Version MCS/3

EGT1
ENGINEERING TRIPOS PART IB

Thursday 6 June 2024 2 to 4.10

Paper 6

INFORMATION ENGINEERING

Answer not more than ***four*** *questions.*

Answer not more than **two** *questions from each section.*

All questions carry the same number of marks.

The ***approximate*** *number of marks allocated to each part of a question is indicated in the right margin.*

Answers to questions in each section should be tied together and handed in separately.

Write your candidate number ***not*** *your name on the cover sheet.*

STATIONERY REQUIREMENTS
Single-sided script paper

SPECIAL REQUIREMENTS TO BE SUPPLIED FOR THIS EXAM
CUED approved calculator allowed
Supplementary pages: one extra copy of Fig. 1 for Question 1
Engineering Data Book

10 minutes reading time is allowed for this paper at the start of the exam.

You may not start to read the questions printed on the subsequent pages of this question paper until instructed to do so.

You may not remove any stationery from the Examination Room.

SECTION A

Answer not more than **two** *questions from this section*

1 (a) What conditions should a physical system satisfy if its Bode diagram is to be determined experimentally? Briefly describe an experimental procedure to determine the diagram. [6]

(b) Figure 1 shows the Bode diagram of a stable linear system with transfer function of the form

$$G(s) = \frac{a(s^2 + bs + c)}{s(s + d_1)(s + d_2)}.$$

(i) Estimate the values of a, b, c, d_1 and d_2. [8]

(ii) The system is to be controlled in a unity gain negative feedback system with pre-compensator

$$K(s) = k\frac{s + 0.6}{s + 6}.$$

For $k = 1$ sketch the Bode diagram of the compensator $K(s)$ and the compensated system $G(s)K(s)$ on the attached figure. [6]

(iii) Use your sketch to estimate the value of k for which the system of part (b)(ii) has a phase margin of 45°. [5]

An additional copy of Fig. 1 is attached to the back of this paper. This should be detached and handed in with your answers.

