Version ACF/3

EGT1
ENGINEERING TRIPOS PART IB

Tuesday 4 June 2024 9 to 11.10

Paper 5

ELECTRICAL ENGINEERING

*Answer not more than **four** questions.*

*Answer not more than **two** questions from each section.*

All questions carry the same number of marks.

*The **approximate** percentage of marks allocated to each part of a question is indicated in the right margin.*

Answers to questions in each section should be tied together and handed in separately.

*Write your candidate number **not** your name on the cover sheet.*

STATIONERY REQUIREMENTS
Single-sided script paper

SPECIAL REQUIREMENTS TO BE SUPPLIED FOR THIS EXAM
CUED approved calculator allowed
Engineering Data Book

10 minutes reading time is allowed for this paper at the start of the exam.

You may not start to read the questions printed on the subsequent pages of this question paper until instructed to do so.

You may not remove any stationery from the Examination Room.

Version ACF/3

SECTION A

*Answer not more than **two** questions from this section*

1 The circuit in Fig. 1 is a differential amplifier constructed using an operational amplifier that may be considered ideal.

(a) State what is meant by the common mode rejection ratio (CMRR) for a differential amplifier. [3]

(b) Determine the output voltage v_{out} in terms of the input voltages v_1 and v_2. [4]

(c) For a differential amplifier with inputs v_1 and v_2, differential gain A_{diff}, and common mode gain A_{cm}, the output voltage v_{out} can be written as:

$$v_{out} = A_{diff}(v_1 - v_2) + A_{cm}\left(\frac{v_1 + v_2}{2}\right)$$

For the differential amplifier in Fig. 1 determine A_{diff} and A_{cm}. [6]

(d) Hence or otherwise show that the CMRR of the differential amplifier is given by:

$$CMRR = \frac{R_4R_1 + 2R_2R_4 + R_2R_3}{2(R_1R_4 - R_2R_3)}$$

[4]

(e) If all resistors have the same nominal value R, but have a precision t such that a resistor with nominal value R is between $R(1-t)$ and $R(1+t)$, explain why the worst case $CMRR$ occurs when $R_1 = R(1+t)$, $R_2 = R(1-t)$, $R_3 = R(1-t)$ and $R_4 = R(1+t)$. [3]

(f) If 0.1 % precision resistors are available, estimate the corresponding CMRR stating any assumptions made. [5]

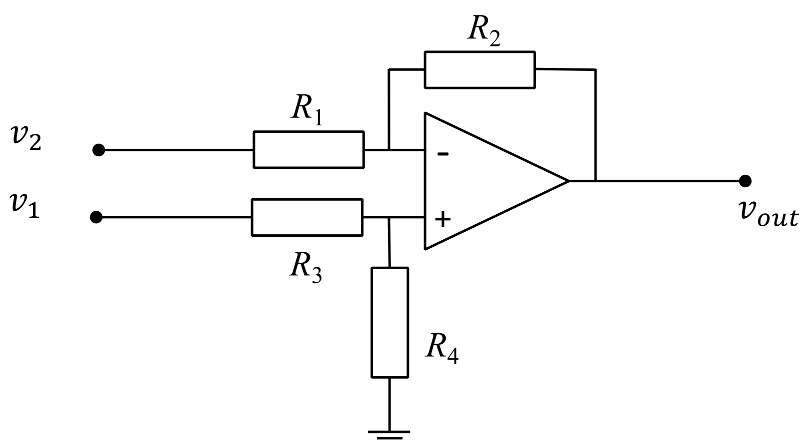

Fig. 1

2 The amplifier circuit in Fig. 2 has two transistors Q_1 and Q_2 arranged as a Darlington Pair. Q_1 is biased such that $I_{B1} = 1$ µA, $I_{E1} = 100$ µA, $V_{BE1} = 0.7$ V and $V_{CE1} = 4.3$ V, and Q_2 is biased such that $I_{B2} = 100$ µA, $I_{E2} = 10$ mA, $V_{BE2} = 0.7$ V, and $V_{CE2} = 5$ V where the number 1 or 2 in the subscript indicates the relevant transistor. At these operating points Q_1 has small signal parameters $h_{ie1} = 50$ kΩ, $h_{fe1} = 100$ and Q_2 has small signal parameters $h_{fe2} = 100$, $h_{ie2} = 500$ Ω. For all transistors in this question the small signal parameters h_{re} and h_{oe} may be neglected.

(a) Determine appropriate values for R_1 and R_E stating any assumptions made. [4]

(b) The Darlington Pair forms a three-terminal device which can be modelled by a single transistor.

(i) Draw the small-signal equivalent circuit for the Darlington Pair.

(ii) Show that the equivalent base resistance h'_{ie} is given by $h_{ie1} + h_{ie2}(1 + h_{fe1})$.

(iii) Show that the equivalent current gain h'_{fe} is given by $h_{fe1} + h_{fe2} + h_{fe1}h_{fe2}$.

