Version CD/6

EGT1
ENGINEERING TRIPOS PART IB

Tuesday 7 June 2022 09.00 to 11.10

09.00 to 10.10 Foreign Language Option

Or Civil Engineering Option

Paper 8

SELECTED TOPICS

If you are not taking the Foreign Language option, answer ***four*** *questions, taken from only two of Sections C–H. Not more than* ***two*** *questions from each section may be answered.*

If you are taking either the Civil Engineering option ***or*** *the Foreign Language option, answer* ***two*** *questions from one of Sections C–H.*
All questions in Sections C–H carry the same number of marks.

The ***approximate*** *number of marks allocated to each part of a question is indicated in the right margin.*

Write your candidate number ***not*** *your name on the cover sheet.*

Section A *has been assessed as coursework*
Section B: *Civil and Structural Engineering has been assessed as coursework*

STATIONERY REQUIREMENTS
Single-sided script paper.

SPECIAL REQUIREMENTS TO BE SUPPLIED FOR THIS EXAM
CUED approved calculator allowed

Engineering Data Book

10 minutes reading time is allowed for the paper at the start of the exam.

You may not start to read the questions printed on the subsequent pages of this question paper until instructed to do so.

You may not remove any stationary from the Examination room.

THIS PAGE IS BLANK

SECTION C: *Mechanics, Materials and Design*

*Answer not more than **two** questions from this section.*

Consider a three-bladed horizontal-axis wind turbine with swept area A operating at a rotational speed ω in a wind of speed V. The air density is ρ.

1 (a) At the blade cross section, the axial air speed is $V(1-a)$, where a is the axial induction factor. Downstream, the axial air speed is $V(1-2a)$. Derive an expression for the turbine power output in terms of ρ, V, A and a. [5]

(b) Figure C.1 shows an element of the blade of length L (out of the page) at a radial position r. The blade has an angle of twist θ. The air's velocity relative to the blade is inclined at an angle α to the blade chord. Forces F_L and F_D due to the air flow act on the blade element in directions along and transverse to the direction of the relative air velocity. Corresponding lift and drag coefficients C_L and C_D for the blade can be approximated by:

$$C_L = 2\pi\alpha \quad \text{and} \quad C_D = 0.05 \quad (\alpha \text{ in radians})$$

Note that:

$$a = \left(\frac{4\sin^2\phi}{\sigma C_N} + 1\right)^{-1} \quad , \quad a' = \left(\frac{4\sin\phi\cos\phi}{\sigma C_T} - 1\right)^{-1} \quad , \quad \sigma = \frac{cB}{2\pi r}$$

where a' is the angular induction factor, C_N and C_T are the normal and tangential force coefficients, ϕ is the angle of relative wind, σ is the blade solidity, c is the chord length and B is the number of blades.
Estimate the contribution to the power generated by the turbine from the three blade elements at this radius, using data given in the table below. [16]

(c) Discuss briefly factors affecting the shape of horizontal-axis wind turbine blades. [4]

Number of blades B	3
Turbine rotational speed ω (rad s^{-1})	4
Radius r (m)	12
Chord length c (m)	1.1
Element length L (m)	0.1
Twist θ (°)	10
Wind speed V (m s^{-1})	16
Air density ρ (kg m^{-3})	1.2

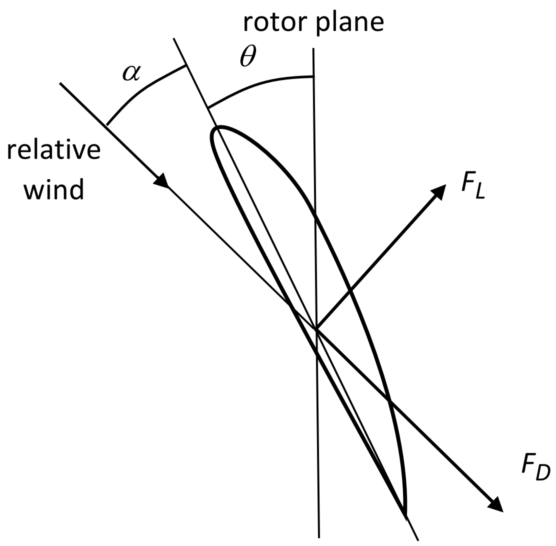

Fig. C.1

2 (a) Discuss the following aspects of wind turbine design:

(i) specialised gear boxes are needed for wind turbines; [2]

(ii) fatigue is a critical mode of failure in wind turbines; [2]

(iii) large blades contain a mixture of carbon and glass fibre composites; [2]

(iv) it is important to take account of noise in designing wind turbines. [2]

(b) Figure C.2 shows a simplified model of a wind turbine blade, comprising three rigid sections each of length L. The sections have masses of $3M$, $2M$ and M, as illustrated, with the centres of mass situated at the midpoints of each section. The connection of the blade to the hub is modelled by a torsional spring of stiffness $3k$, and bending along the blade is modelled by torsional springs of stiffnesses $2k$ and k, as shown

Find an approximate value, in terms of k, M and L, for the lowest resonant frequency of the blade, and sketch the corresponding mode shape. [17]

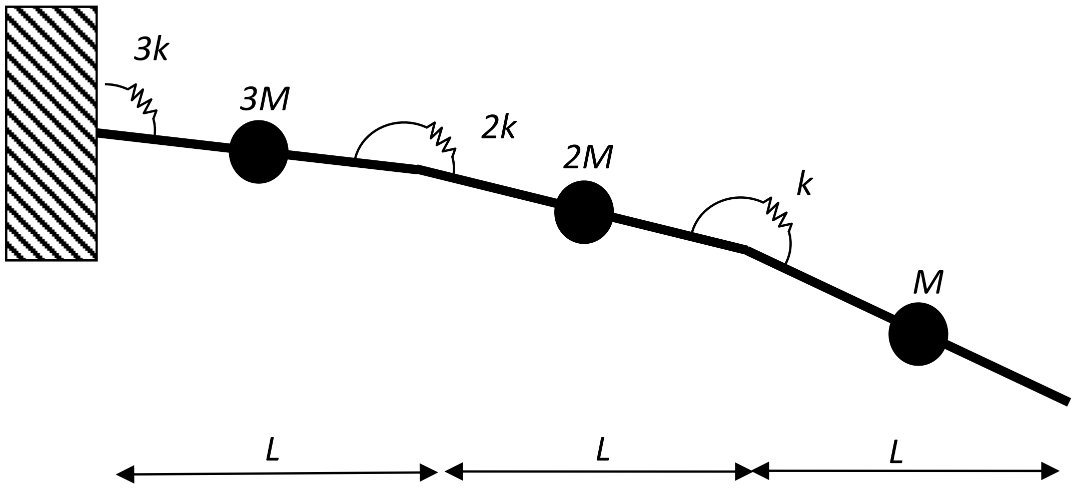

Fig. C.2

3 (a) Crown Estate Scotland recently completed the leasing of seabed for a potential 25 GW of offshore windfarm development. The sustainability of this scheme may be addressed using the five-step methodology outlined in the lecture course.

(i) Outline the generic characteristics of any articulation of a sustainable development and suggest suitable statements for each characteristic. [3]

(ii) Who would be the principal stakeholders to consider in this scheme, and what would be their primary roles or interests in it? [5]

(iii) Part of the fact-finding and synthesis would be to estimate the energy pay-back time of the scheme. Outline the quantities for which data would be needed, and how they would be combined to perform this analysis. [4]

(b) A wind turbine is connected to a 3-phase, 50 Hz 3.3 kV grid. It utilises a 16 pole doubly-fed induction generator (DFIG) coupled via a gearbox to a three-bladed turbine. Its rated power is 5 MW at a wind speed of 12 m s^{-1} . A gearbox has been selected such that the synchronous speed of the DFIG corresponds to the most common wind speed of 7 m s^{-1} . It is assumed that from cut-in to the rated wind speed the rotor speed adjusts to operate at its optimal tip-speed ratio of $\lambda = 9$ for which its power coefficient $C_p = 0.4$.

