EGT1
ENGINEERING TRIPOS PART IB

Thursday 9 June 2022 14.00 to 16.10

Paper 6

INFORMATION ENGINEERING

Answer not more than ***four*** *questions.*

Answer not more than ***two*** *questions from each section.*

All questions carry the same number of marks.

The ***approximate*** *number of marks allocated to each part of a question is indicated in the right margin.*

Answers to questions in each section should be tied together and handed in separately.

Write your candidate number ***not*** *your name on the cover sheet.*

STATIONERY REQUIREMENTS
Single-sided script paper
Supplementary page: one extra copy of Fig. 1 (Question 1)
Supplementary page: graph template for Question 3 (two copies)

SPECIAL REQUIREMENTS TO BE SUPPLIED FOR THIS EXAM
CUED approved calculator allowed
Engineering Data Book

10 minutes reading time is allowed for this paper at the start of the exam.

You may not start to read the questions printed on the subsequent pages of this question paper until instructed to do so.

You may not remove any stationery from the Examination Room.

SECTION A

Answer not more that **two** *questions from this section*

1 (a) Given a physical system, with input $u(t)$ and output $y(t)$, it is required to find a constant k such that when $u(t) = -ky(t)$ the resulting closed loop system has certain desirable properties.

(i) Under what condition on the original system can the information required to plot its Nyquist diagram be experimentally obtained? If this condition is satisfied, describe how this information may be obtained. Comment on any difficulties which might arise. [4]

(ii) Carefully state the Nyquist stability criterion as it applies to this particular scenario. [4]

(b) Figure 1 shows part of the Nyquist diagram for a system with transfer function $G(s)$. $G(s)$ has no poles with a positive real part and satisfies $\lim_{s\to\infty} G(s) = 0$. The system is to be controlled by a proportional controller k as in Fig. 2.

(i) Estimate the gain and phase margins of the system for $k = 1$. Explain briefly what this implies for the robustness of the feedback system if the actual transfer function of the original system differs from $G(s)$. [6]

(ii) For what value of k is the phase margin maximised, and what is the corresponding gain margin? If, for this value of k, the controller is replaced by $K(s) = k\exp(-s\tau)$ then for what range of τ is the feedback system stable? [6]

(iii) If $k = 1$ and $d(t) = 0.1\sin(\omega t)$ for some unknown ω, then what is the maximum possible steady state amplitude of $y(t)$? [5]

An additional copy of Fig. 1 is attached to the back of this paper. It should be detached and handed in with your answers.

