2P5 crib

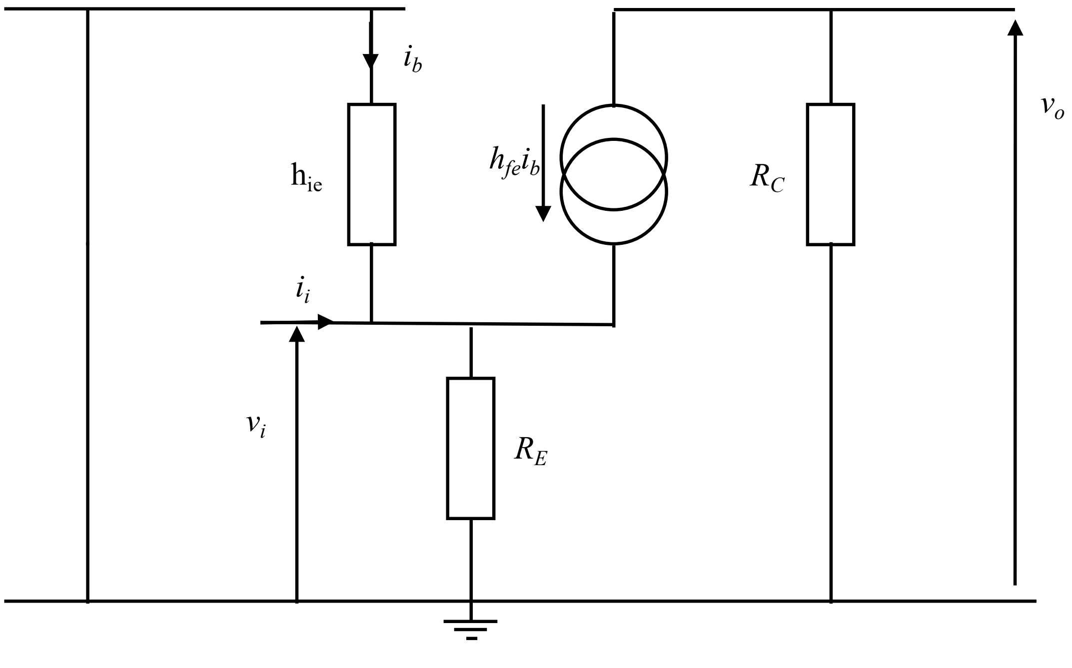
Question 1

a) $R_c = \frac{V_{cc}-V_c}{I_C} = \frac{5}{1\times10^{-3}} = 5\ k\Omega,$
$V_E = V_C - V_{CE} = 5 - 4.7 = 0.3\ V$ therefore $R_E = \frac{V_E}{I_E} = \frac{0.3}{1\times10^{-3}} = 300\ \Omega$
assuming $I_E \approx I_C = 1mA$

b) $I_B = 10\mu A$ therefore current through $R_2 = 1mA$.
$V_B = V_E + V_{BE} = 0.3 + 0.7 = 1$ V

$$R_2 = \frac{V_{cc} - V_B}{1\times10^{-3}} = 9\ k\Omega$$
$$R_1 = \frac{V_B}{1\times10^{-3}} = 1\ k\Omega$$

c)



d)

i) $v_o = -h_{fe}i_bR_c$
$v_i + i_bh_{ie} = 0$

$$\frac{v_o}{v_i} = \frac{h_{fe}R_c}{h_{ie}}$$

With $h_{fe} = 200, R_c = 5\ k\Omega, h_{ie} = 5\ k\Omega$

$$\frac{v_o}{v_i} = 200$$

ii) $i_i + i_b + h_{fe}i_b = \frac{v_i}{R_E}$ but also $v_i = -i_bh_{ie}$ hence $i_b = -v_i/h_{ie}$
therefore

$$i_i = -i_b - h_{fe}i_b + \frac{v_i}{R_E} = \frac{v_i}{h_{ie}} + \frac{h_{fe}v_i}{h_{ie}} + \frac{v_i}{R_E} = v_i\left(\frac{1}{h_{ie}} + \frac{h_{fe}}{h_{ie}} + \frac{1}{R_E}\right)$$