(i) Explain why the DFIG is the most commonly-used generator technology in modern wind turbines. [3]

(ii) Using the rated power and wind speed determine the turbine diameter. [2]

(iii) For the most common wind speed of 7 m s^{-1} determine the gearbox ratio required and the input power to the DFIG. [3]

(iv) The DFIG's stator is star-connected. At a wind speed of 7 m s^{-1} the applied voltage at the rotor slip-rings is zero. The equivalent circuit parameter of the DFIG are: $R_1 = 0.2\,\Omega$, $R_2' = 0.18\,\Omega$, $X_1 = 0.45\,\Omega$, $X_2' = 0.35\,\Omega$. The magnetising reactance and iron loss resistance are large enough to be ignored. Using the simplified torque-slip equation, estimate the input torque to the DFIG, the slip of the DFIG and its input current. [5]

The following may be quoted without proof:

$$P = 0.5 C_p \rho A v^3, \quad \lambda = \frac{\omega R}{v}, \quad T \approx \frac{3V^2 s}{\omega_s R_2'}$$

and take the density of air to be $\rho = 1.23$ kg m^{-3}.

END OF SECTION

SECTION D: Aerothermal Engineering

1 (a) Discuss the advantages and challenges of increasing the bypass ratio of a turbofan engine. In particular, identify the challenges faced by turbomachinery designers. [6]

(b) An aircraft, powered by turbofan engines, cruises at a flight Mach number $M = 0.82$ at an altitude where the ambient conditions are $T = 217$ K and $p = 22.7$ kPa. Calculate the stagnation temperature and stagnation pressure at engine inlet. [3]

(c) The fan pressure ratio is 1.5 and the isentropic efficiency of the inner part of the fan (which compresses the air entering the core) is $\eta = 0.9$. Calculate the stagnation pressure and stagnation temperature at inlet to the core compressor. [3]

(d) The core compressor pressure ratio is 30, and the stagnation temperature at compressor exit is 800 K. If the stagnation temperature at combustor exit is 1500 K, and the high pressure turbine has an isentropic efficiency $\eta = 0.9$, calculate the stagnation conditions at the exit of the high pressure turbine. [4]

(e) The stagnation temperature drop across the low pressure turbine, which also has an isentropic efficiency $\eta = 0.9$, is $\Delta T_{0\mathrm{LPT}} = 390.1$ K. Calculate the core jet velocity. [5]

(f) Using the definition of bypass fan efficiency,

$$\eta_{\mathrm{fb}} = \frac{V_{\mathrm{jb}}^2 - V^2}{2\, c_p\, \Delta T_{0\mathrm{fb}}} = 0.9 \quad ,$$

where V_{jb} is the bypass jet velocity, V is the flight speed and $\Delta T_{0\mathrm{fb}}$ is the stagnation temperature rise, across the fan, of the bypass flow, calculate the bypass ratio. State any assumptions made. [4]

Assume that the working fluid is air throughout with $\gamma = 1.4$ and $R = 287$ J kg^{-1} K^{-1}.

2 (a) An aircraft with four engines has a mass of 500 tonnes. At cruise, the lift-to-drag ratio of the aircraft, $L/D = 20$. Evaluate the net thrust from each engine. [2]

(b) The non-dimensional corrected gross thrust $\tilde{F}_G$ is given by

$$\tilde{F}_G = \frac{F_G + p_a A_N}{A_N p_{02}} \quad ,$$

where A_N is the area of the propelling nozzle and the other symbols have their usual meaning. Explain, physically, why $\tilde{F}_G$ takes this form. [5]

(c) At cruise altitude (where the ambient conditions are $T = 217$ K and $p = 22.7$ kPa), the aircraft of part (a) has a flight Mach number $M = 0.85$ and the mass flowrate of air through each engine is 140 kg s^{-1}. At take-off, the aircraft has speed $V = 90$ ms^{-1} and the ambient conditions are $T = 288.15$ K and $p = 101.3$ kPa. The nozzle area of each engine, $A_N = 1\text{m}^2$. If the engine were to operate at the same non-dimensional condition at take-off and cruise, find,

(i) the engine gross thrust at take-off, [5]

(ii) the engine net thrust at take-off. [4]

(d) When about to take off, the aircraft suffers an engine failure. The aircraft mass may still be taken to be 500 tonnes. Assuming that, at take-off, $L/D = 10$, and the climb angle is such that $\sin\theta = 0.03$, find the net thrust required from each of the remaining three engines and comment on the result. [4]

(e) Explain the design requirements when optimising an engine to operate at cruise and at take-off. Discuss ways in which these requirements are met. [5]

Assume that the working fluid is air throughout with $\gamma = 1.4$ and $R = 287$ J kg^{-1} K^{-1}.