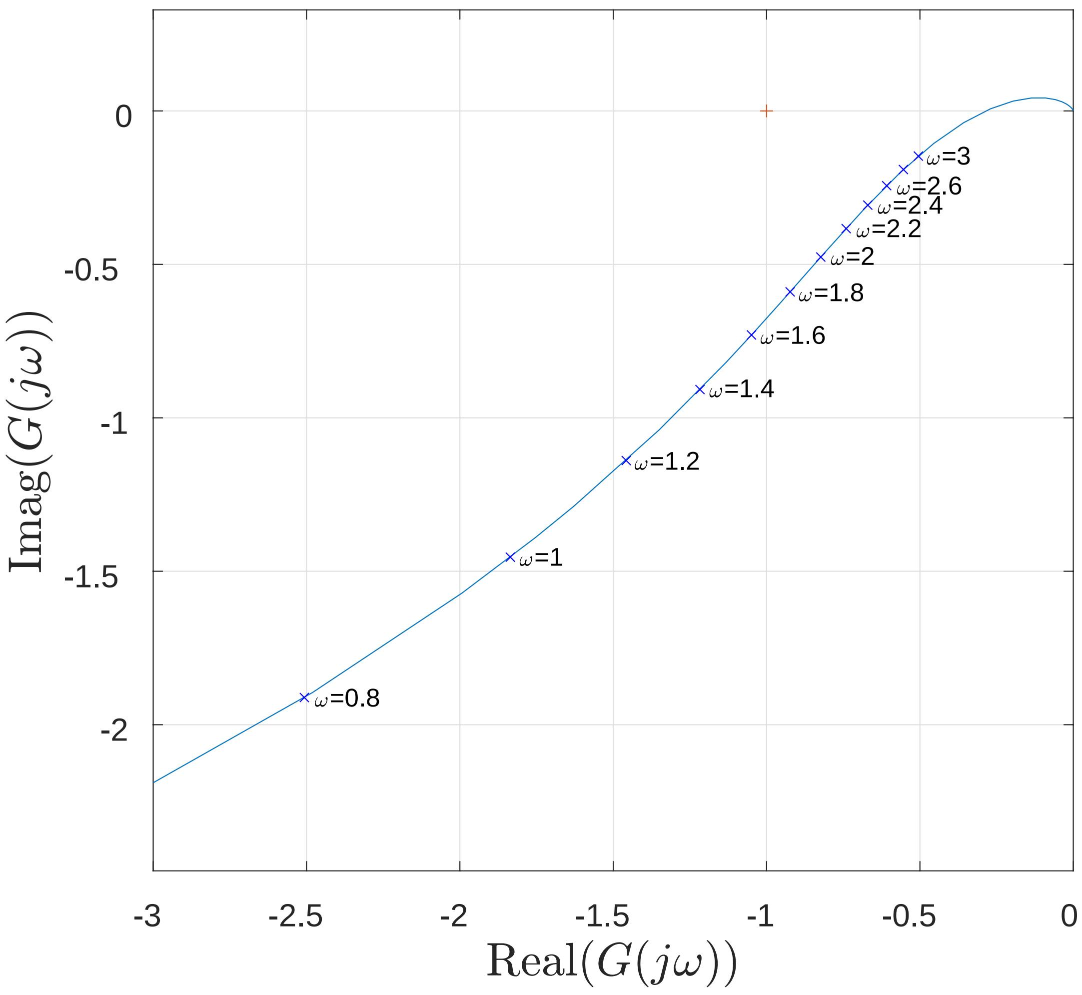

Fig. 1

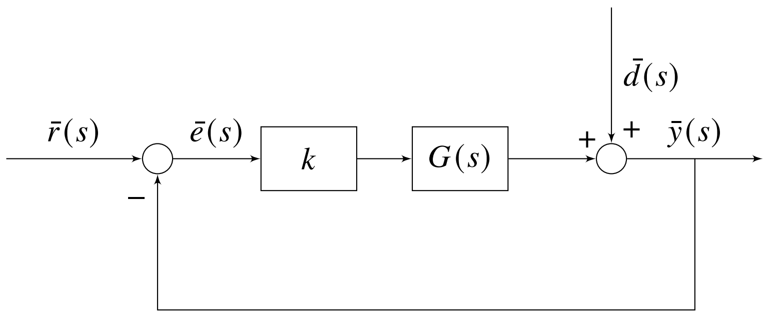

Fig. 2

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2 Consider a feedback combination of a system described by the differential equation

$$\dot{y} + 2y = 11u$$

and a controller

$$\dot{z} - z = y$$

with

$$u = r - z$$

(a) Draw a block diagram which shows the relationship between the Laplace transforms of the signals defined above. [8]

(b) On an Argand diagram, show the positions of the open loop poles (i.e. the poles of the return ratio) and the closed loop poles and comment on the stability of the open and closed loop systems. [5]

(c) Find the closed loop transfer functions from $\bar{r}(s)$ to $\bar{y}(s)$ and from $\bar{r}(s)$ to $\bar{u}(s)$ and sketch the responses of $y(t)$ and $u(t)$ to a unit step on $r(t)$, paying particular attention to the initial value and initial slope and the final value in each case. [12]

3 A particular motor with an inertial load has a transfer function $G(s)$, from current input to position output, where

$$G(s) = \frac{10}{s(s^2 + 20s + 100)}$$

(a) Sketch the Bode diagram of $G(s)$ on the graph template provided at the end of this paper. [7]

Please use the graph template attached to the back of this paper, and hand it in with your answer to this question.

(b) Calculate the value of k such that a feedback controller with transfer function $K(s) = k$, from input position error (demand minus measured position) and output current achieves a phase margin of 30°. What then is the gain margin and what, approximately, is the closed loop bandwidth? Indicate the gain and phase margin on your sketch. [8]

(c) Now consider instead a controller

$$K(s) = k_1 \frac{10(s + 10)}{(s + 100)}$$

Add the Bode diagram of $G(s)K(s)$ to your sketch (for $k_1 = 1$). [5]

(d) If k_1 in part (c) were to be chosen such that the phase margin is again 30° then how would you expect the closed loop response to a step demand on position to compare for the designs of part (b) and part (c)? You are not required to find k_1. [5]

SECTION B

Answer not more that **two** *questions from this section*

4 (a) Suppose that a signal $f(t)$ has Fourier transform $F(\omega)$, defined as follows:

$$F(\omega) = \begin{cases} 1, & |\omega| < \omega_0 \\ 0, & \text{otherwise} \end{cases}$$