Therefore

$$R_{in} = \frac{v_i}{i_i} = \frac{1}{\left(\frac{1}{h_{ie}} + \frac{h_{fe}}{h_{ie}} + \frac{1}{R_E}\right)}$$

With $h_{fe} = 200, R_E = 300\ \Omega, h_{ie} = 5\ k\Omega$

$$R_{in} = \frac{1}{\left(\frac{1}{5 \times 10^3} + \frac{200}{5 \times 10^3} + \frac{1}{300}\right)} = 23\ \Omega$$

iii) Looking from the output $R_{out} = R_C$ since the impendence of an ideal current source is infinite. Therefore with $R_c = 5\ k\Omega$ hence

$$R_{out} = 5\ k\Omega$$

e) The output impedance has the same form as a common emitter amplifier being given by R_C.
The gain of a common emitter amplifier is $-h_{fe}R_C/h_{ie}$ so is the same magnitude as the common-base but inverted (with the common-base being a non-inverting amplifier)
The most notable feature of the common base amplifier is the incredibly low input impedance (23 Ω in this case). In contrast for the common-emitter amplifier it is $h_{ie}||R_1||R_2$ so typically in the $k\Omega$ region.
In practice a common-base amplifier is used for high frequency applications with its low impedance is well matched to co-axial cables.

Question 2

a) i) $v_o = A(v_i - B_1 v_0)$ therefore $v_o + AB_1 v_o = Av_i$ therefore $v_o/v_i = A/(1 + AB_1)$ as required
ii) Current at input $i_i = (v_i - B_1 v_o)/r_i$ but from i) $v_o = v_i A/(1 + AB_1)$ therefore $i_i = (v_i - B_1 v_o)/r_i i_i = (v_i - B_1 v_i A/(1 + AB_1))/r_i$
hence
$r_i i_i/v_i = 1 - B_1 A/(1 + AB_1) = (1 + AB_1 - B_1 A)/(1 + AB_1) = 1/(1 + AB_1)$
Therefore $R_{in} = v_i/i_i = r_i(1 + AB_1)$
iii) Test current in is $i_x = (v_x - A[v_i - B_1 v_x])/r_o$ short input so $v_i = 0$ which gives $i_x = v_x(1 + AB_1)/r_o$ hence $R_{out} = v_x/i_x = r_o/(1 + AB_1)$

b) i) Output of first stage is $v_i A/(1 + AB_1)$. The voltage into the second stage is $\frac{v_i A}{(1+AB_1)} \frac{R_{in}}{R_{in}+R_{out}} = \frac{v_i A}{(1+AB_1)} \frac{r_i(1+AB_1)}{r_i(1+AB_1)+\frac{r_o}{1+AB_1}} = v_i A \frac{r_i(1+AB_1)}{r_i(1+AB_1)^2+r_o}$ hence output voltage is
$v_o = v_i A \frac{r_i(1+AB_1)}{r_i(1+AB_1)^2+r_o} \times \frac{A}{(1+AB_1)} = v_i \frac{A^2 r_i}{r_i(1+AB_1)^2+r_o}$
Hence

$$\frac{v_o}{v_i} = \frac{A^2}{(1 + AB_1)^2 + r_o/r_i}$$

ii) Output impedance will be the same as for a) hence $R_{out} = r_o/(1 + AB_1)$

iii) Input impedance will be same as for a) hence $R_{in} = r_i(1 + AB_1)$

c) i) gain of the concatenated voltage amplifiers is $A_c = A^2 r_i/(r_i + r_o)$ hence $v_o = A_c(v_i - B_2 v_o)$ hence using result from a) gives $v_o/v_i = A_c/(1 + A_c B_2)$ i.e.