3 (a) Show that the mass of fuel burned during cruise m_f is given by

$$m_{\mathrm{f}} = m_0 \left[\exp\left(\frac{s\, g\, \mathrm{sfc}}{V\, L/D} \right) - 1 \right] \quad ,$$

where m_0 is the mass of the aircraft without fuel (combined empty mass and payload mass), s is the distance travelled in cruise at flight speed V with lift-to-drag ratio L/D, and 'sfc' is the thrust-specific fuel consumption. You may assume that all of the fuel is used by the end of cruise. State any other assumptions that you make. [5]

(b) An aircraft with m_0 = 250 tonnes, L/D = 20, sfc = 0.016 kg $\mathrm{s}^{-1}\mathrm{kN}^{-1}$, cruises at M = 0.82 at an altitude where the ambient temperature is constant at T = 220 K. If the cruise distance s = 8000 km, calculate m_{f}. [3]

(c) The aircraft has a lift coefficient C_L = 0.5 and a wing area of A = 550 m^2. Find the ambient pressure at the start of cruise. Hence, using the properties of the International Standard Atmosphere, find the altitude at the start of cruise. [5]

(d) State two advantages and two disadvantages of sub-dividing a long flight into one or more smaller flights. [4]

(e) The aircraft of part (b) now travels the same total distance in two flights, each with s = 4000 km. Evaluate the total fuel burn and briefly comment on any assumptions made. [3]

(f) The reduction in total fuel burn calculated in part (e) is to be obtained by operating a non-stop flight with new engines that have the same thermal efficiency but an improved propulsive efficiency. Evaluate the ratio of the new propulsive efficiency to the original propulsive efficiency. [5]

Assume that the working fluid is air throughout with γ = 1.4 and R = 287 J kg^{-1} K^{-1}.

END OF SECTION

Version GP/4
Numerical answers:
Q1
(b) 246.2 K; 35300 Pa
(c) 279.8 K; 52950 Pa
(d) 979.8 K; 289200 Pa
(e) 397.3 m/s
(f) 6.5
Q2
(a) 61.3 kN
(c) (i) 246.8 kN; (ii) 212.8 kN
(d) 212.6 kN
Q3
(b) 73.4 tonnes
(c) 24510 Pa; 10520 m
(e) 68.7 tonnes
(f) 1.061
Section

THIS PAGE IS BLANK

SECTION E: *Electrical Engineering*

Answer not more than ***two*** *questions from this section*

1 An electron exists in a one-dimensional potential well. The well is infinitely deep in potential ($V = \infty$ outside of the well) but has a finite width $L = 1$ nm.

(a) Explain why the electron can only have certain values of kinetic energy given by

$$E = \frac{n^2 \pi^2 \hbar^2}{2\, m\, L^2}$$

where n is an integer greater than zero and L is the width of the well. [3]

(b) (i) Calculate the minimum momentum that the electron can have in this potential well. [6]

(ii) Show that this is consistent with the Heisenberg Uncertainty Principle which states that

$$\Delta p \Delta x \geq \hbar/2$$ [2]

(c) Calculate the difference in kinetic energy between the electron being in the lowest energy state ($n = 1$) and the next lowest energy state ($n = 2$). Comment on the probability that the electron is not in the lowest energy state at room temperature ($T = 298$ K). [6]

(d) Calculate the probability that the electron is within 0.25 nm of the centre of the well at any moment in time. What would this probability be if quantum mechanics did not apply to this situation and the electron was following the laws of classical mechanics? Comment on your answer. [8]

2 A full-colour (RGB) active matrix liquid crystal display for a desktop monitor has a display area of 60 cm × 33.75 cm, a resolution of 1920 × 1080 and a refresh rate of 60 Hz. The liquid crystal material is 10 μm thick and may be assumed to have a relative permittivity of 8 .

