2P6 Solutions 2025

SECTION A

1. (a) i. A LTI system with impulse response function g is asymptotically stable iff $\int_0^\infty |g(t)|dt$ is bounded. [2]

   ii. T.F. $G(s)$ will be rational ($G(s) = n(s)/d(s)$ for polynomials n, d) and proper ($\deg(d) \geq \deg(n)$). [2]

   (b) The asymptote at low frequencies has magnitude 30dB giving $a = 31.6$. The break point at 0.01 rad/sec (where the magnitude starts to drop at 20dB/dec) accounts for the $(1 + 100s)$ term in the denominator. The gain levels off with a breakpoint at 0.2 rad/s, suggesting $T_3 = 5$. There are resonant and anti-resonant peaks at 1 rad/s and 10 rad/s respectively, giving $1/T_2 = 1$ and $1/T_1 = 10$. With no further poles the magnitude would level off, but there is another breakpoint at 300 rad/s accounting for T_4. Hence the values are

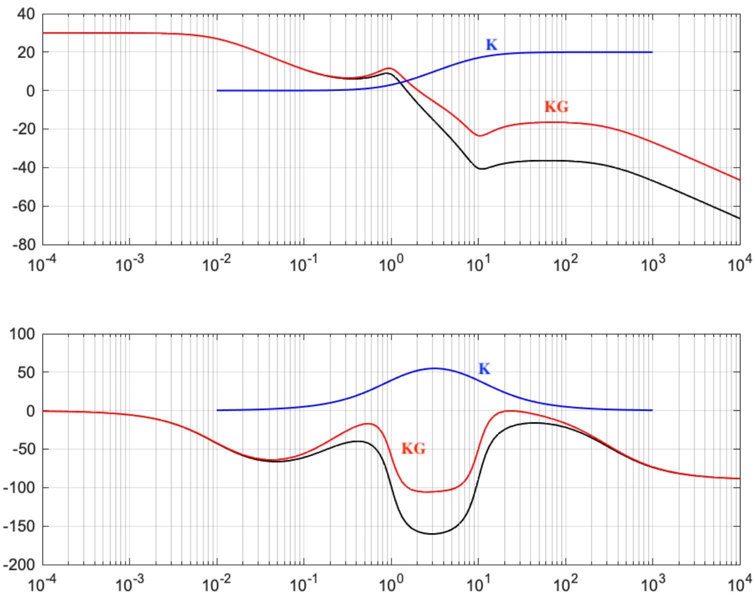
$$a = 31.6,\ T_1 = 0.1,\ T_2 = 1,\ T_3 = 5,\ T_4 = 0.00\dot{3}.$$

[8]

   (c) (i) Phase is always above -180 deg therefore gain margin is infinite. [2]

   (ii) Phase is -100 deg at $\omega = 1$ rad/s, where the gain is 8.5 dB ($= 2.66$), therefore $k_1 = 0.38$. Note that at 10 rad/s, phase is also -100 deg, but system would already be unstable at a phase lag of -80 deg. [2]

   (ii) An accurate plot is shown below: [5]



   (iii) $K(s)$ reduces the slope of the gain by 20 dB/dec between 1 rad/s and 10rad/s, so the gain now hits zero at 2 rad/s. The phase lead due to K is approx 50 deg at 2 rad/s, so the phase margin is now just under 80 deg (accurate value 74.2 deg). [4]

2. (a) Substituting $s := j\omega$ and inspecting the limits as $\omega \to \infty, 0$, we see that:

- $G_1 = $ B because $G_1 \to 2$ as $\omega \to \infty$.
- $G_2 = $ A because $G_2 \to -0.5$ as $\omega \to \infty$.
- $G_6 = $ C because $G_6 \to 1$ as $\omega \to 0$ and B is accounted for.
- $G_5 = $ E because $G_5 \to 2$ as $\omega \to 0$ and $G_5 \to 0$ as $\omega \to \infty$.
- $G_3 = $ D because $\Re(G_3) = \frac{-1/10+2\omega^2}{\omega^2+\omega^4/100} \to 1.9$ as $\omega \to 0$.
- $G_4 = $ F by elimination.