(i) Determine the time function $f(t)$. [3]

(ii) What is the total energy of $f(t)$? [2]

(b) A new signal is made up as the product of functions as follows:

$$f_1(t) = \frac{\sin(\omega_1 t)}{t} \frac{\sin(\omega_2 t)}{t}$$

where $\omega_1 < \omega_2$. Determine the Fourier transform of $f_1(t)$ and sketch it as a function of frequency. [8]

(c) Determine the fraction of the total energy in $f_1(t)$ that lies above frequency $\omega_2 - \omega_1$. [7]

(d) It is desired to sample the signal at regular times, obtaining samples $f_1(nT)$. Make two sketches of the spectrum of the sampled signal, corresponding to the two cases $T = \pi/\omega_2$ and $T = \pi/(\omega_2 + \omega_1)$. What is the maximum allowable sampling period T if $f_1(t)$ is to be perfectly reconstructed from its sampled values? [5]

5 (a) Consider transmitting an information signal $m(t)$ using double-sideband suppressed carrier (DSB-SC) modulation. We wish to generate the DSB-SC signal $x_{\text{dsb-sc}}(t) = m(t)\cos(2\pi f_c t)$, where $m(t)$ is a low-pass signal with bandwidth W much smaller than f_c. To generate the DSB-SC signal, we first feed the sum $(m(t) + \cos(2\pi f_c t))$ into a square-law circuit which produces:

$$y(t) = (m(t) + \cos(2\pi f_c t))^2$$

Explain how $x_{\text{dsb-sc}}(t)$ can be produced by applying a suitable filter to $y(t)$. [7]

(b) A set of 4 information bits from a source is encoded to a 7-bit codeword using a $(7,4)$ Hamming code, where 3 parity-check bits are appended to each block of 4 data bits. Denoting the information bits by $\boldsymbol{s} = (s_1, s_2, s_3, s_4)$, recall that the 7-bit Hamming codeword is given by

$$\boldsymbol{c} = [c_1,\ c_2,\ c_3,\ c_4,\ c_5,\ c_6,\ c_7] = [s_1,\ s_2,\ s_3,\ s_4,\ s_1 \oplus s_2 \oplus s_3,\ s_2 \oplus s_3 \oplus s_4,\ s_1 \oplus s_3 \oplus s_4]$$

where $\oplus$ denotes modulo-two addition.

Each bit of the codeword is mapped to a BPSK symbol in the set $\{2, -2\}$ using the mapping $0 \to 2$, $1 \to -2$, and transmitted over an additive Gaussian noise channel. The received sequence at the output of the channel (after demodulation) is:

$$\boldsymbol{y} = [y_1,\ y_2,\ y_3,\ y_4,\ y_5,\ y_6,\ y_7] = [0.8,\ -1.1,\ -3,\ 1.7,\ -2.5,\ 3.4,\ -2.2]$$

(i) Give the optimal detection rule, and determine the sequence of detected BPSK symbols from $\boldsymbol{y}$. [5]

(ii) Decode the detected sequence to a Hamming codeword, and thereby determine the transmitted information bits. [8]

(iii) Compute the probability of decoding error for a $(7,4)$ Hamming code when used over a binary symmetric channel with crossover probability 0.2. [5]

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6 (a) The DFT for a signal is defined as

$$X_k = \sum_{n=0}^{N-1} x_n \exp(-j2\pi nk/N)$$

(i) If x_n is regularly sampled every 3 seconds from a continuous-time function $x(t)$, what frequency in Hz would component X_3 correspond to when $N = 10$? [3]

(ii) Show that the DFT of a constant signal $x_n = C$ is given by

$$X_k = \begin{cases} CN, & k = 0 \\ 0, & \text{otherwise} \end{cases}$$

[5]

(iii) The inverse DFT is defined as

$$x_n = \frac{1}{N} \sum_{k=0}^{N-1} X_k \exp(j2\pi nk/N)$$

Show that use of this formula to calculate x_n for n outside the range 0 to $N-1$ implies periodicity of the data, i.e.

$$x_{n+N} = x_n$$

and, for the case where X_k is purely imaginary,

$$x_{-n} = -x_n^*$$

[5]

(b) The Fourier transform of a baseband signal $x_b(t)$ is shown in Fig. 3. A passband signal $x(t)$ is generated from $x_b(t)$ and a complex sinusoid of frequency f_c as follows:

$$x(t) = \text{Re}\left[x_b(t)e^{j2\pi f_c t}\right]$$

Here $\text{Re}(a)$ denotes the real part of the complex number a.