(a) How many thin film transistors (TFTs) does the display require? [3]

(b) If the light emitting parts of the display make up 90% of the total area (i.e. 10% is covered by metal tracks and the TFTs through which light does not pass), estimate the capacitance of the liquid crystal that any one TFT has to charge. [4]

(c) Estimate the maximum small-signal resistance of the TFT if it is turned on for 15 μs to charge the liquid crystal to within 5% of the data voltage. [3]

(d) The TFT uses hydrogenated amorphous silicon as the channel semiconductor with a field effect mobility of 1 $cm^2\ V^{-1}\ s^{-1}$ and silicon nitride as the gate dielectric with a relative permittivity of 7.5. $V_{gs} = 3$ V, $V_T = 0.5$ V and $V_{ds} = 0.75$ V when the TFT is turned on. Suggest suitable values for the thickness of the gate dielectric and ratio of the channel width and length (W/L) in the TFT. [10]

(e) The resistance of the TFT when it is turned off is 10^6 times greater than when it is turned on. Estimate the maximum percentage change in the voltage across the liquid crystal that may occur as a result of charge leakage through the TFT before the liquid crystal voltage is refreshed again. [10]

NOTE: For a TFT operating in the linear regime

$$I_{ds} = \mu \frac{W}{L} C\left[\left(V_{gs} - V_T\right) V_{ds} - \frac{V_{ds}^2}{2}\right]$$

[5]

3 A number of 1 µm thick copper tracks need to be produced on a 6 cm × 14 cm glass substrate which will eventually form part of a display. The tracks will be 6 cm long, going across the glass from one edge to the other, and 2 µm wide. Copper will be deposited uniformly over the whole substrate and then a combination of photolithography and wet etching will be used to produce the finished structure.

(a) Suggest a process for cleaning the bare glass substrate before production of the copper tracks is commenced. Justify your choice of process. [4]

(b) A colleague has suggested considering thermal evaporation, sputtering and electroplating as possible methods for depositing the copper. Comment on the appropriateness of each of these techniques for this situation. State which technique you would pick and justify your answer. What process conditions are likely to be important for your chosen technique? [12]

(c) Would you choose a positive or negative tone photoresist for this process? Justify your answer. [3]

(d) A contact printing mask aligner is to be used for exposing the photoresist. The aligner uses 365 nm ultraviolet light and a resolution of 0.5 µm is required to ensure variability between the tracks is within tolerance. Calculate the minimum selectivity that will be required for the wet etching process. [6]

NOTE: The resolution of the contact printing process is

$$R = \frac{3}{2}\sqrt{\lambda\left(s + \frac{z}{2}\right)}$$

END OF SECTION

THIS PAGE IS BLANK

SECTION F: *Information Engineering*

Answer not more than **two** *questions from this section.*

1 In computer vision, point correspondences over different viewpoints are often used to recover an object's pose and 3-D shape. *Keypoints* are first detected in each image and then matched to features in the other viewpoints.

(a) The detection of keypoints first requires the smoothing of a greyscale image, $I(x, y)$ by *low-pass* filtering with a Gaussian filter, $G_{\sigma}(x, y)$.

(i) Why is smoothing required? [2]

(ii) Explain why the Gaussian is a suitable low-pass filter? What is the effect of changing the Gaussian scale parameter, σ, on the frequency response of the low-pass filter. [3]

(iii) How is smoothing performed efficiently at multiple scales? [3]

(b) The keypoint is usually localised in position and scale by filtering the image with a *band-pass* filter over multiple scales, σ.

(i) Show that a filter derived from a Difference of Gaussians, $G_{k\sigma}(x, y) - G_{\sigma}(x, y)$, is a suitable band-pass filter. How is this implemented efficiently? [3]

(ii) Show how the keypoint can be localised in the image and describe how to determine an appropriate scale. [3]

(c) The SIFT (Scale-Invariant Feature Transform) descriptor is used to describe each keypoint. It is computed from a 16×16 patch of pixels around each keypoint centre.

(i) How is the 16×16 patch of pixels sampled at an appropriate scale and orientation? [3]

(ii) Describe the main steps in computing this descriptor. How does it achieve its invariance to lighting, image and viewpoint changes? What are its limitations? [4]

(iii) How are these descriptors used to find correspondences in images from different viewpoints? [2]

(d) What additional processing is needed to recover the 3-D position of each point? [2]

2 (a) In the context of a convolutional layer of a convolutional neural network (CNN):

(i) Define the *stride*, *# input channels*, *# output channels*, and *kernel size*. [4]

(ii) Suppose the input to a convolutional layer is a single image of 32×32 RGB pixels. What shape would the output of the layer be in terms of the terms defined above, assuming stride = 1 and no padding? [3]

(iii) Describe the *activation function* and its role, and give an example of an activation function used in CNNs. [2]

(iv) What are the parameters in a convolutional layer? How are they used to compute the output from the input? [2]

(b) With regard to the role of pooling layers in CNNs:

(i) Name and describe two common types of pooling operation. [1]

(ii) Why is pooling used in convolutional neural networks? Give at least two reasons. [2]

(iii) Why might pooling be harmful to the performance of a CNN? [1]

(c) A dataset of images is collected from a car driving on roads in Canada. Images are labeled based on whether they contain a pedestrian or not.