BADFEC. [8]

(b) The phase ϕ can be computed by simply adding the contributions of $\frac{1}{j\omega}$, $\frac{1}{(1+T_1 j\omega)}$ and $\frac{1}{(1+T_2 j\omega)}$, giving:

$$\phi = -\pi/2 - \tan^{-1}(\omega T_1) - \tan^{-1}(\omega T_2).$$

For the magnitude, note that

$$|G(j\omega)| = |\frac{1}{j\omega(1+T_1 j\omega)(1+T_2 j\omega)}|$$

Evaluating asymptotic frequencies using these expressions, we have:

$$\phi \to -\pi/2 \text{ and } |G(j\omega)| \to \infty \text{ as } \omega \to 0^+$$

$$\phi \to -3\pi/2 = \pi/2 \mod 2\pi \text{ and } |G(j\omega)| \to 0 \text{ as } \omega \to \infty.$$

[5]

(c) Yes. First note that

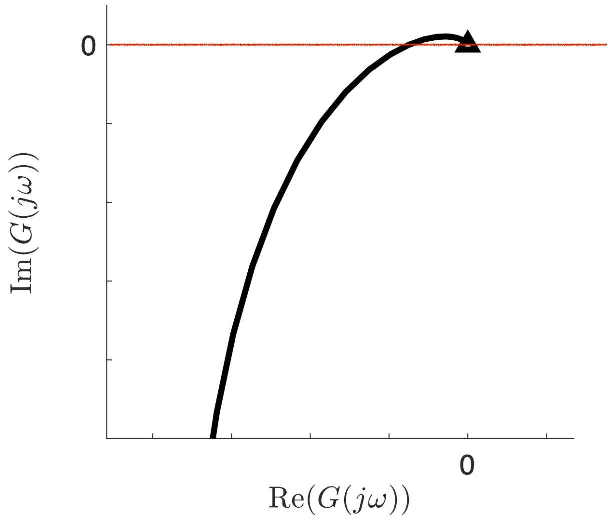
$$G(j\omega) = \frac{1}{j\omega(1+T_1 j\omega)(1+T_2 j\omega)} = \frac{-(T_1+T_2) - \frac{j}{\omega}(1-\omega^2 T_1 T_2)}{1+\omega^2(T_1^2+T_2^2)+\omega^4 T_1^2 T_2^2}$$

which has negative real part for all frequencies.
Second, from part (i) we know that as ω goes from 0 to ∞, the Nyquist locus moves from $-j\infty$ to approach the origin from an angle of $\pi/2$ (i.e. from above the real axis). Therefore there must be a crossing of the real axis. This occurs when the imaginary component is zero, i.e.

$$(1-\omega^2 T_1 T_2) = 0 \implies \omega = 1/\sqrt{T_1 T_2}$$

[4]

(d) Sketch: [3]



(e) Substituting in $\omega = 1/\sqrt{T_1 T_2}$ to the real component of $KG(j\omega)$ (computed in (c)):

$$\Re(G(j\omega)) = \frac{-K(T_1+T_2)}{1+(T_1^2+T_2^2)/(T_1T_2)+1} = \frac{-K(T_1+T_2)T_1T_2}{T_1T_2+(T_1^2+T_2^2)+T_1T_2} = \frac{KT_1T_2}{T_1+T_2}$$

By the Nyquist criterion, for asymptotic stability the locus must avoid encircling the -1 point, requiring:

$$\frac{KT_1T_2}{T_1+T_2} > -1 \implies K < \frac{T_1+T_2}{T_1T_2}$$

[5]

3. (a) Laplace transforming the model equations with zero initial conditions gives:

$$sN_1 = U - F_1 \qquad F_1 = \alpha(N_1 - N_2)$$
$$sN_2 = F_1 - F_2 \qquad F_2 = \beta N_2$$