$$\frac{v_o}{v_i} = \frac{\frac{A^2 r_i}{r_i + r_o}}{1 + \frac{A^2 r_i}{r_i + r_o} B_2} = \frac{A^2 r_i}{r_i + r_o + r_i A^2 B_2} = \frac{A^2}{1 + A^2 B_2 + r_o/r_i}$$

ii) Using result from a) with $A_c = A^2 r_i/(r_i + r_o)$ gives $R_{in} = r_i(1 + A_c B_2)$ hence

$$R_{in} = r_i\left(1 + \frac{A^2 r_i B_2}{r_i + r_o}\right) = r_i\left(1 + \frac{A^2 B_2}{1 + \frac{r_o}{r_i}}\right)$$

iii) Using result from a) with $A_c = A^2 r_i/(r_i + r_o)$ gives $R_{out} = r_o/(1 + A_c B_2)$ hence

$$R_{in} = \frac{r_o}{1 + \frac{A^2 r_i}{r_i + r_o} B_2} = \frac{r_o}{1 + \frac{A^2}{1 + r_o/r_i} B_2}$$

d) (i) For $A \approx 1$ cascading amplifiers will not increase gain it will only increase uncertainty so the arrangement with the fewest voltage amplifiers will be preferable, i.e. arrangement (a)

(ii) For $A \gg 1$ we expect providing feedback around the cascaded voltage amplifiers will be the best arrangement, i.e. arrangement (c), as per Example paper 2, problem 2.

This is all that was required for full marks. A detailed mathematical justification, which was not, is included below for reference.

(a) $\frac{dG_a}{dA} = \frac{d}{dA}\frac{A}{1+AB} = \frac{1+AB-BA}{(1+AB)^2} = \frac{1}{(1+AB)^2} = \frac{G_a^2}{A^2}$

so $\frac{\delta G_a}{G_a} = \frac{G_a}{A}\frac{\delta A}{A} \approx \frac{\delta A}{A}$

(b) $\frac{dG_b}{dA} = \frac{d}{dA}\frac{A^2}{(1+AB_1)^2} = \frac{2A(1+AB_1)^2 - A^2 2B_1(1+AB_1)}{(1+AB_1)^4} = \frac{2A(1+AB_1) - A^2 2B_1}{(1+AB_1)^3} =$
$\frac{2A}{(1+AB_1)^3} = \frac{2}{A^2} G_b\sqrt{G_b}$

So $\frac{\delta G_b}{G_b} = \frac{2\sqrt{G_b}}{A}\frac{\delta A}{A} = \frac{2}{\sqrt{A}}\frac{\delta A}{A}$

(c) $\frac{dG_c}{dA} = \frac{d}{dA}\frac{A^2}{1+A^2B_2} = \frac{(1+A^2B_2)2A - A^2(2AB_2)}{(1+A^2B_2)^2} = \frac{2A}{(1+A^2B_2)^2} = \frac{2}{A^3}\left(\frac{A^2}{1+A^2B_2}\right)^2 = \frac{2}{A^3}G_c^2$

So $\frac{\delta G_c}{G_c} = \frac{2G_c}{A^2}\frac{\delta A}{A} = \frac{2}{A}\frac{\delta A}{A}$

Q3.

a) Balanced three phase has no return path of each phase, reducing current conduction loss.

