

# EGT1

## ENGINEERING TRIPPOS PART 1B

## Paper 8

## SELECTED TOPICS

*Answer **all** questions from section A. In addition:*

*If you are taking the Civil Engineering option AND the Foreign Language option, no further questions should be answered.*

If you are taking **EITHER** the Civil Engineering option **OR** the Foreign language option, answer **two** questions from one of the Sections C-H.

If you are taking **NEITHER** the Civil Engineering option nor the Foreign language option, answer **four** questions from two of the Sections C-H. Not more than two questions from each Section may be answered.

*All questions in Sections C–H carry the same number of marks.*

*The approximate number of marks allocated to each part of a question is indicated in the right margin.*

*Write your candidate number **not** your name on the cover sheet.*

Section A: <i>The Engineer in Business</i> .....	A.1
Section B: <i>Civil and Structural Engineering has been assessed as coursework</i>	
Section C: <i>Mechanics, Materials and Design</i> .....	C.1
Section D: <i>Aerothermal Engineering</i> .....	D.1
Section E: <i>Electrical Engineering</i> .....	E.1
Section F: <i>Information Engineering</i> .....	F.1
Section G: <i>Bioengineering</i> .....	G.1
Section H: <i>Manufacturing and Management</i> .....	H.1

## **STATIONERY REQUIREMENTS**

Single-sided script paper.

## **SPECIAL REQUIREMENTS TO BE SUPPLIED FOR THIS EXAM**

CUED approved calculator allowed

## Engineering Data Book

**10 minutes reading time is allowed for the paper at the start of the exam.**

**You may not start to read the questions printed on the subsequent pages of this question paper until instructed to do so.**

**You may not remove any stationary from the Examination room.**

Version TC/3

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## **SECTION A: The Engineer in Business**

Answer ***all*** questions from this section, on the special answer sheet.

1 Ten equally sized companies make cars for a particular segment of the UK car market. They make low rates of return. We observe one of the companies reducing their prices, when the others do not. This is likely to be an example of what type of pricing behaviour?

- (a) Collusive Pricing;
- (b) Price Discrimination;
- (c) Competitive Pricing;
- (d) Predatory Pricing;
- (e) None of the above.

[2.5]

2 Which of these statements is consistent with a Marxist view of a firm?

- (a) Firms in the consulting sector benefit from economies of scale;
- (b) Firms in the consulting sector pay the employees a salary equal to the value of their output;
- (c) Firms in the consulting sector rotate employees between clients to reduce their client specific knowledge;
- (d) Firms in the consulting sector favour growth of sales over profit maximisation;
- (e) None of the above.

[2.5]

3 Looking across all firms, which of the following situations is least favourable for a firm to grow?

- (a) GDP falls by 2%, technological opportunity increases by 3%, managerial experience increases by 3%;
- (b) GDP falls by 2%, technological opportunity increases by 2%, managerial experience increases by 2%;
- (c) GDP falls by 1%, technological opportunity increases by 2%, managerial experience increases by 3%;
- (d) GDP falls by 1%, technological opportunity increases by 3%, managerial experience increases by 3%;
- (e) None of the above. [2.5]

4 BYD is a Chinese manufacturing firm, initially manufacturing batteries. Recently it has become a major producer of electric vehicles. Which of the following best describes this growth strategy?

- (a) Growth by backward vertical integration;
- (b) Growth by forward vertical integration;
- (c) Growth by merger;
- (d) Growth by diversification;
- (e) None of the above. [2.5]

5 All of the following are accurate descriptions of modern marketing, EXCEPT which one?

- (a) Marketing is the creation of value for customers;
- (b) Marketing is managing profitable customer relationships;
- (c) Selling and advertising are synonymous with marketing;
- (d) Marketing involves satisfying customers' needs;
- (e) Marketing is used by for-profit and not-for-profit organizations.

[2.5]

6 Apple Watch was launched in 2015 and it has quickly become the global market leader in the smartwatch category. Customers described this product as “ease of use”, “stylish”, “powerful”, “convenient”, “cool” and so on. Based on customers feedback of Apple Watch, we can say that Apple Watch successfully satisfy customers':

- (a) tangible and intangible needs;
- (b) radical and incremental needs;
- (c) financial and non-financial needs;
- (d) rational and emotional needs;
- (e) None of the above.

[2.5]

7 Climeworks is the leading Direct Air Capture (DAC) company helping companies to achieve their Net Zero goal by removing the carbon dioxide emissions these companies cannot reduce in other ways. They have recently developed a marketing strategic plan to further expand their business. Which of the following forms a part of their marketing plan?

- (a) Situation analysis;
- (b) Market segmentation;
- (c) Targeting and positioning;
- (d) Marketing mix;
- (e) All of the above.

[2.5]

8 At the start of Microsoft's partnership with OpenAI, this was an example of managers following:

- (a) Porter's ideas about reinforcing activity systems through tradeoffs;
- (b) Barney's ideas about leveraging core resources;
- (c) McGrath's ideas about organisational agility;
- (d) Adner's ideas about ecosystem keystones;
- (e) Both part (c) and part (d).

[2.5]

9 When managers use the tool of five forces analysis, their implicit assumption is:

- (a) They can identify opportunities to grow;
- (b) They can identify new ways to negotiate with suppliers;
- (c) They can identify relevant sets of competitors;
- (d) They can identify core competences;
- (e) They can block new entrants.

[2.5]

10 Which of the following is NOT a product of Value-Rarity-Imitability-Organization (VRIO) analysis?

- (a) A list of core competences;
- (b) Sources of temporary advantage;
- (c) Points of parity with other firms;
- (d) An analysis of the activity system;
- (e) An understanding of what has generated firm profits in the recent past.

