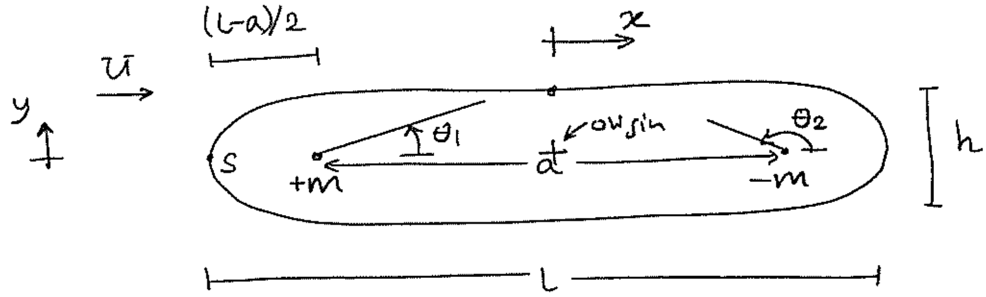
3A1 2022

1. a) Move in a frame of reference moving with the body:



$$\eta = \frac{2m}{\pi U a}$$

i) stagnation point occurs where $u_s = U - \dfrac{m}{2\pi\frac{(L-a)}{2}} + \dfrac{m}{2\pi\frac{(L+a)}{2}} = U - \dfrac{m}{\pi}\left(\dfrac{2a}{L^2-a^2}\right) = 0$

Solve for L/a: $\dfrac{m}{\pi U a}\left(\dfrac{2}{(L/a)^2-1}\right) = 1 \Rightarrow \left(L/a\right)^2 = 1+\eta$

$$\Rightarrow \frac{L}{a} = \pm(1+\eta)^{1/2}$$

Most students answered this well. A few made algebraic mistakes.

[2 marks]

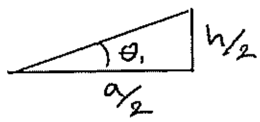
ii) Define streamfunction $\psi = Uy + \dfrac{m\theta_1}{2\pi} - \dfrac{m\theta_2}{2\pi}$

angle from first source

angle from second source.

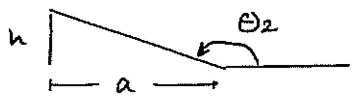
at stagnation point, $y=0$, $\theta_1 = \pi$, $\theta_2 = \pi \Rightarrow \psi = \dfrac{m}{2\pi}(\pi - \pi) = 0$

Find a solution to $\psi = 0$ at $x=0$ and $y \neq 0$.



$\dfrac{h}{a} = \tan\theta_1$

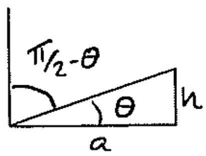
$$\Rightarrow 0 = \frac{Uh}{2} + \frac{m}{2\pi}\left\{\tan^{-1}\left(\frac{h}{a}\right) + \tan^{-1}\left(\frac{h}{a}\right) - \pi\right\}$$



$\theta_2 = \pi - \tan^{-1}(h/a)$

$$\Rightarrow 0 = \frac{h}{a} + \frac{2m}{\pi U a}\left\{\tan^{-1}\left(\frac{h}{a}\right) - \frac{\pi}{2}\right\}$$

$$\Rightarrow \quad = \frac{h}{a} + \eta\left\{-\cot^{-1}\left(h/a\right)\right\}$$



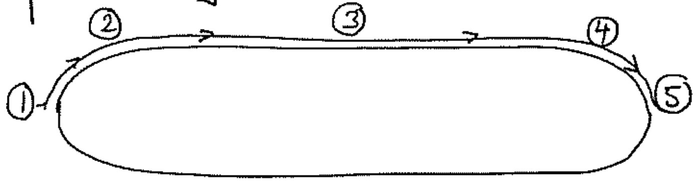
$$\Rightarrow \frac{h}{a} = \eta\cot^{-1}\left(\frac{h}{a}\right)$$

Around 1/2 of the students answered this well, although around 1/4 did not approach this by evaluating the streamfunction and therefore struggled

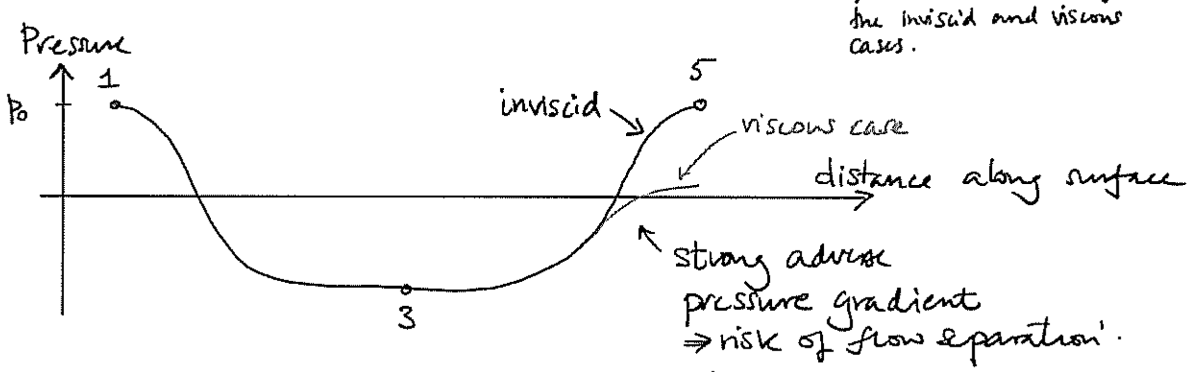
[6 marks]