b) Star

$V_{ph} = 415/\sqrt{3} = 240\,V$

$Z = \sqrt{32^2 + 24^2} = 40\,\Omega$

$I_{ph} = \frac{V_{ph}}{Z} = \frac{240}{40} = 6A$

$P = 3I^2R = 3 \times 6^2 \times 32 = 3.44kW$

$Q = 3I^2\omega L = 3 \times 6^2 \times 24 = 2.58kVAr$

Delta

$V_{ph} = V_{Line}$

$Z = \sqrt{(5^2 + 25^2)} = 25.5\,\Omega$

$I = \frac{V}{Z} = \frac{415}{25.5} = 16.28A$

$P = 3 \cdot I^2R = 3 \times 16.28^2 \times 5 = 3.97kW$

$Q = 3I^2\omega L = 3 \times 16.28^2 \times 25 = 19.87kVA$

$P_{total} = 3.44 + 3.97 = 7.41kw$

$Q_{total} = 2.58 + 19.58 = 22.45kVA$

$S_{total} = \sqrt{P_{total}^2 + Q_{total}^2} = 23.65kVA$

$\cos\theta = \frac{7.41}{22.45} = 0.33$

c) $I = \frac{S}{\sqrt{3}V} = \frac{23.65\times10^3}{\sqrt{3}\times415} = 32.9A$

$P_{loss} = 3I^2R = 3\times(32.9)^2\times0.4 = 1299W$

If PF is corrected to unity. $S = P_1 = 7.41kW$

$I_1 = \frac{P_1}{\sqrt{3}V} = \frac{7.41\times10^3}{\sqrt{3}\times415} = 10.3A$

$P_{loss,1} = 3I_1^2R = 3\times(10.3)^2\times0.4 = 127.3W$

d) i) The maximum torque is achieved when power dissipation is maximised. Using the maximum power transfer theorem, $\frac{R_2'}{s}$ equals to $|jX_m // (R_1+jX_1)+jX_2'|$.

$\frac{R_2'}{s} = |jX_m // (R_1+jX_1)+jX_2'|$

$= |j65 // (0.8+j2.2)+j1.3| = |0.75+j3.44| = 3.517$

$s = \frac{R_2'}{3.517} = \frac{1}{3.517} = 0.284$

$w_r = (1-s)w_s = (1-s)\frac{w}{P} = (1-0.284)\frac{2\pi50}{3} = \underline{74.9rad/s}$

$Z_2 = jX_m // (\frac{R_2'}{s}+jX_2') = j65 // (3.517+j1.3) = 3.37+j1.45$

$E = V_{ph}\cdot\frac{Z}{Z+R_1+jX_1} = \frac{415}{\sqrt{3}}\cdot\frac{3.37+j1.45}{3.37+j1.45+0.8+j2.2} = (151-j48.7)V$

$|E| = 159V$

$$I_2' = \frac{E}{\frac{R_2'}{s} + jX_2'} = \frac{651 - j487}{3.517 + j1.3} = 33.27 - j26.15$$

$$|I_2'| = 42.3A$$

$$T = \frac{3}{\omega_s}|I_2'|\cdot\frac{R_2'}{s} = \frac{3}{104.7}(42.3)^2 \cdot 3.517 = 180.3 N.m$$

ii) $$T_{loss} = \frac{P_{loss}}{\omega_r} = \frac{260}{74.9} = 3.47 \; N.m$$

$$T_{out} = T_{em} - T_{loss} = 180.3 - 3.47 = 176.83 \; Nm$$

$$P_{out} = T_{out}\cdot \omega_r = 176.83 \times 74.9 = 13240 \; W.$$

$$I_1 = \frac{V_{ph}}{Z_{total}} = \frac{V_{ph}}{R_1 + jX_1 + Z_2} = \frac{415/\sqrt{3}}{0.8 + j2.2 + 3.37 + j1.45} = 32.5 - j28.5$$

$$|I_1| = 43.2A$$

$$p.f = \cos\varphi = \cos\left(\tan^{-1}\left(\frac{Im(I_1)}{Re(I_1)}\right)\right) = \cos\left(\tan^{-1}\left(\frac{28.5}{32.5}\right)\right) = \cos 41.2^\circ$$

$$= 0.75 \quad \text{lagging}$$

$$P_{in} = \sqrt{3}V_L I_L \cos\varphi = \sqrt{3}\times 415 \times 43.2 \times 0.75 = 23280W$$

$$\eta = \frac{P_{out}}{P_{in}} \times 100\% = \frac{13240}{23280} \times 100\% = \underline{57\%}$$

Q4.

a) The national grid is so large that no individual generator is able to determine the magnitude or the frequency of the grid's voltage. All generators must operate at the grid voltage in terms of magnitude, frequency, and phase angle to be connected to the grid.