[6]

(c) Hence or otherwise draw a small-signal equivalent circuit for the amplifier valid for mid-band frequencies (where the reactance of the capacitors C_i and C_o may be assumed to be zero). [4]

(d) Using the small-signal circuit, assuming no load is applied, calculate:

(i) the small-signal voltage gain;

(ii) the small-signal input resistance;

(iii) the small-signal output resistance. [6]

(e) Calculate suitable values for C_i and C_o if the circuit is used for signals between 300 Hz and 3.4 kHz with a source resistance of 1 MΩ and load resistance of 20 Ω. [5]

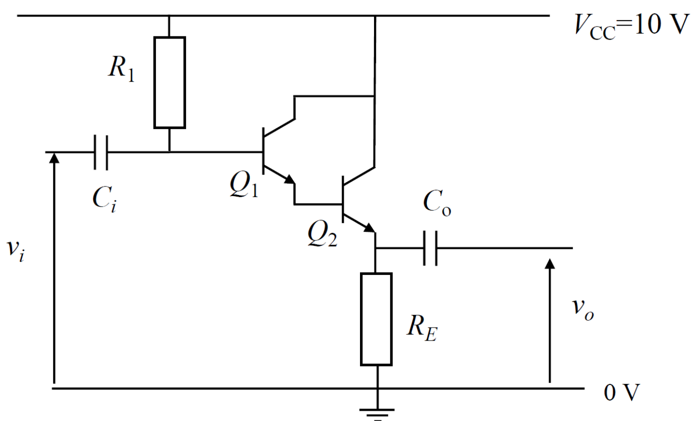

Fig. 2

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3 (a) High-voltage three-phase AC transmission lines typically use only three active wires. Explain why three wires without a fourth are sufficient and advantageous. Also explain why the neutral conductor is required at the distribution level. [5]

(b) A key aspect in electrical systems is protection.

(i) Explain what a symmetrical three-phase fault to earth is. What general properties should the protection system fulfil? [3]

(ii) Explain the difference between circuit breakers and isolators. How does a circuit breaker work? [3]

(c) It is desirable to operate a transmission line so that the magnitudes of the line-line voltages at the source and load ends are the same. Assuming a purely inductive three-phase transmission line, draw a phasor diagram showing the relation between line current and line to earth voltages at the load and source. Hence, show that both load and source contribute to the reactive power consumed by the transmission line. [5]

(d) A power station supplies a town through a 20 km medium-voltage 50 Hz three-phase power line. Each of the three lines is rated for 130 A and has resistance per unit length of 1.2 Ω/km and reactance per unit length of 4.4 Ω/km. Determine the line-line voltage at the power station, assuming that the town consumes only real power, that the line is operating at its rated current, and that the line-line voltage at the town is 33 kV. Also find the real power generated at the power station under these conditions. [4]

(e) Under the conditions of part (d), determine the value of the star-connected capacitors required at the power station so that its voltage and current are in phase. Use your answer to explain the benefits of correcting the power factor at the power station. [5]

SECTION B

*Answer not more than **two** questions from this section*

4 (a) Explain the real and reactive power limits of a synchronous generator and illustrate them graphically via an operating chart. Which element typically limits the generation of reactive power? [5]

(b) Discuss in what ways synchronous generators for hydro and steam power plants are different. Make reference to the different mechanical operating points and how they affect the generator design geometrically as well the rotor winding and number of poles. [4]

(c) What is the per-unit system and how is it used? [3]

(d) A 50 Hz three-phase synchronous generator is star-connected to an 11 kV infinite bus. The synchronous reactance of the generator is $X_S = 0.3\ \Omega$. Find the load angle δ and the excitation voltage E if the prime mover delivers 250 MW and the load has a power factor of 0.75 lagging. [6]

(e) Find the excitation voltage E required for a unity power factor load with the same prime mover power of 250 MW. [4]

(f) Use your answers to parts (d) and (e) to explain the benefits of correcting the load power factor. By what factor is the stator power loss of the generator reduced by correcting the power factor to unity? [3]

5 (a) A plane polarized electromagnetic wave travelling in air in the z-direction has an electric field in the y-direction given by:

$$E_y = E_0\, e^{j(\omega t - \beta z)}\ \mathrm{V\,m^{-1}}$$

Derive an expression for the corresponding magnetic field and for the average power density in $\mathrm{Wm^{-2}}$ carried by the wave. [7]

(b) A circular wire loop of diameter d, oriented in the y-z plane, is immersed in the wave. Obtain an expression for the rms e.m.f. induced in the loop, assuming that d is small compared to the wavelength of the radiation. [6]

(c) A broadcasting station radiates electromagnetic waves at a frequency of 800 MHz. The power emitted is 2 kW, uniformly distributed over a hemisphere. The loop of part (b) has a diameter of 100 mm and is connected to a receiver which requires a minimum power of 5×10^{-9} W for adequate reception. If the receiver has an input resistance of 50 Ω and is power matched to the loop, estimate the maximum distance between transmitter and receiver still enabling adequate reception. [7]

(d) If the minimum power in part (c) is reduced to 2.5×10^{-9} W, estimate the maximum angle that the plane of the loop may take, while still enabling adequate reception. [5]

6 (a) Consider a coaxial cable with outer diameter a and inner diameter b, where the space between the two conductors is filled with a dielectric. Derive an expression for the inductance per unit length. [9]

(b) Describe what is meant by characteristic impedance Z_0 of a coaxial cable. The capacitance per unit length is given by $C = 2\pi\varepsilon_0\varepsilon_r/\ln(a/b)$, with ε_r the relative permittivity of the dielectric. Find an expression for Z_0 and calculate its value for the case a = 3 mm, b = 1.5 mm and $\varepsilon_r = 2$. [9]

(c) When the coaxial cable is terminated by a 50 Ω load, explain which quantity determines the fraction of incident power reflected back from the load. For the case in part (b) calculate the value of this quantity. In order to minimize this value, what size should the inner conductor have? [7]

END OF PAPER

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