Substituting, and eliminating the N_x variables, we get

i. $G_1 := \frac{F_1}{U} = \frac{\alpha(s+\beta)}{s^2+(\beta+2\alpha)s+\alpha\beta}$ [4]

ii. $G_2 := \frac{F_2}{F_1} = \frac{\beta}{s+\beta}$ [3]

iii. $G(s) = G_2(s)G_1(s) = \frac{\alpha\beta}{s^2+(\beta+2\alpha)s+\alpha\beta}$, therefore $P = R = \alpha\beta$ and $Q = (\beta + 2\alpha)$. [3]

(b) First, analyse the system and step response. In Laplace domain $U(s) = A/s$ therefore $F_2(s) = \frac{A}{s}\frac{\alpha\beta}{s^2+(\beta+2\alpha)s+\alpha\beta}$. Using final value theorem, $sF_2(s) \to A$ as $s \to 0^+$ therefore $f_2(t) \to A$ as $t \to \infty$. Similarly, initial value theorem (or elementary reasoning) gives $f_2(0) = 0$.
To assess intermediate behaviour, note that $G(s)$ is a stable second order system, and:

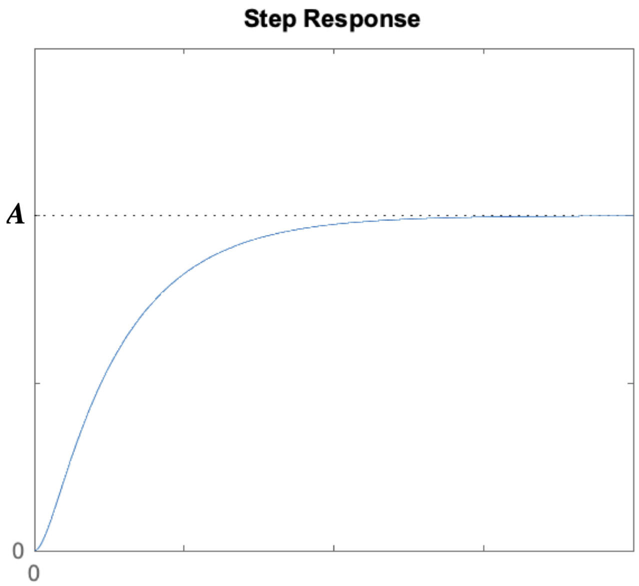
$$G(s) = \frac{\alpha\beta}{s^2 + (\beta + 2\alpha)s + \alpha\beta} = \frac{1}{s^2/(\alpha\beta) + (\beta + 2\alpha)s/(\alpha\beta) + 1}$$

From data book and by inspection of G, we find the time period $T = 1/\sqrt{(\alpha\beta)}$ and following relation for the damping factor, ζ:

$$\zeta = \frac{\alpha + \beta/2}{\sqrt{(\alpha\beta)}} \implies \zeta^2 = \frac{\alpha^2 + \alpha\beta + \beta^2}{\alpha\beta} > 1 \implies \zeta > 1.$$

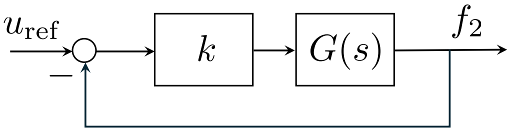
Therefore there is no resonance/overshoot in the response (it is overdamped).

i. From above reasoning, sketch is as follows: [3]



ii. From above analysis, frequency response of G has gain of 1 at zero frequency, monotonically decreasing thereafter. Therefore the steady state outflow rate can never exceed the inflow rate for any realisable input, but may transiently (e.g. input abruptly shut off). [5]

(c) i. Block diagram (NB, students who place k on the feedback path will also get full marks): [3]



ii. For diagram showm, New CLTF is $\frac{kG}{1+kG}$. Plugging in expression for G:

$$\frac{kG}{1+kG} = \frac{k\frac{\alpha\beta}{s^2+(\beta+2\alpha)s+\alpha\beta}}{1+k\frac{\alpha\beta}{s^2+(\beta+2\alpha)s+\alpha\beta}} = \frac{k\frac{\alpha\beta}{s^2+(\beta+2\alpha)s+\alpha\beta}}{\frac{s^2+(\beta+2\alpha)s+\alpha\beta+k\alpha\beta}{s^2+(\beta+2\alpha)s+\alpha\beta}} = \frac{k\alpha\beta}{s^2+(\beta+2\alpha)s+\alpha\beta+k\alpha\beta}.$$