The synchronous generator's rotating speed is precisly set to have the grid's frequency at 50Hz. The rotating speed depends on the poles of the synchronous generator.

i) for 2-pole machine. speed [rpm] = $\frac{50 \times 60}{1} = 3000$

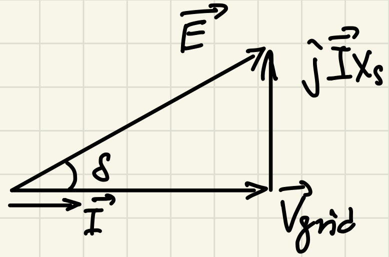
$$\text{speed [rad/s]} = \frac{3000}{60} \cdot 2\pi = 314 \text{ rad/s}$$

ii) For 20-pole machine. speed [rpm] = $\frac{50 \times 60}{10} = 300$

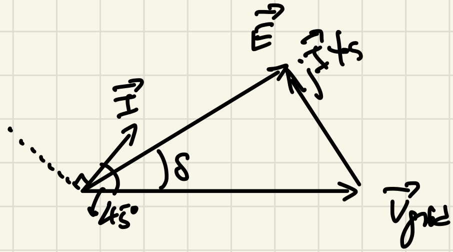
$$\text{speed [rad/s]} = \frac{300}{60} \cdot 2\pi = 31.4 \text{ rad/s}$$

A large number of poles means the actual rotating speed of the synchronous genenerator is slow to generate 50Hz voltage. Large hydro-electric plants have synchronous generators rotating at low speed thus these machines have large pole numbers.

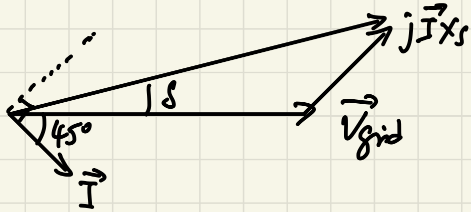
b) i)



ii)



iii)

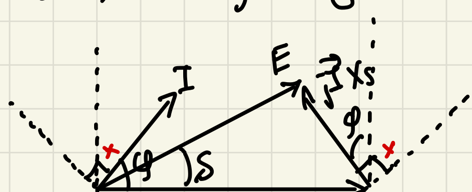


c)

60% rated MVA = $0.6 \times 500 = 300\,\text{MVA}$

$$I_{ph} = \frac{S}{3V_{ph}} = \frac{300\times10^6}{3 \times \frac{22.5}{\sqrt{3}} \times 10^3} = 7873\text{A}$$

$\cos\varphi = 0.6$, $\sin\varphi = 0.8$



$\vec{V} = \frac{22\text{kV}}{\sqrt{3}} = 12.7\text{kV}$

Star connection, $I_{ph} = I_{Line} = 7873\text{A}$

$$I_{ph}X = 7873 \times 0.4 = 3149V$$

$$E^2 = [V_{ph} - (IX_s \cdot \sin\varphi)]^2 + (I_{ph}X_s \cdot \cos\varphi)^2$$

$$= [12700 - 3149 \times 0.8]^2 + (3149 \times 0.6)^2$$

$$\Rightarrow E = 10.35kV$$

$$\sin\delta = \frac{I_{ph}X_s \cdot \cos\varphi}{E} = \frac{3149 \times 0.6}{10.35 \times 10^3} = 0.182$$

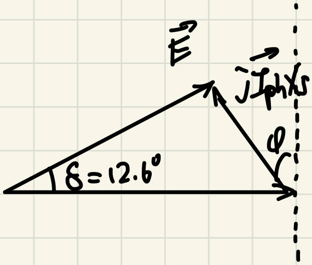
$$\delta = 10.5^\circ$$

d) $P_{new} = 1.2P = 300 \times 10^6 \times 0.6 \times 1.2 = 216MW$

E is constant.

$$\frac{P_{new}}{P_{old}} = 1.2 = \frac{\sin\delta_{new}}{\sin\delta_{old}} \Rightarrow \sin\delta_{new} = 1.2 \times 0.182$$

$$\delta_{new} = 12.6^\circ$$



$$I_{ph}X_s \cos\varphi = E \cdot \sin\delta = 2.26kV$$

$$I_{ph}X_s \sin\varphi = V_{ph} - E\cos\delta = 2.599kV$$

$$\tan\varphi = \frac{\sin\varphi}{\cos\varphi} = \frac{2.599}{2.26} = 1.15$$

$$Q = P \cdot \tan\varphi = 216 \times 1.15 = 248 MVAr$$

Solution Problem #5(a):