[2.5]

**END OF SECTION**

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**SECTION C: Mechanics, Materials and Design**

Answer not more than **two** questions from this section.

1 (a) Flow through a horizontal-axis wind turbine is shown in Fig. C.1(a). It is operating in air of density  $\rho$  and the *uniform* free wind speed is  $V$ . The swept area of the rotor is  $A$  and the axial induction factor is  $a$ . The speed of air at the rotor plane is  $V(1 - a)$  and downstream in the wake the speed is  $V(1 - b)$  as shown in the figure. The pressure difference across the rotor plane is  $\Delta p$  and the Power Coefficient is  $C_p$ .

From first principles (Bernouilli's equation, flow continuity and momentum) show that:

(i)  $b = 2a$ ; [5]

(ii)  $\Delta p = 2\rho V^2 a(1 - a)$ ; [2]

(iii)  $C_p = 4a(1 - a)^2$ ; [3]

(iv)  $a = \frac{1}{3}$  at the Betz limit. [2]

(b) Wind speed varies with height. A model used to account for *non-uniform* wind is shown in Fig. C.1(b). Flow through the wind turbine is divided into two halves. The top half is subject to wind at speed  $V$  and the bottom half is subject to a lower wind speed  $rV$  where  $r < 1$ . The wind turbine is operating with the bottom half at the Betz limit so that  $a_2 = \frac{1}{3}$ . The induction factor in the upper half is  $a_1$ .

(i) Explain why you might expect the pressure drop  $\Delta p$  to be the same for both halves of the turbine. [2]

(ii) Find a quadratic expression for the upper-half induction factor  $a_1$  in terms of  $r$ , and check your answer for the case  $r = 1$ . Explain carefully which of the two roots of the quadratic you should take. [3]

(iii) For the case  $r = \frac{2}{3}$  use your quadratic to find the value of  $a_1$  that satisfies the constraints of this stratified-wind model. [3]

(iv) Estimate the Power Coefficient for the turbine in this stratified condition. [5]

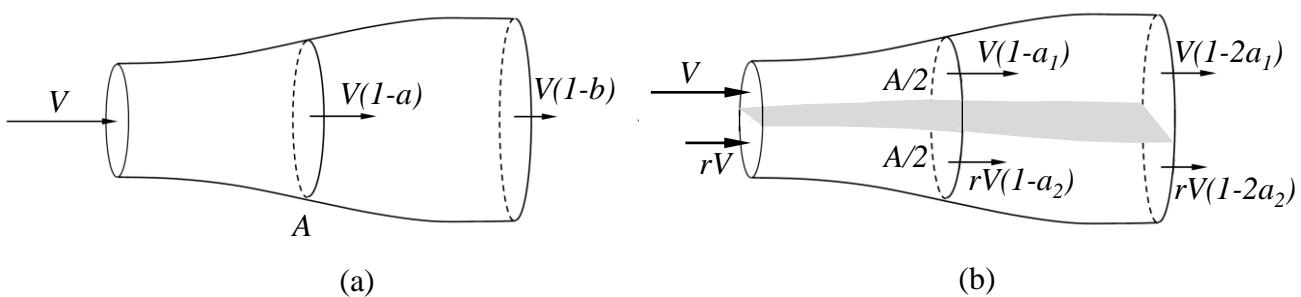


Fig. C.1

2 Consider the composite blade of a large commercial horizontal-axis wind turbine.

- (a) What issues are likely to lead to the blade failure? How can these be avoided? [5]
- (b) Explain why different composite laminates are used in different parts of the blade. [3]
- (c) The spar is made from unidirectional CFRP and the web is made from a CFRP laminate containing 75 % of  $\pm 45^\circ$  plies and 25 % of  $0^\circ$  plies. Use the carpet plot given in Fig. C.2 to estimate the Young's modulus of the two laminates. [2]
- (d) Fatigue failure of the blade is modelled by the equation

$$N = \left(\frac{S}{S_0}\right)^{-M}$$

relating the numbers of cycles to failure  $N$  and the cyclic stress range  $S$ . The parameters  $M = 15$  and  $S_0 = 800$  MPa are material constants.

- (i) The table below gives data for the number of loading cycles per month for the given spread of stress ranges. Estimate the lifetime in years of the blade. [7]

Stress range $S$ (MPa)	0 – 100	100 – 200	200 – 300	> 300
Number of cycles per month (thousands)	864	120	16	0

- (ii) The data from the above table is fitted by the probability density function

$$\phi(S) = \frac{1}{\bar{S}} \exp\left(\frac{-S}{\bar{S}}\right)$$

with  $\bar{S} = 50$  MPa. There is a total of one million cycles of loading per month. Estimate the lifetime in years of the blade using this model of the stress loading. Comment on the difference between this answer and the prediction from part (d)(i). Note that, for integer  $z$

$$\int_0^\infty t^z e^{-t} dt = z!$$

[8]