(i) Explain what supervised learning is. [1]

(ii) Explain how this dataset could be used to train a CNN to predict whether an unseen image contains a pedestrian using supervised learning. [1]

(iii) In one sentence, name and describe a specific loss function that could be used for this. [1]

(iv) In one sentence, name and describe a specific learning algorithm that could be used for this. [1]

(v) Explain the role of *training*, *validation*, and *test* datasets in the above training procedure. [3]

(vi) Suppose the trained CNN does a good job of recognizing pedestrians in unseen images from Canada, but fails on images from the United Kingdom. Why might this be the case? How could it be addressed? [3]

3 Consider the graph shown in Fig. F.1. The aim is to find the path from the start node to either one of the end nodes that maximizes the cumulative reward, where the reward for each step is shown on each edge. Edges can only be traversed in the direction of the arrows.

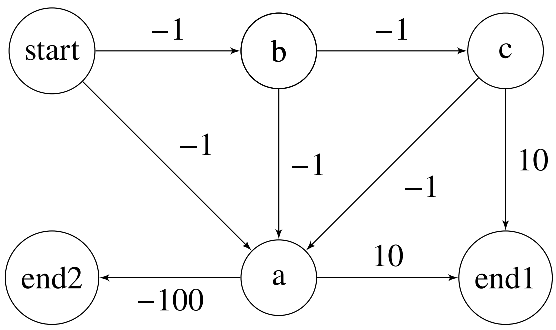

Fig. F.1

(a) Explain why Dijkstra's algorithm is not applicable to this problem. [4]

(b) Write down the solution using Bellman-Ford (you are allowed to choose the order of visiting nodes so as to minimise the computation needed). [5]

(c) Now consider approaching the problem using Q-learning. Assume that all Q-values are initially set to $-\infty$ and that (state,action) pairs happen to be initially visited (updated) in the following order: (a,→end2), (c,→a), (b,→a), (start,→a), (a,→end1), (c,→a) , (b,→c), (start,→b), (c,→end1).

(i) Explain why a learning rate of 1 would be appropriate for this problem. [3]

(ii) Using a learning rate of 1, what are the values of $Q(s, a)$ for each state action pair after the initial visits are completed? (You may find it convenient to show these values as labels on the edges of a copy of the graph in Fig. F.1.) What is the greedy path starting from "start" and how does the actual reward for an agent following that path compare to what it would have expected at the start? [6]

(iii) What further state action pair has to be visited before the greedy path is optimal? [3]

(iv) With $Q(s, a)$ updated for the state and action in part (c)(iii), what is the expected reward if the first action is greedy and subsequent actions are ϵ-greedy, with $\epsilon = 0.1$. [4]

END OF SECTION

THIS PAGE IS BLANK

SECTION G

Bioengineering

1 (a) Explain what affects the visibility of different structures in 2D images of the back of the eye, *other than resolution*, when imaged both with a Fundus Camera and a Scanning Laser Ophthalmoscope (SLO). How can this visibility be improved? [4]

(b) A light beam is focused by a lens. The beam cross-section is a disc of radius r given by:

$$r^2(z) = {r_0}^2 \left(1 + \left(\frac{\lambda z}{\pi {r_o}^2}\right)^2\right)$$

where z is the axial distance from the focal point, r_0 is the radius at the focal point, and λ is the wavelength of light. Assume that all components are in air.

(i) Derive expressions for the lens axial resolution Δz_l (along the z-direction) and radial resolution Δx, each in terms of λ and r_0. [4]

(ii) Show that the numerical aperture, NA, of the lens is given by NA $\approx \frac{\lambda}{\pi r_0}$. [4]

(iii) Hence derive an expression for NA just in terms of Δz_l and Δx. [3]

(c) The lens in (b) is used in an SLO by adding a confocal aperture with diameter d, located close to the focal point. The axial resolution of the SLO is Δz_s.