For the potential flow:

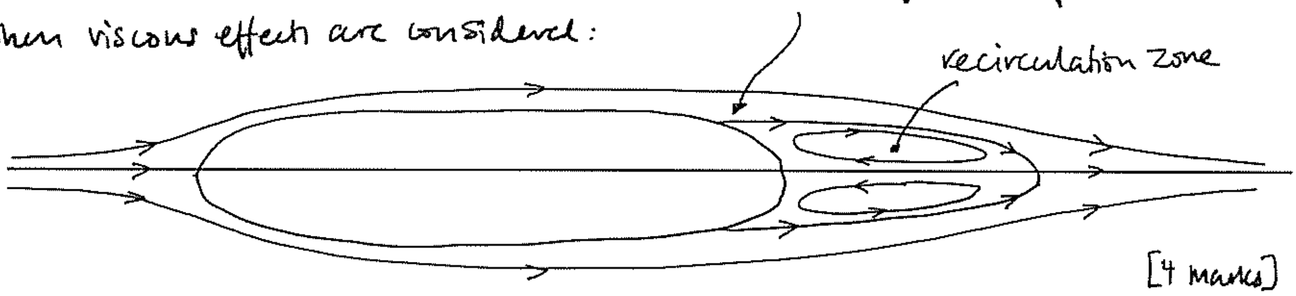
b)



Students answered this section well, showing good knowledge of the qualitative behaviour of the inviscid and viscous cases.



When viscous effects are considered:



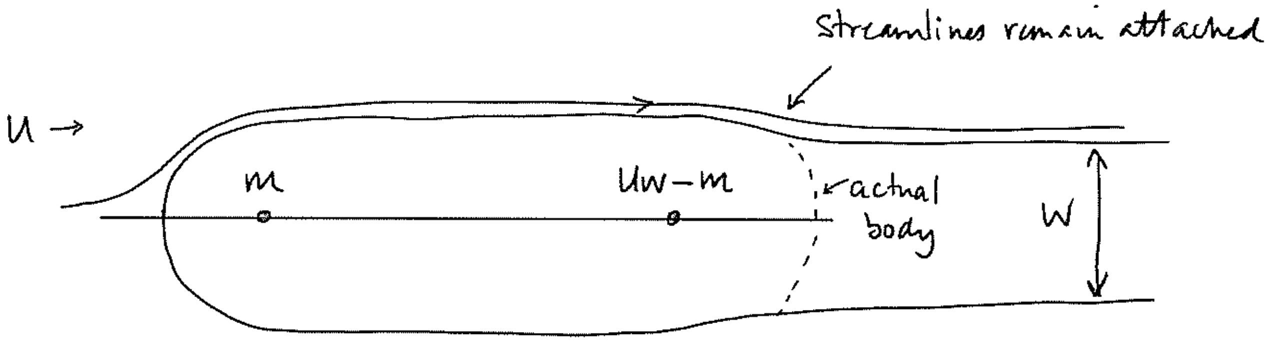
[4 marks]

(c) Base bleed can be modelled by reducing the downstream sink strength. Width w far downstream $\Rightarrow$ vol flowrate Uw within the wake (per unit depth into page). This means that $m_2 = m_1 - Uw$.
m_1 would have to decrease slightly to maintain L/a or increase slightly to maintain h/a. The latter is more important.

Most students answered this well.

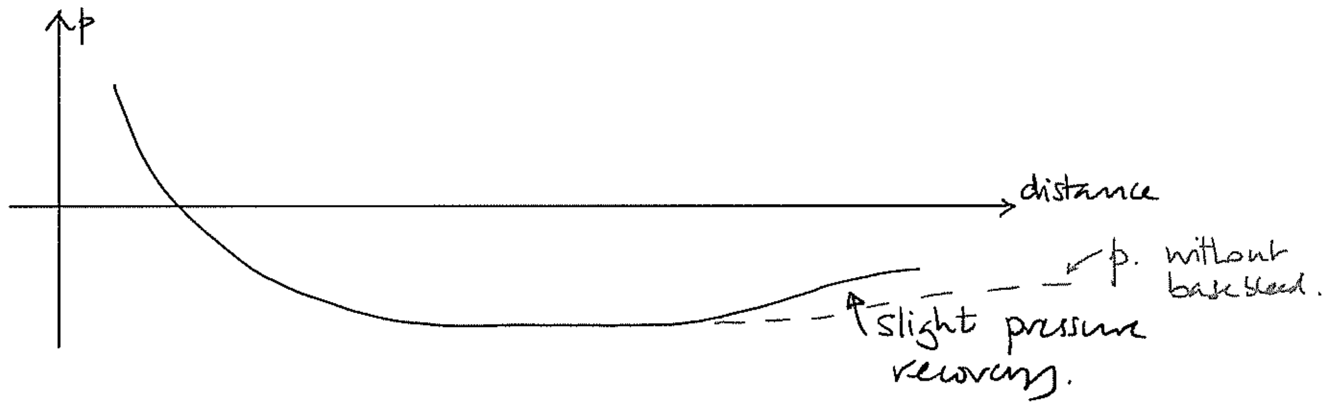
[4 marks]

(d)



pressure distribution over the body

Most students answered this well.