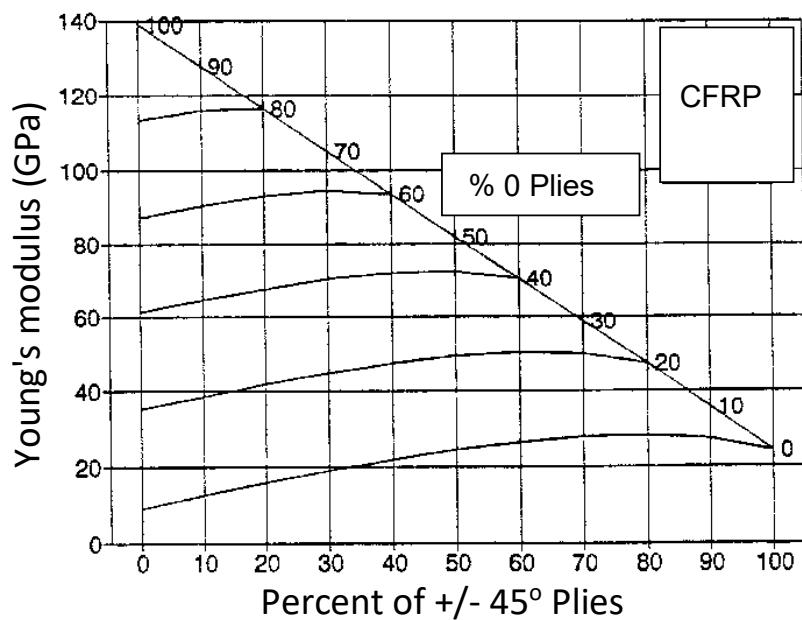


Fig. C.2

3 (a) A wind farm occupies a  $50 \text{ km}^2$  site in Scotland.

(i) The wind farm is rated at 300 MW peak power. Assuming a load capacity factor of 0.33, find the average power density of the wind farm. The wind farm uses turbines with average power 2 MW and blade diameter 100 m. If the turbines are laid out in a square array, what is their average spacing, as a multiple of the blade diameter? Why in practice are wind farms typically laid out in a rectangular array? [4]

(ii) It is proposed to use the same site in Scotland for a solar PV array, occupying one third of the land area. The panel size is 2 m by 1 m, with a peak power output per panel of 350 W, and an overall efficiency of 15%. Find the combined power density of the site, and comment on the implications of the comparison with a 2 GW nuclear power station occupying 1  $\text{km}^2$ . [4]

(iii) Many first generation wind farms are coming to the end of their lifetime. Comment on the end-of-life implications for the following materials used in the construction of a wind turbine: concrete, steel, CFRP, neodymium. [4]

(b) (i) Explain why it is best to operate wind turbines as variable speed devices, and outline the principles of the doubly-fed induction generator to achieve variable speed operation. [2]

(ii) Sketch the torque-speed characteristic of a three-phase cage rotor induction machine, assuming that its stator winding is connected directly to the 50 Hz grid. Hence explain why wind turbines which utilise such generators are essentially fixed-speed systems. [2]

(iii) A three-phase, star-connected, 30-pole cage rotor induction machine is utilised as the generator in a wind turbine of blade radius 43 m. Its stator windings are connected directly to the 3.3 kV, 50 Hz, three-phase grid, and its equivalent circuit parameters are  $R_1 = 0.4 \Omega$ ,  $R'_2 = 0.3 \Omega$ ,  $X_1 = X'_2 = 1 \Omega$ . At a certain wind speed the input mechanical power to the induction generator is 1 MW, and the tip-speed ratio is 5. The following may be quoted without proof:

$$\lambda = \frac{\omega R}{v}; \quad T \approx \frac{3V^2 s}{\omega_s}.$$

Stating any assumptions find:

- the generator speed, torque, slip and phase current (magnitude and angle); [4]

- the generator power losses, output power and efficiency; [3]

- the gearbox ratio required so that the turbine speed is 10 rpm under these conditions, and determine the wind speed. [2]

## END OF SECTION

## SECTION D: Aerothermal Engineering

*Answer not more than two questions from this section.*

1 An aircraft cruises at a flight Mach number of 0.82 at an altitude where the ambient conditions are  $T = 216.7$  K and  $p = 19.4$  kPa. Thrust is provided by single-shaft turbojet engines with an overall stagnation pressure ratio of 45. The stagnation temperature at turbine inlet is  $T_{04} = 1300$  K.

(a) The engine mass flow rate is  $250 \text{ kg s}^{-1}$  and the Mach number of the flow at turbine inlet, where the flow direction is purely axial, is  $M_4 = 0.3$ . Find the annulus area at turbine inlet,  $A_4$ . [6]

(b) The compressor and turbine isentropic efficiencies are both 0.9. Assuming that the flow is again axial at turbine outlet and that the Mach number is  $M_5 = 0.5$ , evaluate the annulus area at turbine outlet,  $A_5$ . [7]

(c) The mid-height radius of the turbine is constant at  $r_{\text{mid}} = 0.6$  m. Calculate the blade height at turbine inlet and at turbine outlet. Comment briefly on the result. [2]

(d) During a static engine test conducted at sea-level, where the ambient temperature  $T_a = 288.15$  K, the shaft speed of the engine was measured at  $N = 7400$  rpm. Stating any assumptions made, calculate the shaft speed at cruise. [4]

(e) It is found that the flow is not axial at turbine outlet (i.e. some swirl is present) and an additional stationary blade row is required to turn the flow to axial. With the aid of sketches, discuss the aerodynamic challenges of designing this blade row. [6]

Assume that the working fluid is air throughout with  $\gamma = 1.4$  and  $R = 287 \text{ J kg}^{-1} \text{ K}^{-1}$ .