The gain at zero frequency is therefore $\frac{k}{1+k}$ (or, alternatively, use final value theorem as above). Thus the steady outflow rate becomes: [4]

$$f_2^{ss} = \frac{Ak}{1+k}$$

Students who placed k on the feedback path will get full marks for a CLTF as $\frac{\alpha\beta}{s^2+(\beta+2\alpha)s+\alpha\beta+k\alpha\beta}$ and a corresponding steady state as $f_2^{ss} = \frac{A}{1+k}$

SECTION B

4. **(a-1)** **Fourier Transform of $x(t) = \text{sgn}(t)e^{-a|t|}$:**

The Fourier Transform of $x(t)$ is given by:

$$X(\omega) = \int_{-\infty}^{\infty} x(t)e^{-j\omega t}dt.$$

Breaking the integral into two parts for $t > 0$ and $t < 0$:

$$X(\omega) = \int_{0}^{\infty} e^{-at}e^{-j\omega t}dt - \int_{-\infty}^{0} e^{at}e^{-j\omega t}dt.$$

For $t > 0$:

$$\int_{0}^{\infty} e^{-t(a+j\omega)}dt = \frac{1}{a + j\omega}.$$

For $t < 0$:

$$-\int_{-\infty}^{0} e^{t(a-j\omega)}dt = -\frac{1}{a - j\omega}.$$

Combining both terms:

$$X(\omega) = \frac{1}{a + j\omega} - \frac{1}{a - j\omega} = -2j\omega/(a^2 + \omega^2).$$

Thus, the Fourier Transform is verified:

$$X(\omega) = \frac{-2j\omega}{a^2 + \omega^2}.$$

(a-2) **Magnitude and Phase Spectra:**

The magnitude spectrum is:

$$|X(\omega)| = \frac{2|\omega|}{a^2 + \omega^2}.$$

The phase spectrum is:

$$\angle X(\omega) = \begin{cases} -\frac{\pi}{2}, & \omega > 0, \\ +\frac{\pi}{2}, & \omega < 0. \end{cases}$$

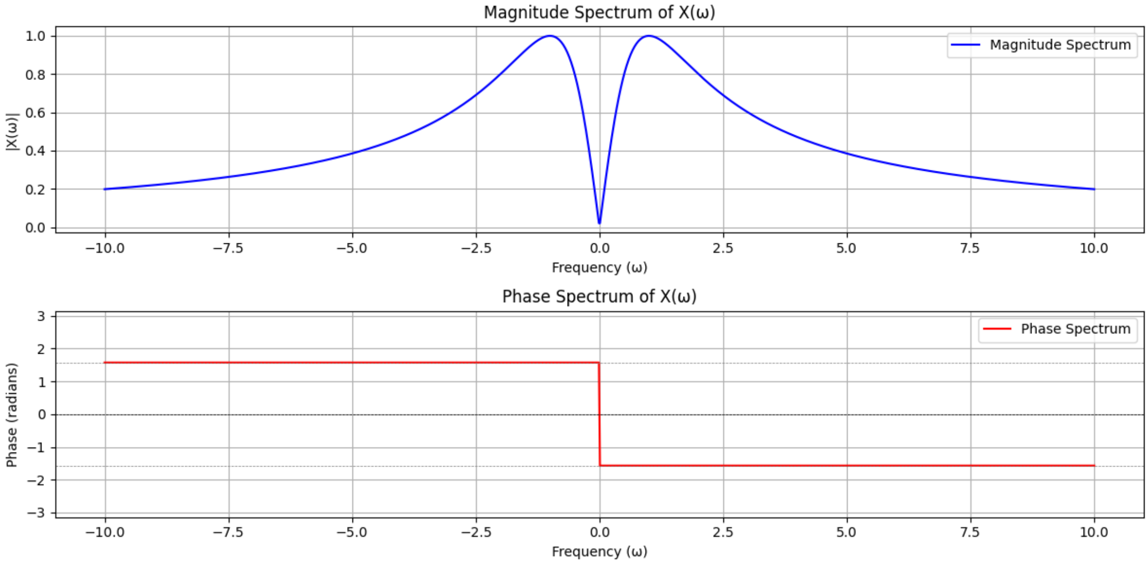


Figure 1: Magnitude and Phase Spectra of $X(\omega) = -2j\omega/(a^2 + \omega^2)$.