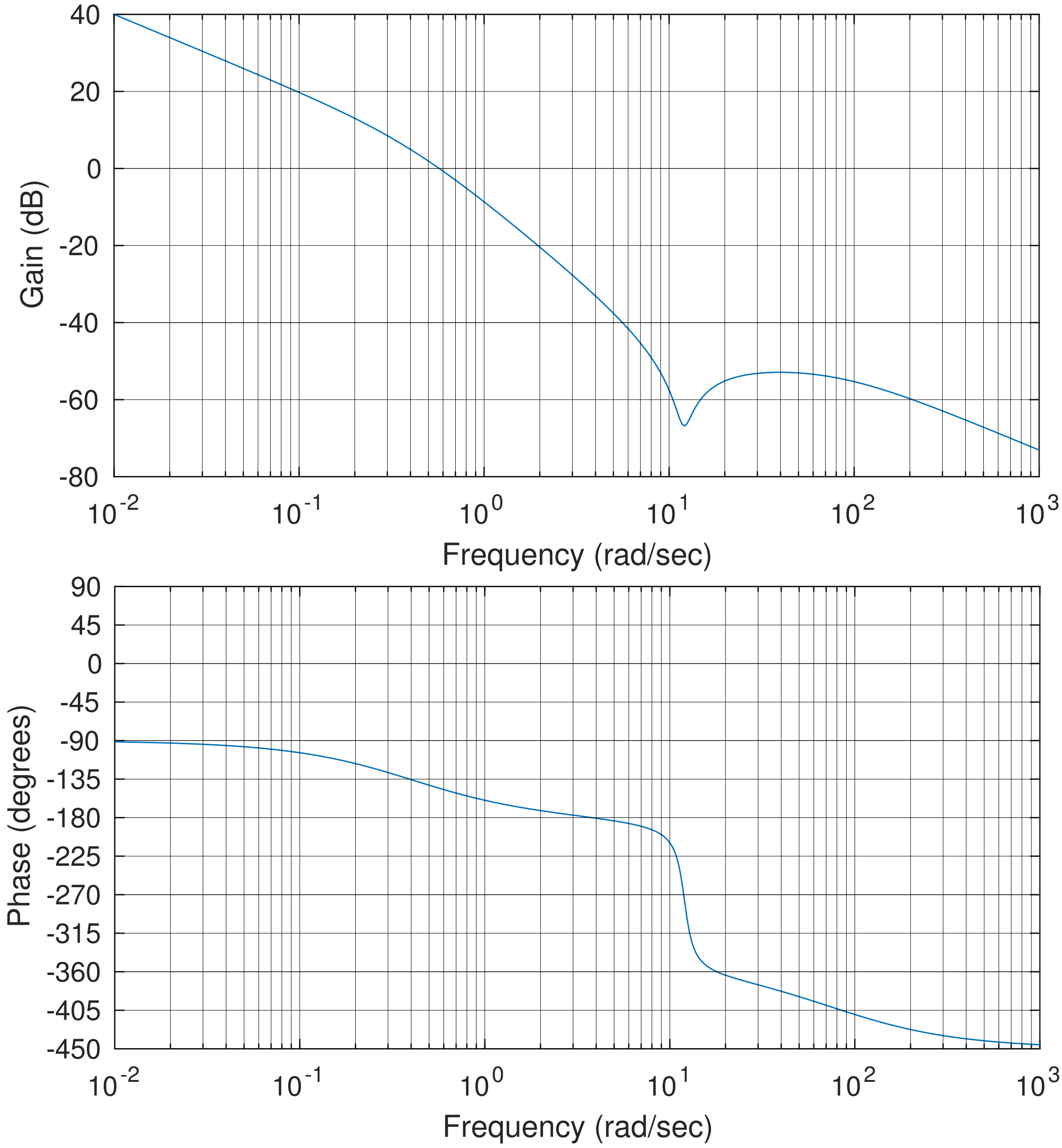
40
20
0
-20
-40
-60
-80
Gain (dB)
10^-2
10^-1
10^0
10^1
10^2
10^3
Frequency (rad/sec)
90
45
0
-45
-90
-135
-180
-225
-270
-315
-360
-405
-450
Phase (degrees)
10^-2
10^-1
10^0
10^1
10^2
10^3
Frequency (rad/sec)

Fig. 1

2 A model of calcium homeostasis in mammals relates the parathyroid hormone concentration $p(t)$ to calcium plasma concentration $c(t)$ and a set-point $r(t)$ by

$$p(t) = k_0(r(t) - c(t))$$

where k_0 is a positive constant. Vitamin D concentration $q(t)$ is determined by

$$\frac{dq}{dt} = k_1 p(t)$$

where k_1 is a positive constant. Calcium plasma concentration satisfies

$$\frac{dc}{dt} = k_2 p(t) + k_3 q(t) - v(t)$$

where $v(t)$ is the calcium clearance rate and k_2 and k_3 are positive constants.

(a) Show that this model can be described by the block diagram of Fig. 2 and find expressions for the constants A and B. [6]

(b) Find the closed-loop transfer function $H_1(s)$ relating $\bar{c}(s)$ to $\bar{r}(s)$ and the closed-loop transfer function $H_2(s)$ relating $\bar{c}(s)$ to $\bar{v}(s)$ in Fig. 2. Show that the closed-loop system is stable when $A > 0$ and $B > 0$. [6]

(c) The real biological system is known to maintain very tight steady-state control of calcium concentration even when parameters vary, and also in the presence of large fluctuations in the steady-state clearance rate. Comment on whether the model exhibits these properties in the following cases:

(i) $A > 0$ and $B > 0$; [4]

(ii) $A > 0$ and $B = 0$. [4]

(d) Let $A = 5$ and $B = 4$. Calculate and sketch the response $c(t)$ to a unit step in $v(t)$. [You may assume that the units are normalised so that t is measured in hours.] [5]