2 (a) Define propulsive efficiency,  $\eta_p$ , and show that,

$$\eta_p = \frac{2V}{V + V_j} \quad ,$$

where  $V$  is the flight velocity and  $V_j$  is the jet velocity. [5]

(b) An aircraft cruises at a flight Mach number of 0.86 at an altitude where the ambient conditions are  $T = 216.7$  K and  $p = 16.6$  kPa. The aircraft is powered by turbofan engines and the propulsive efficiency is  $\eta_p = 0.8$ . Explain the benefits of having equal core and bypass jet velocities. Hence calculate the jet velocity. [4]

(c) A fan efficiency,  $\eta_f$ , can be defined such that,

$$\eta_f = \frac{V_j^2 - V^2}{2c_p \Delta T_{0\text{fan}}^b} \quad ,$$

where  $\Delta T_{0\text{fan}}^b$  is the stagnation temperature rise, across the fan, for the flow entering the bypass duct. If  $\eta_f = 0.9$ , evaluate  $\Delta T_{0\text{fan}}^b$ . [2]

(d) For the flow entering the engine core, the fan isentropic efficiency is 0.9 and the stagnation pressure ratio across the fan is 1.3. If the bypass ratio of the engine is 7, calculate the specific work output of the low pressure turbine. [7]

(e) With the aid of a temperature-entropy diagram for the bypass flow, explain the benefits of reducing fan pressure ratio and briefly discuss four associated challenges. [7]

Assume that the working fluid is air throughout with  $\gamma = 1.4$  and  $R = 287 \text{ J kg}^{-1} \text{ K}^{-1}$ .

3 (a) The total mass of an aircraft at the start of cruise is 170 tonnes. The flight Mach number is 0.85, the lift coefficient  $C_L = 0.5$  and the wing area is  $350 \text{ m}^2$ . Find the altitude at the start of cruise. [3]

(b) Derive the following equation for the distance travelled by the aircraft in cruise,  $s$ ,

$$s = \frac{V L/D}{g \text{ sfc}} \ln\left(\frac{m_1}{m_2}\right) ,$$

where  $V$  is the flight velocity,  $L/D$  is the aircraft lift-to-drag ratio,  $g$  is the acceleration due to gravity,  $\text{sfc}$  is the specific fuel consumption, and  $m_1$  and  $m_2$  are the mass of the aircraft at the start and end of cruise, respectively. State any assumptions made. [5]

(c) Derive an equation for the mass of fuel used in cruise,  $m_f$ , as a function of cruise distance,  $s$ , and fuel Lower Calorific Value, LCV. You may assume that the aircraft empty mass,  $m_e$ , payload mass,  $m_p$  and overall efficiency  $\eta_{ov}$  remain constant. Sketch curves of  $m_f$  against  $s$  for different values of LCV. State any assumptions made. [7]

(d) The aircraft in part (a) uses conventional aviation fuel with a LCV of  $43 \text{ MJ kg}^{-1}$ . The aircraft  $L/D = 20$  and overall efficiency  $\eta_{ov} = 0.42$ . If the cruise distance is 8000 km, find the mass of fuel used during cruise and the altitude at the end of cruise. [4]

(e) Liquid hydrogen, a potential non-hydrocarbon fuel for aviation, has a LCV of  $120 \text{ MJ kg}^{-1}$ . Discuss how you would expect the mass of fuel used, and the increase in altitude during cruise, to change if liquid hydrogen is used. The liquid hydrogen is stored at 20 K and has a density of  $71 \text{ kg m}^{-3}$  (the density of conventional aviation fuel is  $804 \text{ kg m}^{-3}$ ). How would you expect the aircraft empty mass,  $m_e$ , and payload mass,  $m_p$ , to change if an aircraft is designed for liquid hydrogen instead of conventional aviation fuel. [6]

Assume that the working fluid is air throughout with  $\gamma = 1.4$  and  $R = 287 \text{ J kg}^{-1} \text{ K}^{-1}$ .

## END OF SECTION

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## SECTION E: *Electrical Engineering*

*Answer not more than two questions from this section*

1 An electron exists in a one-dimensional potential described by the function

$$V(x) = \frac{cx^2}{2}$$

where  $x$  is the position in one-dimension and  $c$  is a constant equal to  $3 \text{ J m}^{-2}$ .

(a) Why is the lowest energy that the electron can have in this potential well greater than zero? What is the scientific name of this lowest energy state? [4]

(b) The wavefunction of the electron in this lowest energy state is

$$\psi(x) = A_0 \exp\left(\frac{-cmx^2}{2\hbar^2}\right)$$

where  $A_0$  is a constant and  $m$  is the electron rest mass. Show that this is a valid solution to the Time-Independent Schrödinger Equation

$$\frac{-\hbar^2}{2m} \frac{d^2\psi}{dx^2} + V\psi = E\psi$$

You may find making the substitution  $\alpha^2 = cm/\hbar^2$  helpful. [9]

(c) Calculate the energy of the electron in this lowest energy state. [6]

(d) Calculate the value of  $A_0$ . [6]

NOTE:

$$\int_{-\infty}^{\infty} \exp(-ax^2) = \sqrt{\pi/a}$$

2 Figure E1 shows the pixel circuit of an Active Matrix Liquid Crystal Display (AMLCD) and Fig. E2 shows the pixel circuit of an Active Matrix Organic Light Emitting Diode display (AMOLED). The current-voltage characteristics of a Thin Film Transistor (TFT) is given by

$$I_{ds} = \mu \frac{W}{L} C_{ox} \left[ (V_{gs} - V_t) V_{ds} - \frac{V_{ds}^2}{2} \right]$$

(a) Explain the role of each of the TFTs used in the AMLCD and AMOLED pixel circuits. In each case describe how the use affects the design of the TFT. [9]

(b) The TFT in the AMLCD circuit shown in Fig. E1 is required to charge a pixel liquid crystal with a capacitance of  $0.2 \text{ pF}$  at a rate given by a time constant  $\tau = 10 \mu\text{s}$  when the voltages on the TFT are  $V_{ds} = 0.5 \text{ V}$  and  $V_{gs} = 3 \text{ V}$ . The threshold voltage of the TFT is  $V_T = 0.2 \text{ V}$ , the gate capacitance per unit area is  $C_{ox} = 350 \mu\text{F m}^{-2}$  and the field-effect mobility is  $\mu_{FE} = 0.8 \text{ cm}^2 \text{ V}^{-1} \text{ s}^{-1}$ .