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(b) Verification of Parseval's Theorem for $f(t) = e^{-a|t|}$

Given information

- $f(t) = e^{-a|t|}$ where $a > 0$
- We need to verify: $\int_{-\infty}^{\infty} |f(t)|^2 dt = \frac{1}{2\pi}\int_{-\infty}^{\infty} |F(\omega)|^2 d\omega$
- Hint: $\int_{-\infty}^{\infty} \frac{1}{(1+x^2)^2} dx = \frac{\pi}{2}$

Left-Hand Side (LHS)

Calculate $|f(t)|^2$

$$|f(t)|^2 = (e^{-a|t|})^2 = e^{-2a|t|}$$

Set up the LHS integral

$$\int_{-\infty}^{\infty} |f(t)|^2 dt = \int_{-\infty}^{\infty} e^{-2a|t|} dt$$

Split the integral at t = 0

$$\int_{-\infty}^{0} e^{2at} dt + \int_{0}^{\infty} e^{-2at} dt$$

Evaluate each integral

For $\int_{-\infty}^{0} e^{2at} dt$:

$$\frac{1}{2a} e^{2at}\bigg|_{-\infty}^{0} = \frac{1}{2a}(1 - 0) = \frac{1}{2a}$$

For $\int_{0}^{\infty} e^{-2at} dt$:

$$-\frac{1}{2a} e^{-2at}\bigg|_{0}^{\infty} = -\frac{1}{2a}(0 - 1) = \frac{1}{2a}$$

Sum the results

$$\text{LHS} = \frac{1}{2a} + \frac{1}{2a} = \frac{1}{a}$$

Right-Hand Side (RHS)

Step 1: Calculate the Fourier Transform F(ω)

$$F(\omega) = \int_{-\infty}^{\infty} e^{-a|t|} e^{-i\omega t} dt$$

Split the integral:

$$F(\omega) = \int_{-\infty}^{0} e^{at} e^{-i\omega t} dt + \int_{0}^{\infty} e^{-at} e^{-i\omega t} dt$$

Evaluate:

$$F(\omega) = \frac{1}{a + i\omega} + \frac{1}{a - i\omega} = \frac{2a}{a^2 + \omega^2}$$

Step 2: Calculate $|F(\omega)|^2$

$$|F(\omega)|^2 = \left(\frac{2a}{a^2 + \omega^2}\right)^2 = \frac{4a^2}{(a^2 + \omega^2)^2}$$

Step 3: Set up the RHS integral

$$\frac{1}{2\pi} \int_{-\infty}^{\infty} |F(\omega)|^2 d\omega = \frac{1}{2\pi} \int_{-\infty}^{\infty} \frac{4a^2}{(a^2 + \omega^2)^2} d\omega$$

Step 4: Simplify the integral

$$\frac{2a^2}{\pi} \int_{-\infty}^{\infty} \frac{1}{(a^2 + \omega^2)^2} d\omega$$

Step 5: Substitute $x = \frac{\omega}{a}$

$$\frac{2a^2}{\pi} \cdot \frac{1}{a} \int_{-\infty}^{\infty} \frac{1}{(1 + x^2)^2} dx$$

Step 6: Use the given hint

$$\frac{2a}{\pi} \cdot \frac{\pi}{2} = a$$

Step 7: Simplify

$$\text{RHS} = \frac{1}{a}$$

Conclusion

We have shown that both the LHS and RHS evaluate to $\frac{1}{a}$, thus verifying the relation:

$$\int_{-\infty}^{\infty} |f(t)|^2 dt = \frac{1}{2\pi} \int_{-\infty}^{\infty} |F(\omega)|^2 d\omega = \frac{1}{a}$$

This relation is a specific case of Parseval's theorem, which states that the energy of a signal in the time domain equals the energy of its Fourier transform in the frequency domain.