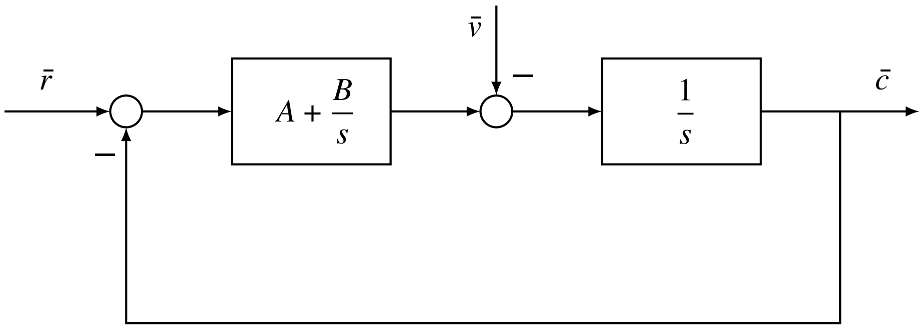

Fig. 2

3 The forward path of the feedback amplifier of Fig. 3 employs n identical first order lags connected in series to provide a transfer function

$$G(s) = \left(\frac{k}{Ts+1}\right)^n$$

where $k > 0, T > 0$ and n is a positive integer.

(a) Sketch the Nyquist diagram of $G(s)$ for $n = 1, 2$ and 3. Explain why the feedback amplifier is stable for any $k > 0$ and $T > 0$ when $n = 1$ or 2 but not if $n > 2$. [7]

(b) Assuming $n > 2$ show that $G(j\omega)$ has its first crossing of the negative real axis when

$$\omega T = \tan\left(\frac{\pi}{n}\right).$$

Hence show that the feedback amplifier is stable if and only if

$$k < \sec\left(\frac{\pi}{n}\right).$$

[6]

(c) For any postive integer n and any $k > 1$ show that $|G(j\omega)| = 1$ when

$$\omega T = \sqrt{k^2 - 1}.$$

[3]

(d) Let $n = 3$, $k = \sqrt{3}$ and $T = 1/\sqrt{300}$.

(i) Find the phase margin of the feedback amplifier. [4]

(ii) Find the steady state response $y(t)$ of the feedback amplifier to an input $r(t) = \sin(30t)$. [5]

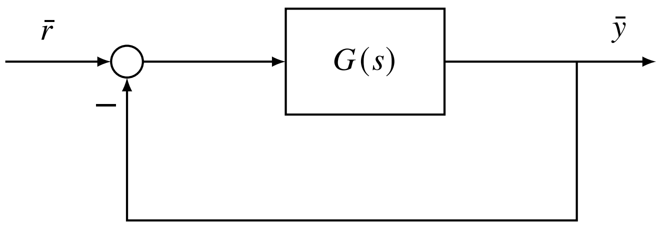

Fig. 3

SECTION B

Answer not more than **two** *questions from this section*

4 (a) A real-valued signal $x(t)$ has Fourier transform $X(\omega)$. Show the following result:

$$\int_{-\infty}^{\infty} x(t)^2 dt = \frac{1}{2\pi} \int_{-\infty}^{\infty} |X(\omega)|^2 d\omega$$

[5]

(b) A linear system obeys the following equation:

$$a\frac{dy(t)}{dt} + y(t) = x(t)$$

for a constant $a > 0$. Use the Fourier transform to determine the frequency response of the system, i.e. the ratio $Y(\omega)/X(\omega)$. Explain whether it is possible for the total energy of $y(t)$ to be greater than the total energy of $x(t)$. [5]

(c) The input signal is defined as

$$x(t) = \begin{cases} \exp(-t), & t > 0 \\ 0, & \text{otherwise} \end{cases}$$

Show that its Fourier transform is

$$X(\omega) = \frac{1}{1 + j\omega}$$

and that its total energy is 0.5. [5]

(d) Consider the linear system in part (b) with the input signal in part (c). Determine the total energy of the output $y(t)$ and find the value of a for which the total energy of the output $y(t)$ is equal to half of the energy at the input $x(t)$. [10]