(i) Estimate the small-signal resistance required of the TFT under these conditions. [3]

(ii) Determine the  $W/L$  ratio of the TFT that is required for this application. [7]

(c) The current drive TFT in the AMOLED circuit shown in Fig. E2 is to be used to pass a current of  $1 \mu\text{A}$  through the OLED when the voltages on the TFT are  $V_{ds} = V_{gs} - V_T$  and  $V_{gs} = 2 \text{ V}$ . The threshold voltage of the TFT is  $V_T = 0.5 \text{ V}$ , the gate capacitance per unit area is  $C_{ox} = 800 \mu\text{F m}^{-2}$  and the field-effect mobility is  $\mu_{FE} = 10 \text{ cm}^2 \text{ V}^{-1} \text{ s}^{-1}$ . Determine the  $W/L$  ratio of the TFT that is required for this application. [6]

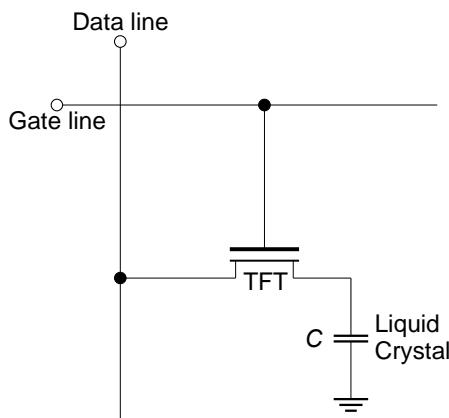


Fig. E1

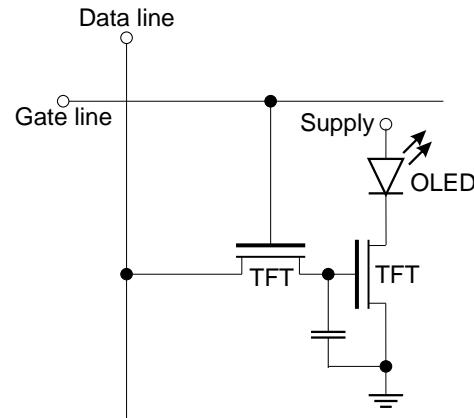


Fig. E2

3 The *Néel relaxation time* for magnetic materials is given by

$$\tau_N = \tau_0 \exp\left(\frac{Kv}{kT}\right)$$

where the attempt time constant  $\tau_0$  is typically 1 ns.

(a) State the three basic requirements of a binary digital data storage system and explain how ferromagnetic materials satisfy these requirements. [6]

(b) It is desired to make a hard disk drive with a data storage density of  $1.6 \times 10^{15}$  bits  $m^{-2}$  and using a ferromagnetic layer that is no more than 50 nm thick. If the data on the hard disk drive should be stable for at least 10 years at room temperature (298 K), calculate the minimum magnetic anisotropy energy density that the ferromagnetic material that is chosen must possess. [9]

(c) Figure E3 shows the basic cross section of a hard disk drive. Suggest a suitable method for depositing the magnetic material and explain the most important processing parameters that must be controlled for the method you have chosen. [4]

(d) Why is a seed layer usually required underneath the magnetic material? [2]

(e) Hard disk drives usually employ a read head based on tunnelling magnetoresistance. Briefly describe how such a read head works and how it differs from an inductive read head. [4]

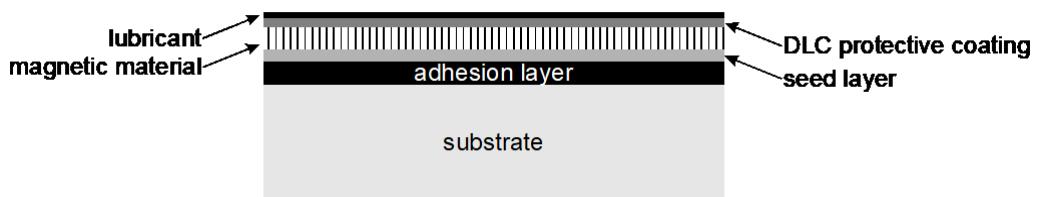


Fig. E3

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## SECTION F: *Information Engineering*

*Answer not more than two questions from this section.*

1 A photographer with a hand-held camera has captured two greyscale images,  $I_1(x, y)$  and  $I_2(x, y)$ , of the Baker Building in the Cambridge Engineering Department. The photographs are taken a few seconds apart, from a similar pose and location. Each image has a size of  $640 \times 480$  pixels.

(a) Describe an operation that can be used to reduce noise in each image. You should include details of how truncation is used to efficiently implement the operation on a computer, and any other strategies that contribute to efficiency. [4]

(b) Suppose that we now wish to find correspondences between the two full images,  $I_1(x, y)$  and  $I_2(x, y)$ .