The boundary layer remains attached over the rear of the body because the adverse pressure gradient is reduced. The pressure increases slightly compared with the case without base bleed. The pressure in the recirculating zone is ∴ slightly higher and the net drag force is ∴ lower.
(Base bleed also prevents vortex shedding but this is not in the course).

(M. Juniper)

[4 marks]

2 (a)

(i) $$F(z) = \sum_{n=-\infty}^{+\infty} -\frac{i\Gamma}{2\pi}\ln(z+nd)$$

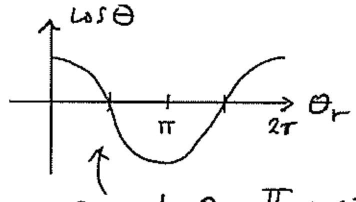
the sign does not matter because n runs from $-\infty$ to $+\infty$.

$\boxed{\text{Almost every student obtained this.}}$

[2 marks]

ii) Stagnation points lie where $u = v = 0$

$$u - iv = \frac{dF}{dz} = \sum_{n=-\infty}^{\infty} -\frac{i\Gamma}{2\pi}\frac{1}{z+nd} = -\frac{i\Gamma}{2\pi}\frac{\pi}{d}\cot\left(\frac{\pi z}{d}\right) = -\frac{i\Gamma}{2d}\cot\left(\frac{\pi z}{d}\right)$$



$$\cot\theta = \frac{\cos\theta}{\sin\theta} \Rightarrow \cot\theta = 0 \text{ where } \cos\theta = 0$$

zero at $\theta_r = \frac{\pi}{2} + n\pi$ & $\theta_i = 0$

$$\Rightarrow \theta = \frac{\pi z}{d} = \frac{\pi}{2} + n\pi = \pi\left(n+\frac{1}{2}\right)$$

$$\Rightarrow y = 0 \text{ and } \frac{x}{d} = \pi\left(n+\frac{1}{2}\right)$$

There are many ways to solve this question. Most students managed to do so.

Alternatively, note that, at $x/d = n + 1/2$ there is an equal number of vortices to the left as there is to the right. By symmetry their velocities will sum to zero at $x/d = n + 1/2$.

[4 marks]

iii) $$u - iv \to -\frac{i\Gamma}{2d}\cot\Big(\underbrace{i\pi\frac{y}{d}}_{\theta}\Big) \text{ as } \frac{y}{d} \to +\infty \text{ or } -\infty$$

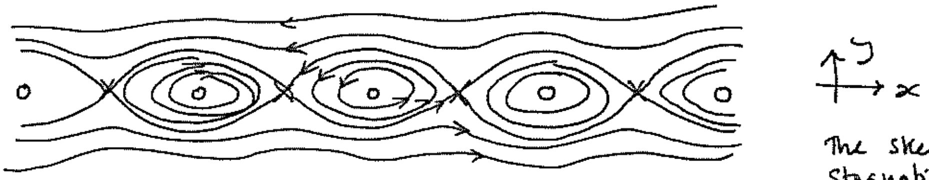
$$\cot i\theta = i\,\frac{e^{-\theta}+e^{\theta}}{e^{-\theta}-e^{\theta}} \begin{cases} \to -i \text{ as } \theta \to \infty \\ \to +i \text{ as } \theta \to -\infty \end{cases}$$

So $\quad u \to -\frac{\Gamma}{2d}$ as $y/d \to \infty$ and $+\frac{\Gamma}{2d}$ as $y/d \to +\infty$

$v \to 0$

There are many ways to solve this and most students did.

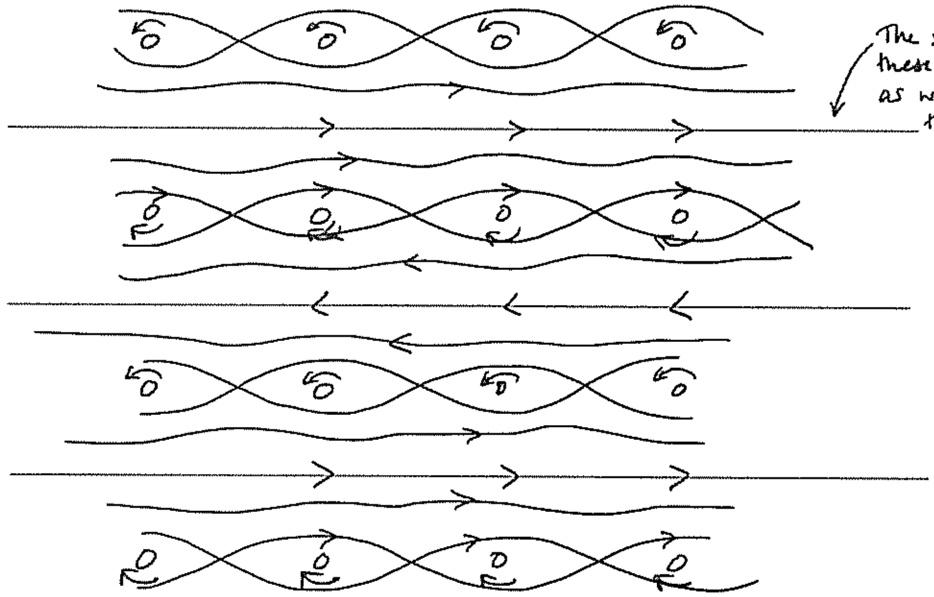
[2 marks]



[4 marks]

The sketch needs to show the stagnation points and some streamlines inside and outside the stagnation streamlines. Many students' sketches were not sufficiently careful or detailed.