5 (a) A continuous time signal is specified as

$$x(t) = \text{sinc}^2(t).$$

(i) Determine the largest frequency for which the spectrum of $x(t)$ is non-zero. [3]

(ii) It is desired to sample the signal with sampling interval $T = 2\pi/3$ s. Determine and sketch the spectrum of the sampled signal. Explain whether $x(t)$ may be reconstructed perfectly from its digital samples $x_n = x(nT)$. [4]

(iii) With $T = \pi/2$, find an expression for obtaining the continuous signal $x(t)$ from its digital samples $x_n = x(nT)$ in the form

$$f(t) = \sum_{n=-\infty}^{+\infty} c_{n,t} x_n$$

where $c_{n,t}$ are coefficients that should be determined. [*Hint:* Recall that the sampled signal may be represented in the time domain as a train of δ-functions, each weighted by the signal values x_n, and that reconstruction involves filtering the sampled signal with an appropriate reconstruction filter $H_r(\omega)$.] [6]

(b) A passband Quadrature Amplitude Modulation (QAM) signal $x(t)$ is generated using a complex-valued baseband waveform $x_b(t) = \sum_k X_k p(t - kT)$ as

$$x(t) = \mathsf{Re}(x_b(t)) \cos(2\pi f_c t) - \mathsf{Im}(x_b(t)) \sin(2\pi f_c t).$$

Here f_c is the carrier frequency, $p(t)$ is a unit-energy rectangular pulse of duration $T = 1$ second, and the symbols X_k are drawn from the constellation $C = \{1,\ j,\ -1, -j\}$.

(i) Suppose that pairs of bits are mapped to constellation symbols according to the rule $00 \to 1,\ 01 \to j,\ 11 \to -1,\ 10 \to -j$. Sketch the (baseband) waveform modulating the carrier $\cos(2\pi f_c t)$ for the following sequence of information bits: 11 00 10 01. [5]

(ii) Suppose that the complex-valued constellation C is replaced by the real-valued constellation $\{-3A, -A, A, 3A\}$. Compute the value of A so that the average energy per symbol is the same as that for the complex-valued constellation. [3]

(iii) Compare the probability of error for the complex-valued constellation C versus the real-valued one in part (b)(ii), when used over a Gaussian noise channel. You do not need to compute the error probabilities, but state which of the two probabilities you expect to be larger, justifying your answer. [4]

6 (a) Consider the digitisation of a signal $x(t)$ given by

$$x(t) = 3\sin(2000\pi t) + 4\cos(2000\pi t).$$

The signal is sampled every T seconds, and each sample quantised using a uniform quantiser with step-size $\Delta = 0.2$.

(i) What is the amplitude of $x(t)$? [2]

(ii) Compute the power of the quantisation noise, stating any assumptions on the distribution of the noise. [4]

(iii) Compute the average signal to quantisation noise ratio, assuming the quantiser has enough levels to cover the range of $x(t)$. Express your answer in dB. [3]

(iv) What is the minimum number of bits required to represent each quantised sample, assuming the quantiser has enough levels to cover the range of $x(t)$. [3]

(v) If T is such that the signal is sampled at 1.2 times the Nyquist rate, and each sample is represented using the number of bits calculated in part (iv), what is the bit rate of the digitised sequence? [4]

(b) Consider a Frequency-Division Multiple Access (FDMA) system with a total bandwidth of 100 MHz. Each user transmits information at a rate of $R = 500$ kbit/s, using Pulse Amplitude Modulation with a 4-symbol constellation and a unit-energy rectangular pulse of duration T seconds.