(i) If Δz_s is defined as the range over which at least half the beam area will pass through the aperture, derive an expression for Δz_s in terms of λ, r_0 and the aperture diameter d. [3]

(ii) Bearing in mind your answers to (b) and (c)(i), what is the approximate minimum achievable axial resolution of the system in practice, just in terms of r_0? What value of d would achieve this? [5]

(iii) Other than resolution, what are the constraints on the size of the confocal aperture d? [2]

2 (a) Delivery of liquid drug formulation to the retina is usually achieved through the sclera instead of through the cornea of the eye.

(i) Describe the anatomy, functions and extracellular matrix organisation of the cornea and the sclera. Use sketches to assist your answers. [5]

(ii) Based on your answer in (i), briefly explain why a poroelasticity model could be useful in describing fluid transport behaviour as a result of intraocular pressure (IOP) for the sclera, but not for the cornea. [2]

(iii) Figure 1 below shows the time dependent response of a sclera which was subjected to an indentation test. Based on the information given in this figure, estimate the hydraulic permeability κ of the tested sclera specimen, assuming the material time constant $\tau = \frac{h^2}{E\kappa}$, where the specimen thickness h is 1 mm and the sclera material Young's modulus E is 2 MPa. [4]

(iv) Discuss the conditions and assumptions under which the estimation of hydraulic permeability of sclera in (iii) is valid. [3]

(v) The diffusion constants D for different drug molecules in sclera were found to range from $4 \times 10^{-13}\,\mathrm{m^2s^{-1}} \lesssim D \lesssim 2 \times 10^{-10}\,\mathrm{m^2s^{-1}}$. Along with the information from (ii) to (iv), comment on whether drug transport in sclera is dominated by diffusion or convection at IOP=20 mmHg. (1 mmHg = 133.3 Pa). [3]

(b) Aided by a sketch, describe lens accommodation in young, normal, healthy eyes. [3]

(c) Intraocular lenses, and daily disposable contact lenses, can both be used to augment the eye for correcting 'near-sightedness' (or, myopia). For each case, discuss the material selection criteria and list the typical materials used. Comment on the advantage(s) and disadvantage(s) for using intraocular lenses versus daily disposable contact lenses for eyesight corrections. [5]

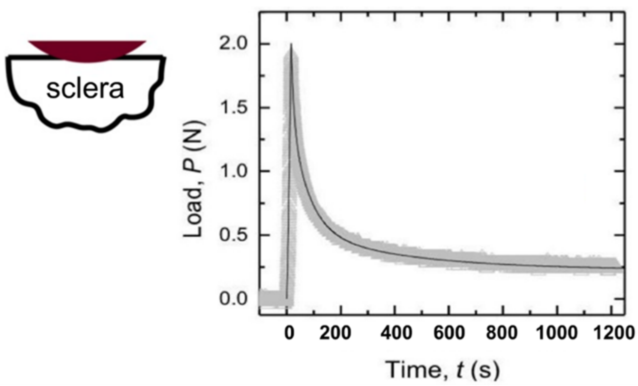

Figure 1.

3 This question is about efficient coding. A neuroscientist discovers a neuron in the brain of an adult primate that responds linearly to the scalar activities s_{L} and s_{R} of two retinal ganglion cells, from the left and right eyes respectively. For simplicity, the activities s_{L} and s_{R} are modelled as correlated Gaussian variables, each with zero mean and variance σ^2, and with correlation $\rho > 0$. The response of the neuron is modelled as $x = w_{\mathrm{L}} s_{\mathrm{L}} + w_{\mathrm{R}} s_{\mathrm{R}} + \epsilon$, where w_{L} and w_{R} are some weight parameters and ϵ is independent Gaussian noise with zero mean and unit variance.

(a) What condition(s) on the two ganglion cells' receptive fields would justify the assumption of a positive correlation $\rho > 0$ between s_{L} and s_{R}, under natural visual stimulation? Additionally, why is it unrealistic to envisage a perfect correlation, $\rho = 1$? [5]

(b) For each of these statements, explain, with reason, whether it is likely wrong or could well be true:

(i) The neuron was discovered in LGN, $w_{\mathrm{L}} > 0$, and $w_{\mathrm{R}} > 0$;

(ii) The neuron was discovered in V1, $w_{\mathrm{L}} > 0$, and $w_{\mathrm{R}} > 0$;

(iii) The neuron was discovered in V1 of the right hemisphere, $w_{\mathrm{L}} = 0$, and $w_{\mathrm{R}} > 0$.