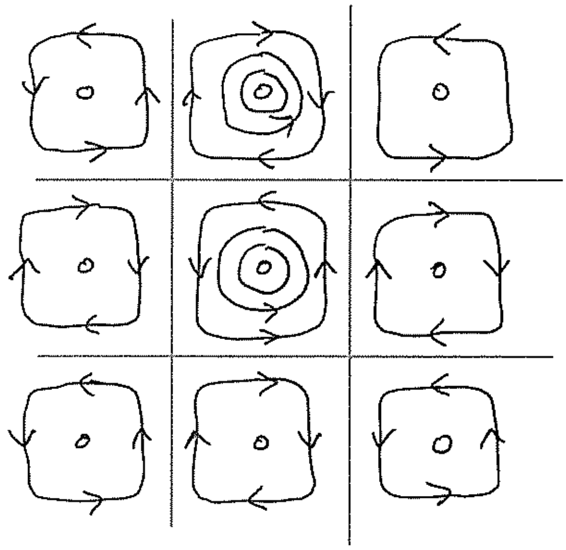
(b) Note the image system created by the vortices. This allows us to draw some straight streamlines without calculation.



The sketch needs to show these straight streamlines as well as the features on the previous sketch.

[2 marks]

(c)



The sketch needs to show the stagnation streamlines.

[2 marks]

Many students omitted the description of how the vortices would mix the fluids.

(d) The flow in (b) draws in one fluid from the right and another from the left. It interleaves the two fluids. If columns of vortices have opposite sign, the vortices draw in one fluid from the top and one from the bottom. The flow in (c) does not mix. Each cell around each vortex is self contained. Weakening vortices in turn would allow fluid to be pushed from one cell to the adjacent cells, allowing for precise control of the amount of each fluid in each cell.

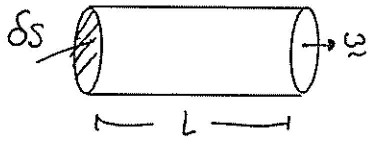
This model is inviscid and ∴ does not account for the influence of viscosity on the velocity field. It also doesn't account for turbulence or unsteady phenomena, such as switching on or switching off vortices. [4 marks]

Almost all students mentioned the lack of viscosity in the model.

M. Juniper

3.

(a) Consider a vortex tube of length L containing uniform vorticity. It must have constant cross-sectional area because $\int \omega . dA$ = const. along the tube.



conservation of volume: $L\delta s$ = const.

Even if L changes, $\Gamma = \int \omega . ds = \omega \delta s$ = const.

But $\delta s = \frac{\text{const}}{L} \Rightarrow \Gamma = \omega \frac{\text{const}}{L}$ = const [4 marks]

$\Rightarrow \frac{\omega}{L}$ = const.

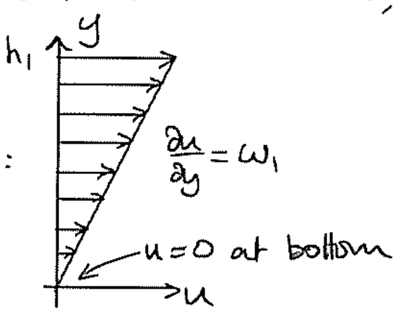
Most students answered this well. A line of reasoning was required. It is not enough to quote the Helmholtz laws and then say "therefore ω/L = const."

(b) vorticity $= \frac{\partial u}{\partial y} = \omega_1$ = uniform.

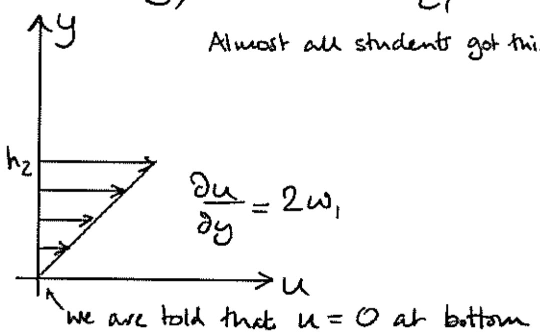
The width doubles so, due to vortex stretching, $\omega_2 = \omega_1 \frac{L_2}{L_1} = 2\omega_1$

Almost all students got this

section 1:



section 2:



$$\text{vol. flowrate}_1 = \int_0^{h_1} u\,dy$$

$$= L_1 \int_0^{h_1} \omega_1 y\,dy = L_1 \omega_1 \frac{h_1^2}{2}$$

$$\text{vol flowrate} = L_2 \int_0^{h_2} \omega_2\,dy$$

$$= L_2 \omega_2 \frac{h_2^2}{2}$$

These must be the same $\Rightarrow h_2^2 = \frac{L_1 \omega_1}{L_2 \omega_2} h_1^2 = \frac{1}{4} h_1^2$

Many students made algebraic errors but marks were given mainly for the method.

$\Rightarrow h_2 = \frac{1}{2} h_1$ [8 marks]

(c) The flow is inviscid so there are no mechanisms that dissipate mechanical energy. Therefore $\dot{E}$ will be conserved between 1 and 2.