(*i*) The expression for the voltage $\tilde{V}\,(z)$ traveling in the $+z$ direction contains the factor $e^{-\gamma z}$, where $\gamma = \alpha + i\beta$ is the propagation constant, and α and β are real-valued constants. Therefore, the ratio of voltage at a distance l from some other point on the transmission line is:

$$\frac{\tilde{V}\,(z+l)}{\tilde{V}\,(z)} = \frac{e^{-\gamma(z+l)}}{e^{-\gamma z}} = e^{-\gamma l} = e^{-\alpha l}\, e^{-j\beta l}$$

The magnitude of this difference is just the first term, *i.e.*, $e^{-\alpha l}$. We also know that:

$$\gamma = \sqrt{(R + j\omega L)(G + j\omega C)}$$

At 100 MHz, we find $\gamma = 0.00850 + i3.14468\ \mathrm{m}^{-1}$. Therefore, $\alpha = 0.00850\ \mathrm{m}^{-1}$, and the voltage after $l = 1$ m is:

$$(1\ \mathrm{V})\exp[-(0.00850\ \mathrm{m}^{-1})(1\ \mathrm{m})] = 0.9915\ \mathrm{V}$$

(*ii*) From Part (*i*), we know that this phase difference is just the phase of the factor $e^{-j\beta l}$. Since $\beta = 3.14468\ \ \mathrm{rad/m}$, the phase of $e^{-j\beta l}$ is 180° for $l = 1$ m.

Alternatively, $\lambda = \frac{2\pi}{\beta} = \frac{2\pi}{3.14} = 2$ m, therefore, for $l = 1$ m, phase is 180°

Solution Problem #5(b):

(*i*) The total voltage on the transmission line can be expressed as the sum of incident and reflected fields:

$$\tilde{V}\ (x) = \tilde{V}_F\left[\ e^{-j\beta x} +\ \rho_L\ e^{j\beta x}\right] \quad \text{where, } \rho_L = |\rho_L| e^{j\phi}.$$

$$\tilde{V}\ (x) = \tilde{V}_F\left[\ e^{-j\beta x} +\ |\rho_L|\ e^{j(\beta x+\phi)}\right]$$

$$\tilde{V}\ (x) = \tilde{V}_F[\cos(\beta x) + j\sin(\beta x) + |\rho_L|\cos(\beta x + \phi) - j\ \ |\rho_L|\sin(\beta x + \phi)]$$

Therefore, the magnitude of $\tilde{V}\ (x)$ is given by:

$$\left|\tilde{V}\ (x)\right| = \left|\tilde{V}_F\right|\sqrt{[(\cos(\beta x) + |\rho_L|\cos(\beta x + \phi))^2 + (\sin(\beta x) -\ |\rho_L|\sin(\beta x + \phi))^2]}$$

Simplifying,

$$\left|\tilde{V}\ (x)\right| = \left|\tilde{V}_F\right|\sqrt{1 + |\rho_L|^2 + 2\ |\rho_L|\cos\ (2\beta x + \phi)} \qquad (1)$$

(*ii*) The variations in the plot have the periodicity of half the wavelength:

$$\frac{\lambda}{n} = \frac{\lambda}{\sqrt{\epsilon_r}} = \frac{\lambda}{\sqrt{3}} = 8 \text{ mm};\ \lambda = 13.856 \text{ mm}$$

Therefore, frequency of the volage source, $f = \frac{c}{\lambda} = 21.65$ GHz

(*iii*) VSWR $= \frac{1+|\rho_L|}{1-|\rho_L|}$, from the voltage plot this value is VSWR $= 6/2 = 3$, therefore $|\rho_L| = \frac{1}{2} = 0.5$