(i) Compute the value of T. [2]

(ii) Denoting the spectrum of the rectangular pulse by $P(f)$, sketch its magnitude $|P(f)|$. [4]

(iii) Assuming that the band-pass spectrum of each user does not cause interference beyond the first side-lobe, how many users can be accommodated in the FDMA system? You may assume that the carrier frequency is much larger than 100 MHz. [3]

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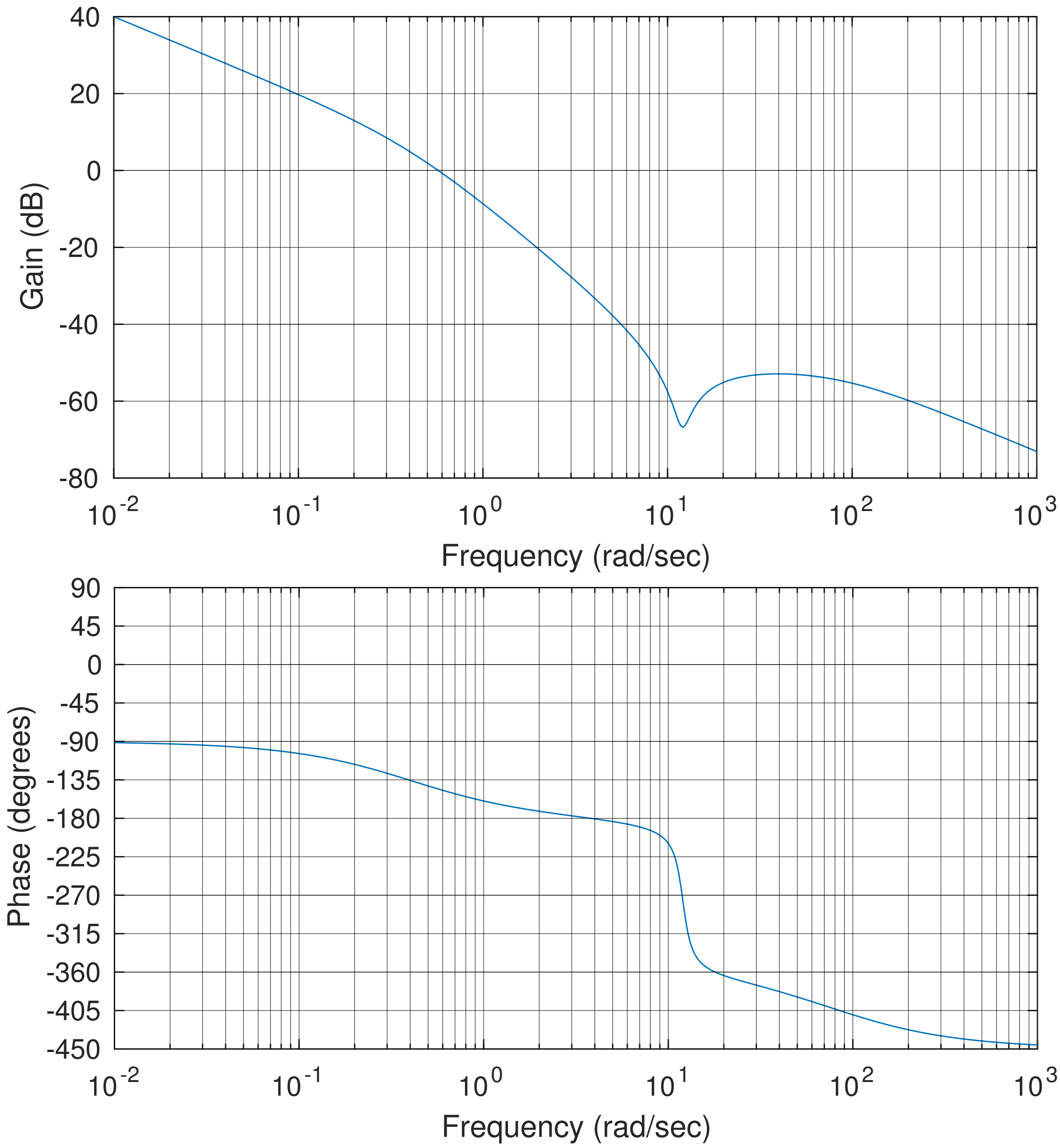
Thursday 6 June 2024, Paper 6, Question 1.



Extra copy of Fig. 1 for Question 1.

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