At section 1, $\dot{E}_1 = \int_0^{h_1} (p + \rho g y + \frac{1}{2}\rho u^2)\, dQ$

now, $p = p_a + \rho g(h_1 - y)$ and $dQ = u_1 L_1 dy$

$$\Rightarrow \dot{E}_1 = \int_0^{h_1} (p_a + \rho g h_1)\, dQ + \frac{1}{2}\rho L_1 \int_0^{h_1} u_1^3 dy$$

Note that $u_1 = \omega_1 y$ so $Q = \int_0^{h_1} u_1 L_1 dy = \omega_1 \frac{h_1^2}{2} L_1$

and $\frac{1}{2}\rho L_1 \int_0^{h_1} u_1^3 dy = \frac{1}{2}\rho L_1 \omega_1^3 h_1^4/4 = \rho Q \frac{\omega_1^2 h_1^2}{4}$

$$\Rightarrow \dot{E}_1 = Q\left\{ p_a + \rho g h_1 + \rho \frac{\omega_1^2 h_1^2}{4} \right\}$$

note height of channel bottom

At section 2, $\dot{E}_2 = \int_0^{h_2} (p + \rho g(y + b_2) + \frac{1}{2}\rho u^2)\, dQ$

$$\Rightarrow \dot{E}_2 = Q\left\{ p_a + \rho g(h_2 + b_2) + \rho \frac{\omega_2^2 h_2^2}{4} \right\}$$

From part (b), $\omega_2 = 2\omega_1$ and $h_2 = h_1/2$ so $\omega_2 h_2 = \omega_1 h_1$

$$\Rightarrow \dot{E}_1 - \dot{E}_2 = \rho g h_1 - \rho g(h_2 + b_2) = 0$$

$\Rightarrow b_2 = h_1 - h_2$ i.e. the water level is the same in both sections.

If $b_2 = h_1 - h_2$ then the constraint on the vorticity, on the vol. flowrate, and on the mechanical energy dissipation are consistent.

[8 marks]

No students completed this successfully but marks were given for the method and for reasoning.

M. Juniper

4. (a) The key point of Prandtl's boundary layer theory is that the inertia term and the viscous term are of the same order. The inertia term is $u\,\partial u/\partial x$, which is of order U^2/L. The viscous term is $\nu\,\partial^2 u/\partial y^2$, which is of order $\nu U/\delta^2$. Therefore $\dfrac{U^2}{L} \sim \nu\dfrac{U}{\delta^2} \Rightarrow \dfrac{\delta^2}{L^2} \sim \dfrac{\nu}{UL} \Rightarrow \dfrac{\delta}{L} \sim \dfrac{1}{\sqrt{Re_L}}$

Most students obtained the desired scaling but few gave a satisfactory reasoning. [4]

(b) The no-slip condition at $y=0 \Rightarrow u(0)=0$

The momentum equation at $y=0$ reduces to $\displaystyle 0 = -\frac{\partial p}{\partial x} + \nu\left.\frac{\partial^2 u}{\partial y^2}\right|_{y=0}$

$\partial p/\partial y \simeq 0$ in a boundary layer so p is the free-stream pressure. From Bernoulli's equation in the free stream, $\partial p/\partial x = -U\,\partial U/\partial x$ so the second boundary condition is $\displaystyle \left.\frac{\partial^2 u}{\partial y^2}\right|_{y=0} = -\frac{U}{\nu}\frac{\partial U}{\partial x} = -\frac{U}{\nu}\frac{dU}{dx}$ because U is a fn only of x

All students quoted the no slip condition. Many correctly identified the other condition but a common mistake was to assume that $dU/dx = 0$. [3]

(c) To match the free stream velocity, $u(\delta) = U(x)$.

To enforce a smooth approach at $y=\delta$, $\partial u/\partial y = 0$ at $y=\delta$. [2]

Almost all students gave these boundary conditions with reasoning.

(d) $u(0)=0 \Rightarrow a_0 = 0$; Elementary algebraic manipulation gives: $a_1 = \frac{3}{2}$; $a_3 = -\frac{1}{2}$

$\displaystyle \left.\frac{\partial^2 u}{\partial y^2}\right|_{y=0} \Rightarrow a_2 = 0$

Almost all students correctly answered this. [3]

(e) Momentum thickness $\displaystyle = \theta = \int_0^\delta \frac{u}{U}\left(1-\frac{u}{U}\right)dy = \delta\int_0^1 \left(\frac{3}{2}\eta - \frac{\eta^3}{3}\right)\left(1-\frac{3}{2}\eta + \frac{\eta^3}{3}\right)d\eta$ where $\eta = \dfrac{y}{\delta}$

Elementary manipulation gives $\theta = (39/280)\,\delta$

$\displaystyle \tau_w = \mu\left.\frac{\partial u}{\partial y}\right|_{y=0} = \frac{3\mu U}{2\delta}$; $\displaystyle C_f' = \frac{\tau_w}{\frac{1}{2}\rho U^2} = \frac{3\nu}{\delta U}$

Most students answered this correctly. [4]

(f) In (d) we were told to set $dU/dx = 0$. The momentum integral eq. reduces to:

$$\frac{d\theta}{dx} = \frac{C_f'}{2} \Rightarrow \frac{39}{280}\frac{d\delta}{dx} = \frac{3}{2}\frac{\nu}{\delta U} \Rightarrow \frac{1}{2}\frac{d\delta^2}{dx} = \frac{140}{13}\frac{\nu}{U} \Rightarrow \delta = \left(\frac{280}{13}\right)^{1/2}\left(\frac{\nu x}{U}\right)^{1/2}$$ [4]

$\displaystyle \Rightarrow \frac{\delta}{x} = 4.64\,Re_x^{-1/2}$

Many students answered this correctly.