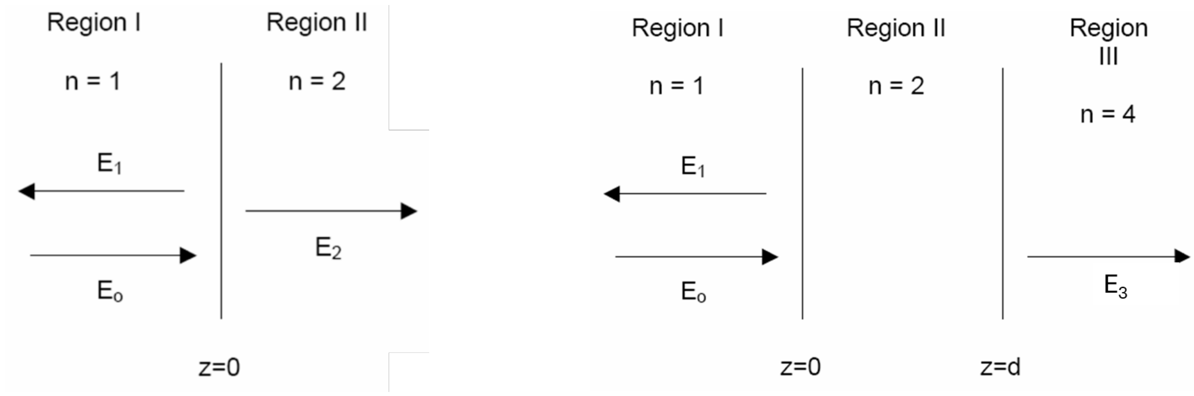
(*iv*) The minimum value of $\left|\tilde{V}\ (z)\right|$ occurs for the first time at $z = -3$ mm, where, from equation (1), this must imply that the term: $(2kz + \phi) = -\pi$. Solving:

$$\left(-2\frac{2\pi}{\lambda}3 + \phi\right) = -\pi$$

$$\left(-\frac{3\pi}{2} + \phi\right) = -\pi$$

$$\phi = \frac{\pi}{2} \qquad \text{Implies, } \rho_L = j/2$$

Solution Problem 6:



Since $\theta_i = 0°$, lets write down all the reflection and transmission coefficients:

$r_{ij} = \frac{n_i - n_j}{n_i + n_j}$ and $t_{ij} = \frac{2n_i}{n_i + n_j}$ where subscripts *i* and *j* represent the regions 1 and 2, or 2 and 3.

Therefore: $r_{12} = -1/3$ and $r_{23} = -1/3$

and $t_{12} = 2/3$, $t_{23} = 2/3$ and $t_{21} = 4/3$

(a) $E_2 = t_{12}\, E_0 = \frac{2}{3} E_0$

(b) $E_1 = r_{12}\, E_0 = -\frac{1}{3} E_0$

(c) $E_3 = t_{12}\, t_{23}\, E_0\, e^{ik_2 d}$, where $k_2 = \frac{2\pi}{\lambda_0} 2 = \frac{4\pi}{\lambda_0}$

Therefore: $E_3 = \frac{4}{9}\, E_0\, e^{i(\frac{4\pi}{\lambda_0})d}$

(d) Similarly, $E_1 = r_{12} E_0 + t_{12}\, e^{ik_2 d} r_{23}\, e^{ik_2 d} t_{21} E_0$

$$E_1 = -\frac{1}{3}E_0 - \frac{8}{27}\, e^{i(\frac{8\pi}{\lambda_0})d} E_0$$

(e) If $d = \lambda_0/8$, then $E_1 = -\frac{1}{3}E_0 - \frac{8}{27}\, e^{i\pi}E_0 = -\frac{1}{27}\, E_0$

Since the reflected electric field is almost zero implies that a thin slab of thickness $d = \lambda_0/8$ of refractive index $\mathrm{n} = 2$ (Region II) serves as an anti-reflection coating for light propagating from a low refractive index medium ($\mathrm{n} = 1$, region I) to a high refractive index medium ($\mathrm{n} = 4$, region III).