 3-phase, 450 V , star-connected induction motor is tested under no-load conditions such that the slip is zero. Find the iron loss resistance and the magnetising reactance of this motor, given the following no-load measurements:

$$
\begin{equation*}
V_{\text {line }}=450 \mathrm{~V}, I_{\text {line, noload }}=15 \mathrm{~A}, P_{\text {in,noload }}=5 \mathrm{~kW} \tag{4}
\end{equation*}
$$

(d) A locked-rotor test is now carried out on the same motor at 50 Hz . Given that the stator winding resistance is $0.1 \Omega$, and the ratio of stator to rotor leakage reactance is $2: 3$, find the referred rotor resistance, the referred rotor leakage reactance and the stator leakage reactance using the following measurement data:

$$
V_{\text {line }}=250 \mathrm{~V}, I_{\text {line,locked }}=300 \mathrm{~A}, P_{\text {in,locked }}=3 \times 30 \mathrm{~kW}
$$

What would be the slip $s$ and the speed $n$ here? What would be the frequency of the rotor currents?
(e) The motor is star-connected to a 450 V , three-phase, 50 Hz supply and is used to drive the wheels of an electric vehicle. The car is driving at $190 \mathrm{~km} / \mathrm{h}$ (kilometres per hour) slightly downhill so that the motor is very close to its synchronous speed. The car has tyres with an 1800 mm circumference and a fixed gear ratio in the transmission of 17:20 (motor:wheels). Using this information, find the number of poles of the motor, its synchronous speed, its actual speed, and the slip.
(f) Sketch the relationship between torque and speed, clearly marking out the three regimes of operation of the motor and in which range the machine is typically operated in steady state.

## Version ACF/4

5. The Telegrapher's Equations for a lossless transmission line are

$$
\frac{\partial V}{\partial x}=-L \frac{\partial I}{\partial t} \quad \frac{\partial I}{\partial x}=-C \frac{\partial V}{\partial t}
$$

where $L$ is the inductance per unit length and $C$ is the capacitance per unit length.
(a) Starting from the Telegrapher's Equations, show that a travelling electrical wave will propagate along a lossless transmission line with a velocity $\quad v=1 / \sqrt{L C}$
(b) A simple sinusoidal electrical signal propagating along a lossless transmission line in the absence of reflections may have its voltage and current expressed as

$$
V_{F}=\mathbb{R} e\left\{\overline{V_{F}} e^{j(\omega t-\beta x)}\right\} \quad \text { and } \quad I_{F}=\mathbb{R} e\left\{\overline{I_{F}} e^{j(\omega t-\beta x)}\right\}
$$

Using these expressions and the Telegrapher's Equations, show that the characteristic impedance of the transmission line is given by $Z_{0}=\sqrt{L / C}$
(c) A lossless coaxial cable has a characteristic impedance of $50 \Omega$ and a capacitance per unit length of $150 \mathrm{pF} \mathrm{m}^{-1}$. It is used to transmit a sinusoidal signal at a frequency of 150 MHz .
(i) Calculate the velocity with which an electrical signal will propagate along the transmission line. How would the design of the coaxial cable need to be changed to increase this velocity?
(ii) The coaxial cable is to be terminated with a resistive load so that a standing wave is produced in the cable with a voltage standing wave ratio (VSWR) of 1.7. Calculate the required load resistance to achieve this.
(iii) Assuming that the coaxial cable has a non-zero length, what is the minimum length that the cable must be if the input impedance to the cable is to be the same as the load resistance?

## Version ACF/4

6 (a) The electric field strength of an electromagnetic plane wave travelling through free space is given by

$$
\boldsymbol{E}=\boldsymbol{u}_{x} E_{0} e^{j(\omega t-\beta z)}
$$

where $\boldsymbol{u}_{x}$ is the unit vector in the $x$ direction. Find the condition on $\omega$ and $\beta$ for this wave to satisfy the wave equation:

$$
\frac{\partial^{2} E}{\partial z^{2}}-\varepsilon_{0} \mu_{0} \frac{\partial^{2} E}{\partial t^{2}}=0
$$

and hence show that the speed of propagation of the wave in free space is $1 / \sqrt{\varepsilon_{0} \mu_{0}}$.
(b) Sketch the relation between the direction of propagation of the electric and magnetic fields of this plane electromagnetic wave, and define the impedance of free space, $\eta_{0}$. Hence write an expression for the corresponding magnetic field strength and derive an expression for the power per unit area transmitted by the wave.
(c) A 100 kW femtosecond fusion laser emits 10000 pulses per second, each with an effective duration of $20 \times 10^{-15} \mathrm{~s}$. Assume a circular beam diameter of 1 mm and the simplification that the pulses have practically a rectangular envelope shape. Determine the magnitude of the electric and magnetic fields in free space.
(d) The laser output is perpendicularly coupled into a glass body with two times the relative permittivity of vacuum for the relevant frequency range (same magnetic permeability). What are the magnitudes of the electric and magnetic fields right behind the interface where the light entered the material? Use, for instance, transmission line theory and impedances to estimate reflected and transmitted parts of the electric field first.
(e) The glass body is not lossless but dampens the wave to $90 \%$ after 1 m . Derive an equation for the electric field equivalent to the one from part (a).

## END OF PAPER