Jie Li & M. Juniper

5. a) Apply eq 1 (continuity) with $U = Ar$, $V = -Bz$:

Almost all students answered part (a) correctly.

$$\frac{\partial (ru)}{\partial r} + r\frac{\partial v}{\partial z} = \frac{\partial (Ar^2)}{\partial r} + r\frac{\partial(-Bz)}{\partial z} = 2rA - rB = 0 \quad \Rightarrow \quad B = 2A$$

[2]

b) $\psi(r,z) = F(r) f(\eta)$ where $\eta \equiv z/g(r)$.

Almost all students answered part (b) correctly. Some left f' in the answer.

$$u = \frac{1}{r}\frac{\partial \psi}{\partial z}\bigg|_r = \frac{1}{r} F f' \frac{\partial \eta}{\partial z}\bigg|_r = \frac{1}{r}\frac{Ff'}{g} \rightarrow U(r) \text{ as } \eta \rightarrow \infty.$$

Without loss of generality, set $f'(\infty) \to 1 \Rightarrow F(r) = r g(r) U(r)$ [4]

(c) We could follow the Falkner-Skan solutions in the lecture notes to work out $f(\eta)$ and $g(r)$ simultaneously but it is easier to use physical insight to guess : $g(r) \sim \left(\frac{\nu r}{U}\right)^{1/2} = \left(\frac{\nu r}{Ar}\right)^{1/2} = \left(\frac{\nu}{A}\right)^{1/2}$ hence $\frac{\partial \eta}{\partial z} = \left(\frac{A}{\nu}\right)^{1/2}$, $\frac{\partial \eta}{\partial r} = 0$

This insight greatly simplifies the problem because g is constant (i.e. not a function of r).
No student spotted this, which meant that their expressions contained g' and were long and tedious to derive.

$$\Rightarrow \psi(r,z) = F(r) f(\eta) = r g U f(\eta) = Ar^2 (\nu/A)^{1/2} f(\eta)$$

$$\Rightarrow u = \frac{1}{r}\frac{\partial \psi}{\partial z} = Ar(\nu/A)^{1/2} f'(1/g) = Arf' \qquad \Rightarrow \frac{\partial u}{\partial r} = Af' \; ; \frac{\partial u}{\partial z} = Arf''\left(\frac{A}{\nu}\right)^{1/2}$$

$$v = -\frac{1}{r}\frac{\partial \psi}{\partial r} = -2A(\nu/A)^{1/2} f \qquad \Rightarrow \frac{\partial^2 u}{\partial z^2} = Arf'''\left(\frac{A}{\nu}\right)$$

Some students correctly commented that their expressions did not contain ν. [8]

(d) Substitute these into the momentum equation :

Most students persevered and used the correct method. They obtained an expression that contained g'. Some students then assumed that all terms containing g' should be zero.

$$\cancel{Ar}f'Af' - 2\cancel{A}\cancel{(\nu/A)^{1/2}} f\cancel{Ar}f''\cancel{\left(\frac{A}{\nu}\right)^{1/2}} = \cancel{A}^{2}\cancel{r} + \cancel{\nu}\cancel{Ar}f'''\left(\frac{\cancel{A}}{\cancel{\nu}}\right)$$

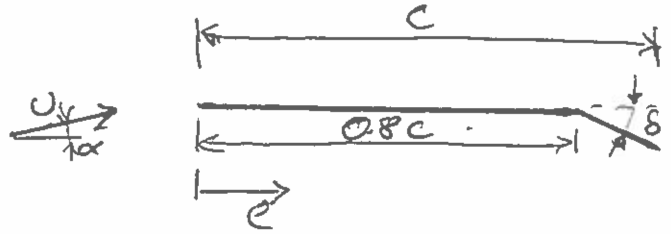
$$\Rightarrow f''' + 2ff'' + 1 - (f')^2 = 0$$ with boundary conditions $f(0) = f'(0) = 0$; $f'(\infty) = 1$

One student obtained the correct expression for the governing eq., having assumed that terms in g' were zero.

Most students who attempted (d) correctly stated the boundary conditions. [6]

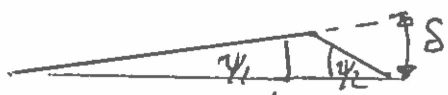
Jie Li & M. Juniper

Q6



a) Lowering the flap has two (!) effects: increase camber and increase angle of attack ($\Delta\alpha = \delta\left(1 - \frac{0.8c}{c}\right) = 0.2\delta$)

The standard approach to solve this is to calculate the camber effect by considering the following shape:



This has two different slopes: $\psi_1 = \frac{\delta}{5}$ $\quad \psi_2 = \frac{4}{5}\delta \ (<0)$

From lifting line theory, using $\frac{e}{c} = \frac{1+\cos\theta}{2}$

with flap hinge line at $\theta_f = \cos^{-1} 0.6 = 0.927$

we get:

$$g_0 = \frac{1}{\pi}\left[\int_{\theta_f}^{\pi}\left(-2\frac{\delta}{5}\right)d\theta + \int_0^{\theta_f}\left(2\frac{4}{5}\delta\right)d\theta\right] = 0.19\delta$$

$$g_1 = \frac{2}{\pi}\left[\int_{\theta_f}^{\pi}\left(-2\frac{\delta}{5}\cos\theta\right)d\theta + \int_0^{\theta_f} 2\frac{4}{5}\delta\cos\theta\, d\theta\right] = 1.02\delta$$

For lift, include additional $\Delta\alpha$ term

$$\Delta C_\ell = 2\pi\, 0.2\delta + \pi\left(g_0 + \frac{g_1}{2}\right) = 3.45\delta$$