(c) Using the Convolution Theorem, we evaluate the convolution of two signals in the frequency domain:

$$x(t) = \text{sgn}(t)e^{-a|t|}, \quad h(t) = u(t) - u(t-T)$$

where $u(t)$ denotes the unit step function.

Step 1: Compute Fourier Transforms of $x(t)$ and $h(t)$ For $x(t) = \text{sgn}(t)e^{-a|t|}$: From previous results, the Fourier Transform is:

$$X(\omega) = \frac{-2j\omega}{a^2 + \omega^2}$$

For $h(t) = u(t) - u(t-T)$: This is a rectangular pulse of duration T. Its Fourier Transform is:

$$H(\omega) = \frac{1 - e^{-j\omega T}}{j\omega}$$

Step 2: Apply the Convolution Theorem

The convolution $y(t) = x(t) * h(t)$ in the frequency domain becomes:

$$Y(\omega) = X(\omega)H(\omega) = \frac{-2j\omega}{a^2 + \omega^2} \cdot \frac{1 - e^{-j\omega T}}{j\omega} = \frac{-2(1 - e^{-j\omega T})}{a^2 + \omega^2}$$

Step 3: Compute Inverse Fourier Transform

Break $Y(\omega)$ into two terms:

$$Y(\omega) = \frac{-2}{a^2 + \omega^2} + \frac{2e^{-j\omega T}}{a^2 + \omega^2}$$

Using the Fourier Transform pair $\frac{2a}{a^2+\omega^2} \leftrightarrow e^{-a|t|}$, we find:

First term:

$$\mathcal{F}^{-1}\left\{\frac{-2}{a^2 + \omega^2}\right\} = -\frac{1}{a}e^{-a|t|}$$

Second term ($e^{-j\omega T} \to$ time shift):

$$\mathcal{F}^{-1}\left\{\frac{2e^{-j\omega T}}{a^2 + \omega^2}\right\} = \frac{1}{a}e^{-a|t-T|}$$

Combine results:

$$y(t) = \boxed{\frac{1}{a}\left(e^{-a|t-T|} - e^{-a|t|}\right)}$$

Step 4: Case Analysis

The solution simplifies for different ranges of t: For $t < 0$:

$$y(t) = \frac{1}{a}\left(e^{a(t-T)} - e^{at}\right)$$

For $0 \leq t < T$:

$$y(t) = \frac{1}{a}\left(e^{-a(T-t)} - e^{-at}\right)$$

For $t \geq T$:

$$y(t) = \frac{1}{a}\left(e^{-a(t-T)} - e^{-at}\right)$$

Final Result

$$y(t) = \boxed{\frac{1}{a}\left(e^{-a|t-T|} - e^{-a|t|}\right)}$$

5. (a) **(a.i)** The original frequency in Hz is:

$$f = \frac{2000\pi}{2\pi} = \boxed{1000\,\text{Hz}}$$

(a.ii) The Nyquist frequency is:

$$f_{\text{Nyquist}} = \frac{f_s}{2} = \frac{1500}{2} = \boxed{750\,\text{Hz}}$$

(a.iii)

- **Comparison:** Original frequency = $1000\,\text{Hz}$, Nyquist frequency = $750\,\text{Hz}$. Since $1000\,\text{Hz} > 750\,\text{Hz}$, aliasing occurs.
- **Calculation:** Observed frequency due to aliasing:

$$f_{\text{observed}} = f_s - f_{\text{original}} = 1500 - 1000 = \boxed{500\,\text{Hz}}$$

- **Explanation:** The reconstruction filter (cutoff = $750\,\text{Hz}$) cannot resolve frequencies above $750\,\text{Hz}$. The aliased frequency folds back into the range $[0, 750\,\text{Hz}]$, resulting in $500\,\text{Hz}$.