[5]

(c) Express the variance of the neuron's response x in terms of ρ, σ^2, w_{L} and w_{R}. [3]

(d) Express the mutual information (in bits) between x and the stimulus vector $\vec{s} = (s_{\mathrm{L}}, s_{\mathrm{R}})$ in terms of ρ, σ^2, w_{L} and w_{R}. [Hint: do this by first finding expressions for the marginal and conditional entropies of the response: H_x and $H_{x|\vec{s}}$.] [5]

(e) Now suppose the weight vector $(w_{\mathrm{L}}, w_{\mathrm{R}})$ is constrained to have unit length, but otherwise has unconstrained direction in this two-dimensional parameter space. Which direction yields maximal coding efficiency? State your answer in terms of the angle θ such that $w_{\mathrm{L}} = \cos\theta$ and $w_{\mathrm{R}} = \sin\theta$. [5]

(f) Assume $w_{\mathrm{L}} > 0$. With the weight vector $(w_{\mathrm{L}}, w_{\mathrm{R}})$ constrained to have unit length as in part (e), we now add the further constraint that the response variance should not exceed the upper bound $b > 0$. Describe qualitatively how the optimal direction of the weight vector (*i.e.*, the angle θ that yields maximal coding efficiency under the new additional constraint) changes as b is lowered from very large values. [2]

END OF PAPER

Numerical Answers:

Q1:	(b) (i)	$\Delta x = 2r_0$, and $\Delta z_l = \frac{2\pi r_o^2}{\lambda}$
	(b) (iii)	$\mathrm{NA} = \frac{\Delta x}{\Delta z_l}$
	(c) (i)	$\Delta z_s = \frac{2\pi r_0}{\lambda}\sqrt{\left(\frac{1}{2}d^2 - {r_0}^2\right)}$
	(c) (ii)	$\Delta z_s = d = 2r_0$
Q2:	(a) (iii)	$\kappa \approx 5 \times 10^{-15}\ \mathrm{m^4Ns}$
Q3:	(c)	$\mathrm{Var}[x] = 1 + \sigma^2(w_\mathrm{L}^2 + w_\mathrm{R}^2 + 2\rho w_\mathrm{L} w_\mathrm{R})$
	(d)	$I[x;\vec{s}] = \frac{1}{2}\log_2[1 + \sigma^2(w_\mathrm{L}^2 + w_\mathrm{R}^2 + 2\rho w_\mathrm{L} w_\mathrm{R})]$
	(e)	$\theta = 45^\circ$ or $\theta = 225^\circ$

SECTION H*: Manufacturing and Management*

Answer not more than **two** *questions from this section.*

1 (a) Explain at least four types of *market pull* sources of innovation, providing examples for each. [4]

(b) You have been asked to develop the design for a new low-cost bicycle for commuters.

(i) Describe the design process that you should follow to generate the product idea. [4]

(ii) Describe how prototyping could help in the design process, discussing examples of prototyping techniques which could be used. [10]

(c) Explain the characteristics and uses of these two forms of *Intellectual Property Rights (IPR)*:

(i) patents;

(ii) copyrights. [7]

2 (a) Explain how *Intellectual Property Rights (IPR)* systems work. [6]

(b) Two amateur inventors have designed a new infrared and optical camera system which makes use of standard and bespoke components. They are planning to create a new business and target the security systems market for home and small businesses users.

(i) Discuss the advantages and disadvantages of *selling the IP* and *licensing the IP* for their start-up company, illustrating practically how the two inventors can utilise these two business models. [9]

(ii) After much consideration and research, the inventors decide to develop a production facility for this invention. They anticipate that the market for this device will be around 10000 units initially with the view of expanding production and market after the first few years. Discuss the issues that would need to be considered in designing a production system for this device. [10]

3 (a) Discuss the role of *product specifications* in the design process. [8]

(b) Describe the advantages and disadvantages of the following types of business models for a large company in generating commercial value from an invention. Provide examples of these business models in real life:

(i) product and consumables;

(ii) service enabled by a product;

(iii) service only. [7]

(c) Discuss the fundamental characteristics of the *Penthatlon framework,* providing examples of how companies deploy activities to manage innovation. [10]

END OF PAPER