- (i) Describe what is meant by the terms *invariance* and *distinctiveness*. Explain the relationship between these properties in the context of feature matching. [3]
- (ii) What kind of invariance can be established through the use of *blobs* that is challenging to achieve with *edges* or *corners*? [1]
- (iii) Describe in detail how the Scale-Invariant Feature Transform (SIFT) detects *keypoints* in an image and highlight the components of the algorithm that contribute to its efficiency with respect to memory and computation. [8]

(c) (i) Describe four potential nuisance factors that could make it challenging to bring the two images of the Baker Building into correspondence. [3]

- (ii) Explain which of the stages of the SIFT *descriptor* aim to achieve invariance to the factors you gave in your previous answer. Alternatively, if the SIFT descriptor struggles to achieve invariance to one or more of your given nuisance factors, identify these as weaknesses of SIFT. [4]
- (iii) What is the motivation for examining the two nearest neighbours of a SIFT descriptor when performing matching? How is a match determined in practice? [2]

2 (a) Consider a logistic regression classifier with weights  $w$  trained on a data set of  $N$  images using gradient descent and the negative log-likelihood loss. Define  $\|w\|$  as the  $L_2$  norm of the weights,  $\|w\| = \sum_i w_i^2$ .

(i) Consider the case where the model predicts all of the examples in the training set correctly for a given value of  $w$ . Draw a plot of loss as a function of  $\|w\|$  that shows how the training loss changes qualitatively as a function of  $\|w\|$  and label the point where  $\|w\| = 0$  according to its loss value. [4]

(ii) Now consider the case where the model predicts *most*, but not *all*, of the examples in the training set correctly for a given value of  $w$ . Again, draw a plot of loss as a function of  $\|w\|$  that shows how the training loss changes qualitatively as a function of  $\|w\|$  and label the point where  $\|w\| = 0$  according to its loss value. Describe the limiting behaviour of the loss as  $\|w\| \rightarrow \infty$ . [4]

(iii) Consider again the case where the model predicts *all* of the examples in the training set correctly for a given value of  $w$ , and suppose we also include a regularizer  $\beta\|w\|^2$ ,  $\beta > 0$  in the loss. Again, draw a plot of loss as a function of  $\|w\|$  that shows how the training loss changes qualitatively as a function of  $\|w\|$  and label the point where  $\|w\| = 0$  according to its loss value. Describe the limiting behaviour of the loss as  $\|w\| \rightarrow \infty$ . [4]

(iv) In light of the above plots explain how the proposed regularizer might help prevent overfitting, what effect this would have on the models predictions, and why this might improve the test loss. [4]

(b) In machine learning, the term *inductive bias* refers to assumptions encoded by the choice of learning algorithm (e.g. the model architecture used), which can be used to encode prior knowledge of the problem. For instance, in logistic regression, the decision boundary is assumed to be linear; for convolutional neural networks, this is not the case. Three properties of convolutional neural networks that influence their inductive bias are *local connectivity* (convolutional filters that are smaller than the input image); *tied weights* (where the same filter is applied at multiple locations within an image); and *pooling*.

(i) For each of the above three properties, explain in one sentence why it might encode a useful inductive bias for images. [6]

(ii) For each of the above three properties, explain in one sentence how it reduces the number of parameters or the amount of computation required. [3]

3 Figure F.1 shows a Markov decision process with the rewards for each state/action pair labelled (e.g. the reward for taking action 2 in state 2 is 0).

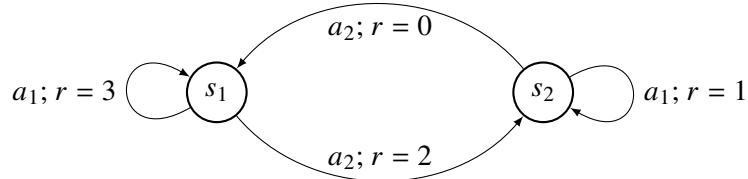


Fig. F.1

- (a) Describe the Q-learning algorithm as it applies to finding the optimal discounted reward for this process. What would be an appropriate learning rate? [5]
- (b) Assuming a discount factor  $\gamma = 0.9$  write down the value function and the associated optimal control by inspection. Verify that this value function satisfies the Bellman equation at each state. [5]
- (c) Write down the sequence of states, actions and rewards that follow from starting in state 1 and applying action 1, and subsequently following the policy of taking action 2 in state 1 and action 1 in state 2. [5]
- (d) Find the resulting action-values that would result from applying Q-learning to the data in Part (c), using a learning rate of 1 and assigning an initial value of 0 to all action-values. You should use all of the available data sequentially, in the order that it was generated. [4]
- (e) For each set of tentative action-values derived in Part (d) write down the sequence of states, actions and rewards that would follow from starting in state 1 and following the greedy policy. [4]
- (f) Comment on the convergence of Q-learning with greedy action selection for this problem. How might this be improved? [2]

**END OF SECTION**

Version GV/2

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## SECTION G: *Bioengineering*

*Answer not more than two questions from this section*

1 (a) A spectral Optical Coherence Tomography (OCT) system designed for imaging the eye has a centre wavelength  $\lambda$  and bandwidth  $\Delta\lambda$ , and uses a linear array of 1000 photo-diodes.