Alternatively, you could include the $\Delta\alpha$ term in the camber calculation (this is allowed because of small angles + linearisation).

e.g. $g_0 = \frac{1}{\pi}\left[0 + \int_0^{\theta_f} 2\delta\, d\theta\right]$

This gives the same answer, more quickly

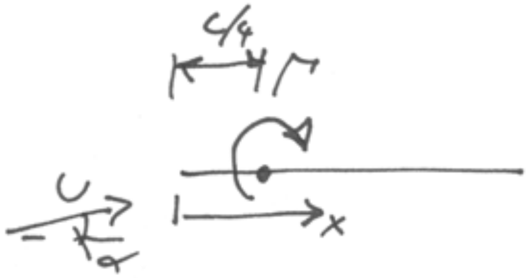
Q6 cont

b) Using the values gives $\Delta C_\ell = 0.6$

c) $\Delta C_\ell = 2\pi \Delta\alpha_{eff}$ $\Delta\alpha_{eff} = 0.096 \triangleq 5.5^\circ$

d) An increase in α gives a strong LE suction peak which risks separation & stall. The deployment of a flap gives additional low pressure (on upper surface) further along the chord which spreads the adverse pressure gradient and is generally preferred. There is, however, an increased risk of TE separation, but this is normally more benign.

a) see notes (section 2.5)



Lift (per span) $= \rho U \Gamma = \frac{1}{2}\rho U^2 c \cdot c_L$

assuming $c_L = 2\pi\alpha$ gives: $\Gamma = \pi c U \alpha$

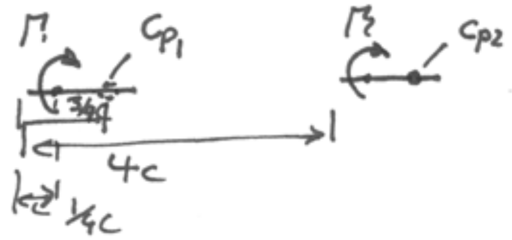
Downwash along plate: $\dfrac{\Gamma}{2\pi(x-\frac{c}{4})} = \dfrac{Uc\alpha}{2(x-\frac{c}{4})}$ (varies along x

Boundary condition: Flow along plate, so downwash due to vortex is equal opposite to $U\alpha$. This can only be true at one point x

thus:

$$U\alpha = U\alpha \cdot \frac{c}{2x-\frac{c}{2}} \quad \therefore \quad \underline{\underline{x = \tfrac{3}{4}c}}$$

b)



Apply BC at C_{p_1} and C_{p_2}:

At collocation point C_{p_1}: $U\alpha = \dfrac{\Gamma_1}{2\pi\frac{c}{2}} - \dfrac{\Gamma_2}{2\pi(4c-\frac{c}{2})} = \dfrac{\Gamma_1}{\pi c} - \dfrac{\Gamma_2}{7\pi c}$

at C_{p_2}: $U\alpha = \dfrac{\Gamma_2}{2\pi\frac{c}{2}} + \dfrac{\Gamma_1}{2\pi(4c+\frac{c}{2})} = \dfrac{\Gamma_2}{\pi c} + \dfrac{\Gamma_1}{9\pi c}$

using $\Gamma_0 = \pi\alpha U c$ gives:

$$\Gamma_1 = \frac{9}{8}\Gamma_0$$

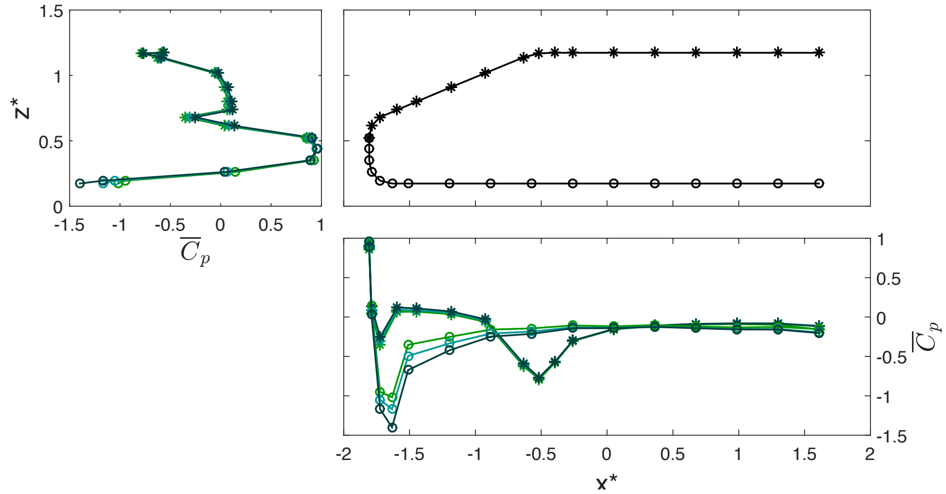
$$\Gamma_2 = \frac{7}{8}\Gamma_0$$

c) The front wing has greater circulation due to upwash from rear wing increasing effective α. (the opposite is true for rear wing).
Thus, when aircraft approaches stall, front wing would stall first. This would generate a nose-down pitching moment, taking aircraft out of stall. Hence, this is desireable / safe.

Note, that total circulation is the same as for a single wing (of twice the span). However in traditional aircraft tailplane produces downforce so canard aircraft may have a (slight) advantage.