(i) Show that the Fourier transform of $x_b^*(t)$ is $X_b^*(-f)$. (Here a^* denotes the conjugate of a complex number a.) [3]

(ii) Determine an expression for $X(f)$, the Fourier transform of $x(t)$, in terms of $X_b(f)$. [Hint: For a complex number a, $\text{Re}(a) = (a + a^*)/2$.] [4]

(iii) Sketch $X(f)$. [5]

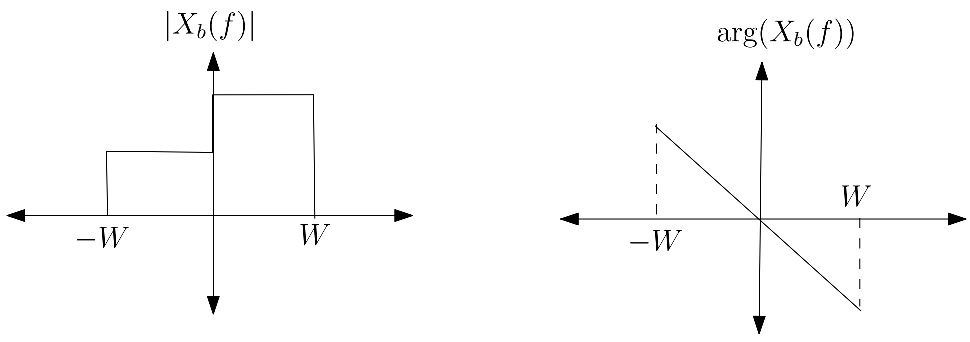

Fig. 3

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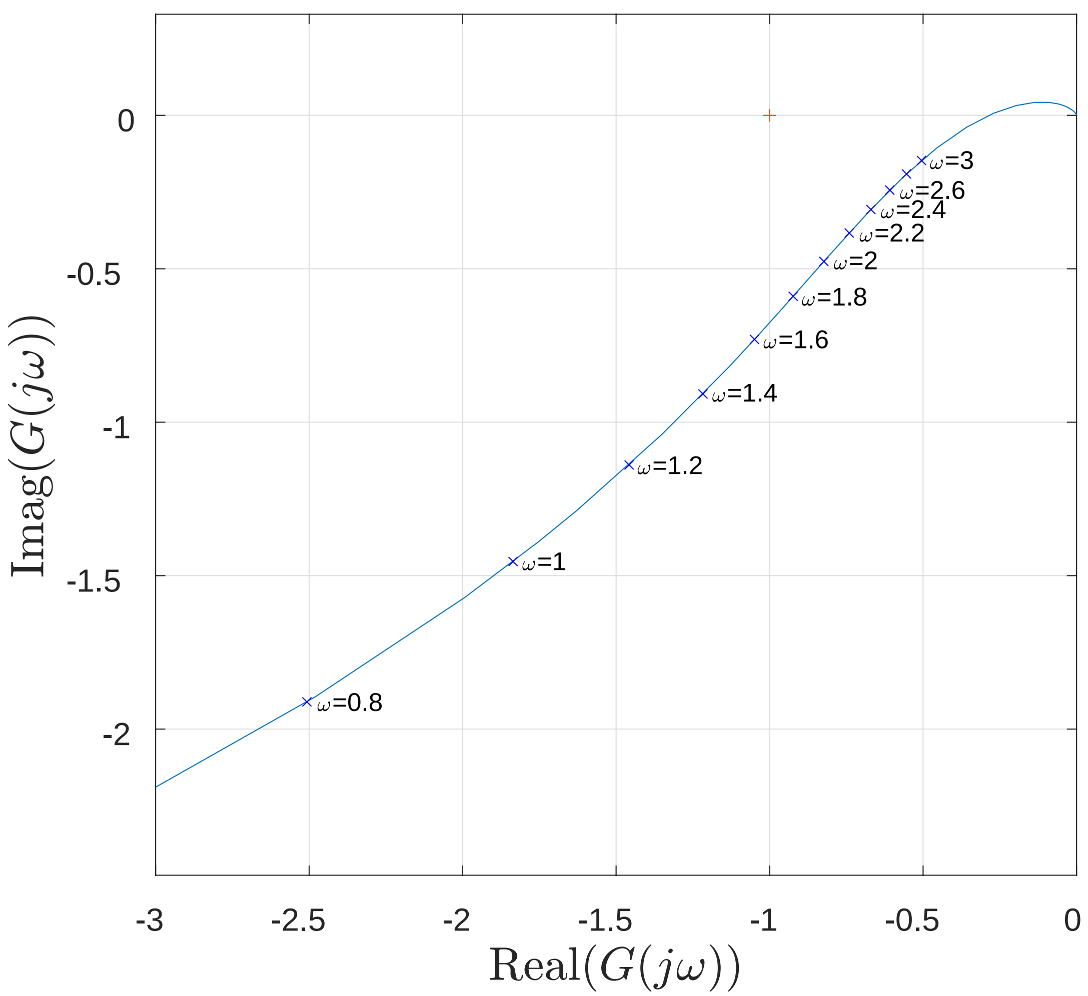