(i) What range of  $\lambda$  would be appropriate for this task, and what are the consequences of using different values for  $\lambda$ ? [3]

(ii) Derive an approximate expression for the spacing of depth samples, in terms of  $\lambda$  and  $\Delta\lambda$ . Assume that the photo-diodes are set up to sample the reflected light appropriately, and that  $\Delta\lambda \ll \lambda$ . [5]

(iii) What approximate bandwidth  $\Delta\lambda$  should be used if it is required to image to a depth of 6 mm, and is this achievable? Presume a sensible choice for  $\lambda$ . [2]

(iv) What else affects the resolution of this system other than the photo-diode array? [2]

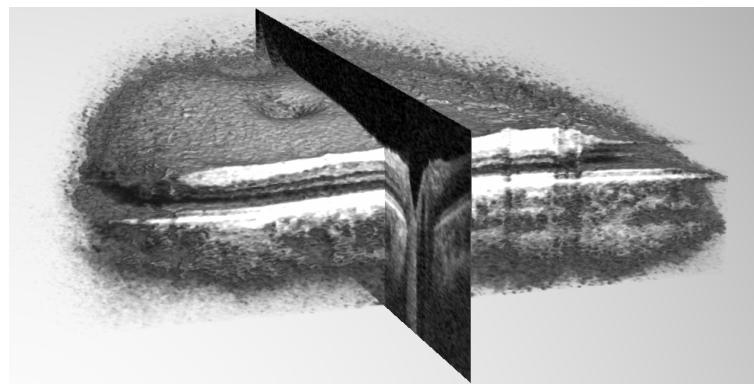


Fig. G.1

(b) Figure G.1 shows a visualisation of some OCT data.

(i) What anatomy is shown in the image and what are the approximate dimensions? Also name four different anatomical structures which are visible in this image and describe where you can see them. [3]

(ii) Explain how this visualisation has been created from the image data. [6]

(iii) Describe two imaging artefacts you can see in the image and explain how they arise. [4]

2 (a) Write short notes on the visual contrast sensitivity function, including how it can be measured experimentally using a two-alternative forced choice task. [4]

(b) Locusts see the world through a pair of compound eyes (Fig. G.2, on the next page; viewed from the top). The inter-ommatidial angle in the horizontal plane,  $\alpha$ , is estimated to be  $0.5^\circ$  (vastly exaggerated in the figure for clarity), which is small. The horizontal separation between the forward-looking regions of the two eyes is  $2s = 4$  mm.

(i) Suppose a small prey is located at a distance  $d$  from the eyes in the horizontal plane, such that it projects onto a single ommatidium. If the prey was located at a distance  $d + \Delta d$ , it would recruit the neighbouring ommatidium. Derive an expression for  $\frac{\Delta d}{d}$  as a function of  $d$ ,  $s$  and  $\alpha$ . You may neglect the size of the eye relative to  $d$ , and use a small-angle approximation for  $\alpha$ . What do you conclude about relative depth perception at long distances? [4]

(ii) Explain why, according to Fig. G.2 on the next page, locusts should in fact be unable to perceive *any* relative depth beyond a critical distance  $d_{\max}$ , and give the value of  $d_{\max}$  to the nearest centimetre. [3]

(iii) As it turns out, locusts are capable of accurately jumping onto small preys located up to a metre away. These jumps are mostly ballistic with little room for in-flight adjustments; therefore, locusts do in fact perceive depth accurately even at such distances. Indeed, just prior to jumping, locusts engage in a stereotypical behaviour called ‘peering’. An example of peering is depicted in Fig. G.3 (on the next page: see in particular the time course of angles  $\beta$  and  $\gamma$ ). In the diagram, B and H are fixed centres of rotation on the body and head respectively, and the two dashed lines are parallel. Explain by what principle ‘peering’ might enhance depth perception, and give a brief explanation of this principle. [4]

(iv) Suppose you have access to a camera that can track the movements of a locust in real time, and that you can show this locust virtual preys anywhere on a horizontal monitor. Outline an experiment that could test your answer to (b)(iii). [4]

(c) For each of the following optical solutions, name one animal whose eyes use it:

- (i) negative lens;
- (ii) reflective mirror;
- (iii) three lenses in the same eye;
- (iv) an eye in which all the optical power is in the lens;
- (v) scanning telescope; and
- (vi) more than ten lenses in the same eye. [6]