(b) DFT The DFT is given by the formula:

$$X[k] = \sum_{n=0}^{N-1} x[n]e^{-j\frac{2\pi}{N}kn}, \quad k = 0, 1, 2, \ldots, N-1$$

—

Calculate Each $X[k]$
For $k = 0$:

$$X[0] = \sum_{n=0}^{3} x[n]e^{-j\frac{2\pi}{4}(0)n}$$

Simplify:

$$= 1 + 2 + 0 + (-1)$$

Evaluate:

$$X[0] = 2$$

—

For $k = 1$:

$$X[1] = \sum_{n=0}^{3} x[n] e^{-j\frac{2\pi}{4}(1)n}$$

Substituting each term:

$$= 1 + 2e^{-j\frac{\pi}{2}} + 0 + (-1)e^{-j\frac{3\pi}{2}}$$

Evaluate each exponential:

$$e^{-j\frac{\pi}{2}} = -j, \quad e^{-j\pi} = j$$

Substituting:

$$= 1 + 2(-j) + 0 + (-1)j$$

Simplify:

$$X[1] = 1 - 3j$$

—

**For $k = 2$:

$$X[2] = \sum_{n=0}^{3} x[n] e^{-j\frac{2\pi}{4}(2)n}$$

Simplify each term:

$$= 1 + 2e^{-j\pi} + 0 + (-1)e^{-j3\pi}$$

Evaluate:

$$e^{-j\pi} = -1, \quad e^{-j3\pi} = -1$$

Substituting:

$$= 1 + 2(-1) + 0 + (-1)(-1)$$

Simplify:

$$X[2] = 1 - 2 + 1 = 0$$

—

For $k = 3$:

$$X[3] = \sum_{n=0}^{3} x[n] e^{-j\frac{2\pi}{4}(3)n}$$

Simplify each term:

$$= 1 + 2e^{-j\frac{3\pi}{2}} + 0 + (-1)e^{-j\frac{9\pi}{2}}$$

Evaluate:

$$e^{-j\frac{3\pi}{2}} = j, \quad e^{-j\frac{9\pi}{2}} = -j$$

Substituting:

$$= 1 + 2(j) + 0 + (-1)(-j)$$

Simplify:

$$X[3] = 1 + 3j$$

—

The DFT coefficients are:

$$X[k] = \{2, 1 - 3j, 0, 1 + 3j\}$$

(b) (i) Recall that the Fourier transform of $\cos(2\pi f_0 t)$ is $\frac{1}{2}[\delta(f - f_0) - \delta(f + f_0)]$. Thus the Fourier transform of $m(t) = 2\cos(1000\pi t) - \cos(2000\pi t)$ is [3]

$$\delta(f - 500) + \delta(f + 500) \; - \; \frac{1}{2}[\delta(f - 1000) + \delta(f + 1000)].$$

(ii) The modulation factor $\beta = \frac{\Delta f}{W}$, where W is the signal bandwidth and $\Delta f = k_\mathrm{f} \max|m(t)|$ is the frequency deviation. The maximum absolute value of $m(t)$ is 3. This can be seen either by inspection (say by observing that $m(t) = -3$ at $t = 1/1000)$) or by differentiating and setting $m'(t) = 0$. Therefore, $\Delta f = 3000 \cdot 3 = 9000$. From part (b)(i), the signal bandwidth is $W = 1000$ Hz. Therefore. $\beta = \frac{\Delta f}{W} = 9$. [5]
By Carson's rule, the bandwidth of the FM signal is approximately $2(1+\beta)W =$ 20kHz.

(iii) To recover $m(t)$ from the FM signal, we can use a differentiator followed by an envelope detector. Indeed, differentiating $s(t)$, we obtain

$$s'(t) = \frac{\mathrm{d}s(t)}{\mathrm{dt}} = 2\pi(f_c + k_\mathrm{f} m(t)) \cos\left(2\pi f_c t \; + \; 2\pi k_\mathrm{f} \int_0^t m(u) du\right).$$

Since $f_c \gg k_\mathrm{f} \max|m(t)|$, we can use a standard AM-style envelope detector to recover $m(t)$ from $s'(t)$. The differentiator can be implemented via a filter with frequency response $H(f) = \mathrm{j}2\pi f$. [5]