Extra copy of Fig. 1 for Question 1.

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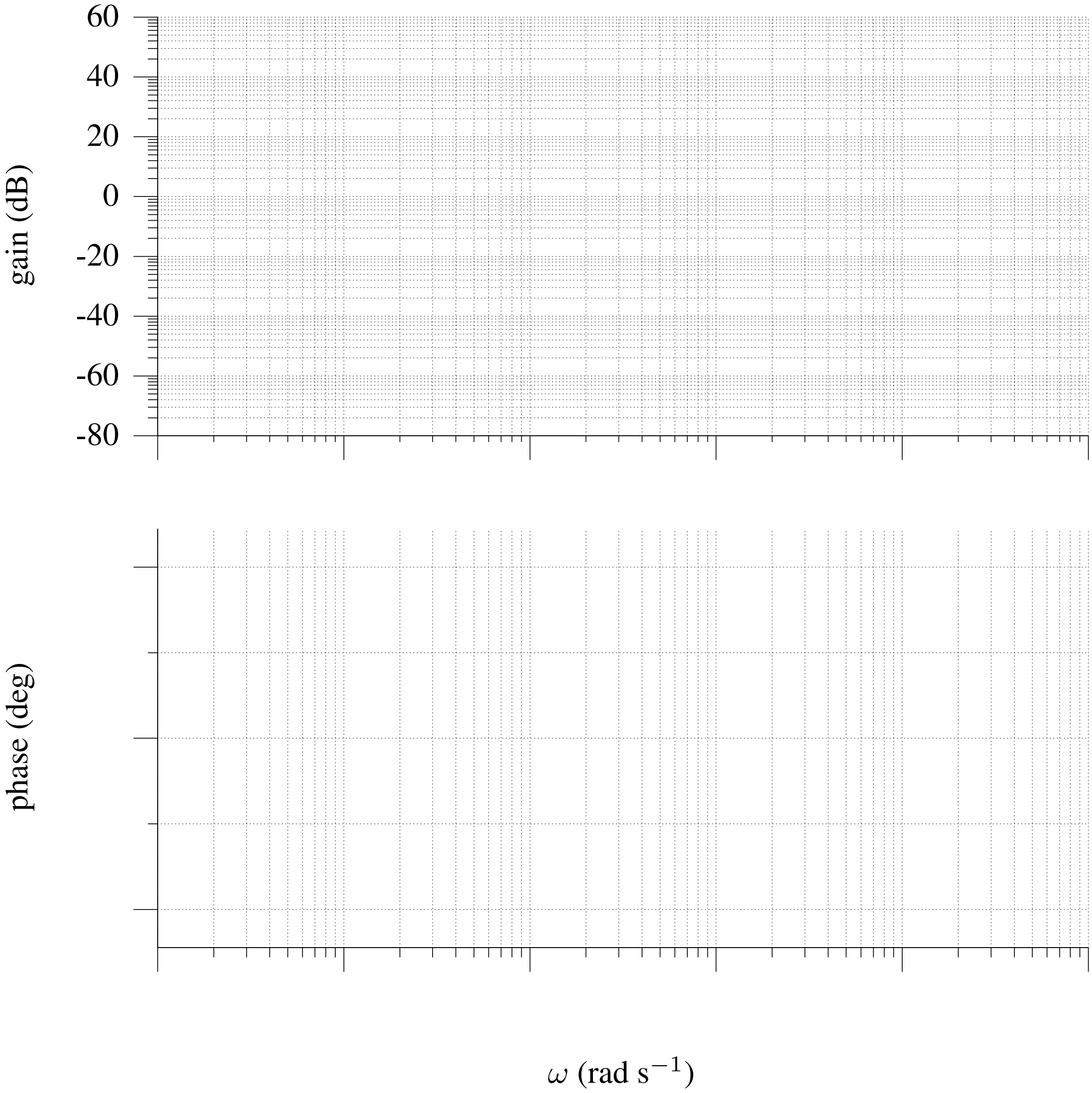