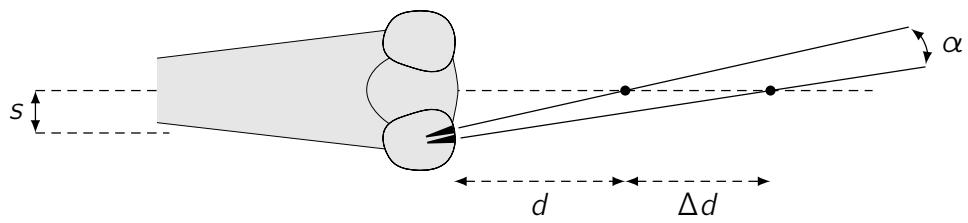


Fig. G.2

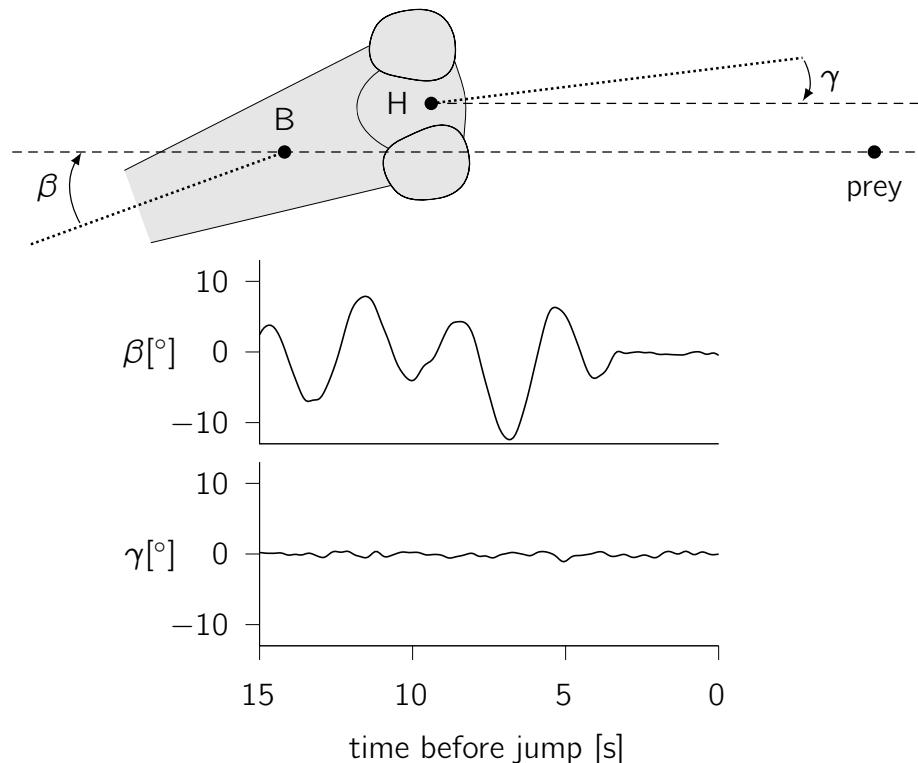


Fig. G.3

3 (a) (i) How is IntraOcular Pressure (IOP) regulated in the eye? [2]

(ii) Describe the clinical procedure for *Goldmann tonometry* in measuring IOP. [4]

(iii) How does the attachment of a contact lens to a normally-functioning eye change the eye's IOP? Use sketches to illustrate the change of IOP over time for a contact lens made of: I. hydrogel; and II. PolyMethyl MethAcrylate (PMMA). Label key features (e.g. approximate IOP values and time scales) and state your reasoning. [6]

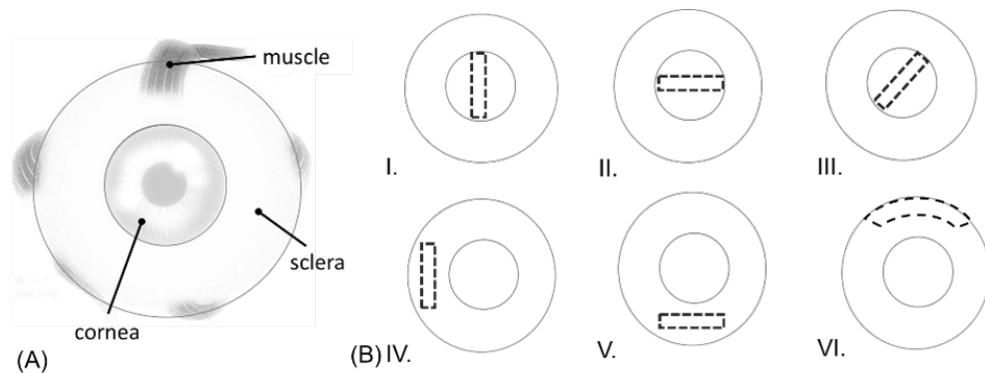


Fig. G.4

(b) Figure G.4 shows selected external features of an eyeball. Sample strips are taken out of the Fig. G.4(A) eyeball configuration, from the cornea region (shown in Fig. G.4(B)I to III) and from the sclera region (shown in Fig. G.4(B)IV to VI). The sample strips are then flattened and clamped at the two ends of the long-axis, and subjected to uniaxial tensile testing to determine each sample's stress-strain curve.

(i) Sketch, on one plot, the stress-strain curves for samples I to III. Describe and explain the key features and the differences between the three curves. [5]

(ii) Sketch, on one plot, the stress-strain curves for samples IV to VI. Describe and explain the key features and the differences between the three curves. [3]

(iii) Sketch, on one plot, the stress-strain curves for samples III and IV. Explain the differences between the two curves. [2]

(iv) List the major uncertainties associated with using this testing method to determine the elastic moduli of sclera and cornea in general. [3]

**END OF SECTION**

## Section H: Manufacturing and Management

*Answer not more than two questions from this section.*

1 (a) Explain what is meant by *market pull* and *technology push*. Give four sources for each. Provide one example for each source. [5]

(b) Discuss the relationship between *product specification* and the design of a *manufacturing process* for that product. [10]

(c) Explain why it is so challenging for a manufacturing business to balance supply and demand. Discuss possible strategies for managing supply and managing demand to keep things in balance. Use examples to support your answer. [10]

2 You have designed a novel bicycle lock that is going to be targeted at students.

(a) Describe the four ways in which you can segment any market. [4]

(b) Draw a *perceptual map* and explain your rationale for the axes chosen. Show how your design fits this map. [7]

(c) Discuss the strengths and weaknesses of different forms of *prototyping*. Describe how each form of prototyping could be used in the development of your new bicycle lock design. [6]

(d) Discuss the options you would have for protecting the *intellectual property* relating to your new bicycle lock. Explain the advantages and disadvantages of each type of *intellectual property right* relevant for this product. [8]

3 An early-stage, start-up company with five employees has developed a new low-cost, internet-connected sensor for monitoring air quality on public transport. It will allow passengers to monitor the air quality via their mobile phones. The sensors will be made in very high volumes and sold to train and bus companies using a product plus service business model. The start-up company has just completed a successful trial of the technology with a large potential customer.

(a) Explain the advantages and disadvantages of a *product plus service* business model in comparison to other possible business models. [4]

(b) The start-up company is seeking to raise funding to support the growth of the business. List and compare the funding options available to the start-up. [8]

(c) Discuss some of the issues likely to arise in the design and operation of a manufacturing system to produce these low-cost sensors in very high volumes. [8]

(d) Discuss the type of challenges that might arise when this very small company works in partnership with the very large companies who operate the buses and trains. [5]

**END OF PAPER**