All marks were allocated as indicated in exam paper.
Generally, this was well done, although often candidates provided only poor sketches and weak explanations for their workings.
The straightforward initial derivation (similar to one performed in the lectures) was not always well answered, whereas most students did well on (b).
The discussion of the effects of wing configuration on wing loading and stall behaviour (c) was sometimes not sufficiently detailed. Not many were able to explain their result from (b) or put this into context.

a) Expect low pressures around the (rounded) corners (particularly the lower front edge because of ground effect) and approximately something slightly below free-stream pressure at the top rear corner. There are high pressures around the stagnation area. An actual result is shown here (crosses are along upper surface and open symbls are at lower surface). I do not expect correct answers for the pressures below the vehicle, as long as a suction peak is suggested at the lower front edge.

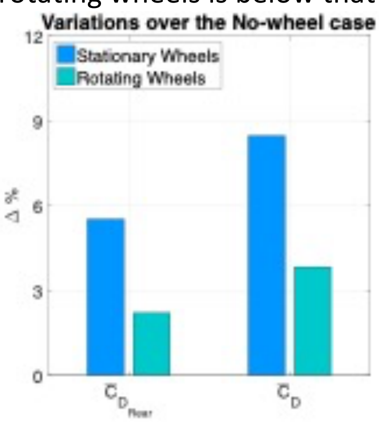


b) Separation might occur downstream of each corner (suction peak) and of course all across the rear.

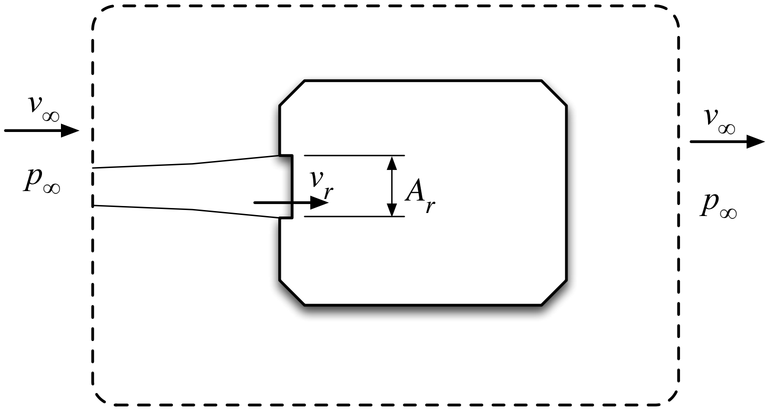
c) The vehicle has a large separation at the rear, like a typical bluff body. Thus, any answer in the region of Cd=0.3-0.45 is acceptable. This can be with reference to the drag of 3-dimensional bluff bodies with a rounded front (eg semi-sphere, which as Cd=0.42 in the notes, however the body here is more elongated and thus would be expected to be lower drag) or to hatchback cars with a large rear separation (various references in the notes are also around 0.4). The presence of wheels and actual roughness in real vehicles might suggests that an idealised body as shown here is a bit lower than 0.4.

d) The obvious improvement is the addition of a boat-tail (ensuring that the boat-tail angle is below 20deg). This can make the vehicle very aerodynamic, possibly even down to values around 0.2 in an idealised case. However, the question suggests that this is a commercial vehicle (i.e. van) and these generally require large internal volume and rear doors for access. Boat-tails would have an adverse effect on both, which is why current vans are relatively ‘box-shaped’.

e) Wheels obviously increase the drag do to the additional frontal area and their not exactly streamlined shape. However, the exposed parts of rotating wheels is generally moving in the direction of the flow and therefore the additional drag for rotating wheels is below that of stationary ones. Here is actual data:



d) Following the derivation in the notes (simple first year fluid mechanics):
Take a control volume approach. Here we assume a radiator area A_r with a local flow velocity on entry v_r. We also assume that the flow is incompressible and steady.



We can assume that the mass flow entering the car through the front radiator (which has a stagnation pressure equivalent to free stream) eventually leaves with negligible momentum (relative to the car). Thus, its momentum is 'lost' which causes drag. A momentum balance then provides the following additional cooling drag coefficient:

$$\boldsymbol{F = \dot{m}_r(0 - v_\infty)}$$
$$\boldsymbol{F = -\rho\, A_r v_r v_\infty}$$

$$\boldsymbol{c_{D,r} = \frac{\rho\, A_R v_r v_\infty}{\frac{1}{2}\rho v_\infty^2 A} = 2\frac{v_r}{v_\infty}\frac{A_R}{A}}$$

or

$$\boldsymbol{c_{D,r} = \frac{2\,\dot{m}_r}{\rho v_\infty A}}$$

where A is the cross-sectional area of the car.

Using the values given,

$$\boldsymbol{c_{D,r} = \frac{2 \times 0.2kg/s}{1.2\frac{kg}{m^3} \times \frac{30m}{s} \times\ 2.5m^2} = 0.004}$$

For a typical passenger vehicle $c_D \approx 0.3$, thus radiator drag is about 1.5% of the total..

Comments:
Marks were allocated as indicated on the exam paper.

On the whole, this was well done by many. Unfortunately, quite a number of candidates produced very poor sketches of the pressure distribution in (a), often missing regions of adverse pressure gradient altogether. In (c), some candidates (erroneously) used 2-dimensional drag coefficients as reference, giving a much too high drag coefficient estimate. Not many candidates were able to recognize the difference in drag between stationary and rotating wheels. Almost all of those who attempted the last part on radiator drag (f) achieved near-perfect answers.