The graph template above is provided for Question 3. It should be annotated with your constructions and handed in with your answer to this question

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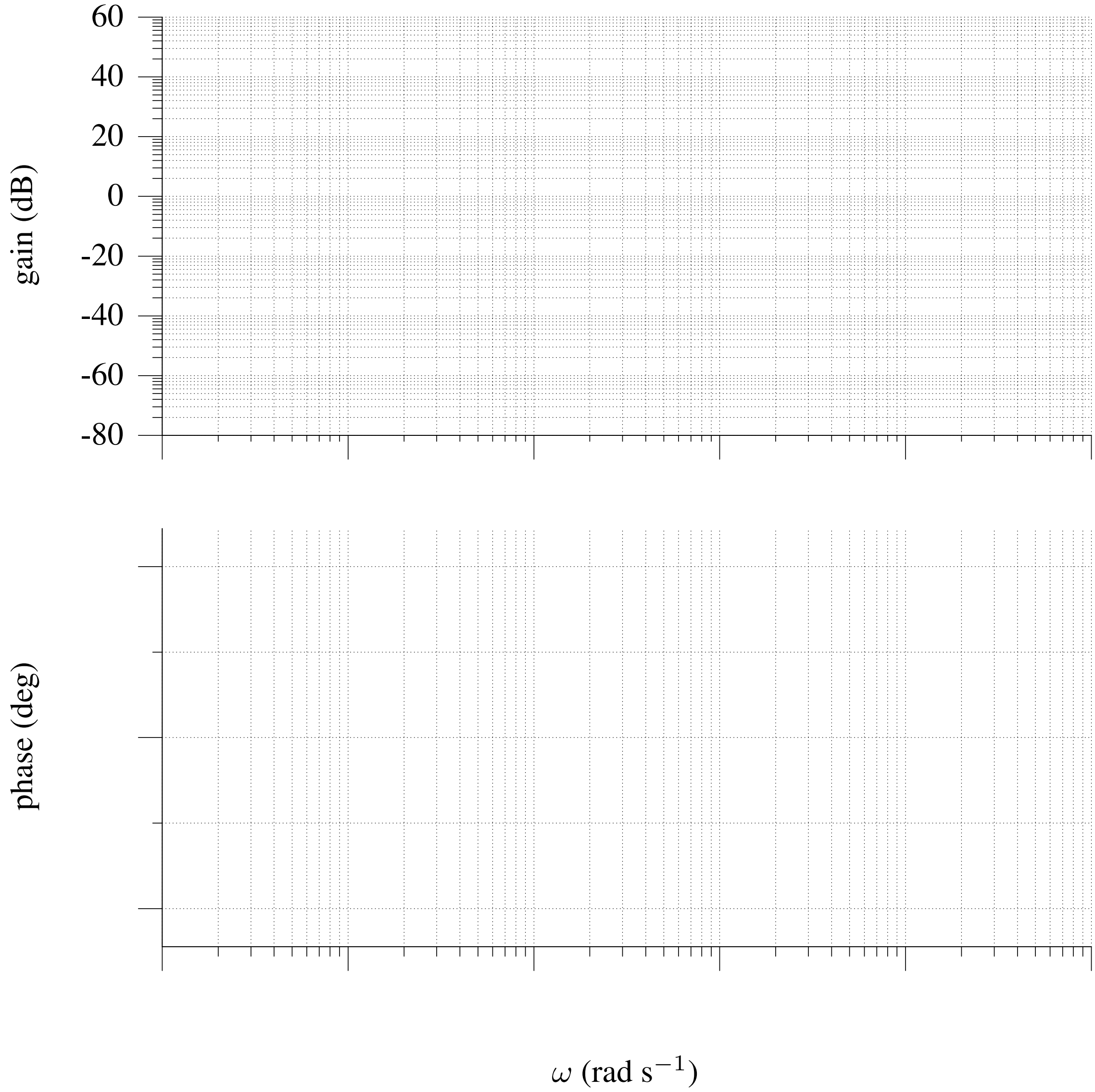

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