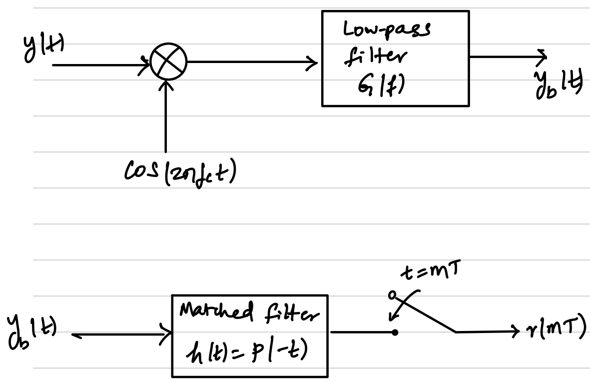
6. (a) The rate of transmission is $1/T = 1000$ bits/s. [2]

(b) From the duality theorem (Information databook), we know that the Fourier transform of $p(t) = \frac{1}{\sqrt{T}}\text{sinc}\left(\frac{\pi t}{T}\right)$ is a rectangular function (in frequency) given by: [5]

$$P(f) = \begin{cases} \sqrt{T}, & -\frac{1}{2T} \leq f \leq \frac{1}{2T} \\ 0, & \text{otherwise} . \end{cases}$$

(An answer in terms of $P(\omega)$ is also fine.) Therefore, the baseband bandwidth of $x_b(t)$ is $\frac{1}{2T}$ (since for a fixed sequence of symbols $X_b(f) = \left(\sum_k X_k e^{-\text{j}2\pi f k T}\right) P(f)$). Hence the passband bandwidth of $x(t)$ is $\frac{1}{T} = 10$kHz.

(c) (i) The block diagram is shown below. [5]



The low-pass filter $H(f)$ has constant gain for $f \in [-\frac{1}{2T}, \frac{1}{2T}]$ and a cutoff frequency that is a bit larger (say $\frac{1}{T}$ to allow for a roll-off, enablling practical implementation). The impulse response of the matched filter is $h(t) = p(-t) = \frac{1}{\sqrt{T}}\text{sinc}(\pi t/T)$.

(ii) The optimum decision rule is $\hat{X}_m = A$ if $Y_m \geq A/2$, and $\hat{X}_m = 0$ otherwise. [4]
The probability of decision error is

$$\begin{aligned} P_e &= P(X=0)P(Y \geq A/2 \mid X=0) \; + \; P(X=A)P(Y < A/2 \mid X=A) \\ &= \frac{1}{2}P(N \geq A/2) + \frac{1}{2}P(N < -A/2) \\ &= P(N \geq A/2) = Q\left(\frac{A}{2\sigma}\right) = Q\left(\sqrt{\frac{A^2}{4\sigma^2}}\right). \end{aligned}$$

(iii) For a $\{0, A\}$ constellation, the average energy per bit is $A^2/2$. Therefore $P_e = Q\left(\sqrt{\frac{E_b}{2\sigma^2}}\right)$. Using the approximation, we have $P_e \approx \frac{1}{2}\exp(-E_b/(4\sigma^2))$. Setting this to 0.01, we obtain $E_b/\sigma^2 = 4 * \ln 50 = 15.65$, which is 11.95dB. [4]

(iv) The probability of each symbol being detected in error in 0.01 (since the snr is the same as in part (iii)). The Hamming code can correct up to one detection error in each block of 7 symbol, so the probability that an information bit is decoded in error is

$$p_{\text{Hamming error}} = 1 - \binom{7}{0}(0.99)^7 - \binom{7}{1}(0.99)^6(0.01) = 0.002.$$

With a $(7, 4)$ Hamming code, we need 7 binary symbols to transmit 4 information bits. Therefore, the effective transmission rate is $R_{\text{eff}} = \frac{4}{7}\frac{1}{T} = \frac{4000}{7} =$ 571.43 bits/second. [5]

Therefore, with the Hamming code the probability of an information bit being decoded wrongly is smaller by a factor of 5 compared to the uncoded case, but the effective transmission rate is $\frac{4}{7}$th of the uncoded system.

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