Version MPJ/5

EGT2
ENGINEERING TRIPOS PART IIA

Monday 25 April 2022   9.30 to 12.40

Module 3A1

FLUID MECHANICS I

*Answer not more than **five** questions.*

All questions carry the same number of marks.

*The **approximate** percentage of marks allocated to each part of a question is indicated in the right margin.*

*Write your candidate number **not** your name on the cover sheet.*

STATIONERY REQUIREMENTS
Single-sided script paper

SPECIAL REQUIREMENTS TO BE SUPPLIED FOR THIS EXAM
CUED approved calculator allowed
Engineering Data Book
Attachments:

- Incompressible Flow Data Card (2 pages);
- Boundary Layer Theory Data Card (1 page);
- 3A1 Data Sheet for Applications to External Flows (2 pages).

10 minutes reading time is allowed for this paper at the start of the exam.

You may not start to read the questions printed on the subsequent pages of this question paper until instructed to do so.

You may not remove any stationery from the Examination Room.

1 (a) The potential flow around an object moving at speed U is to be modelled as the 2D planar flow around a Rankine body in which the source and sink have volumetric flowrates m as shown in Fig. 1. The distance between the source and sink is a. In this question, express your results in terms of the dimensionless parameter $\eta = 2m/(\pi U a)$.

(i) Derive an *explicit* expression for l/a in terms of η, where l is the length of the object. [10%]

(ii) Derive an *implicit* expression for h/a in terms of η, where h is the maximum height of the object. [30%]

(b) Sketch the pressure distribution of the potential flow around the object. Then sketch the streamlines of the flow when viscous effects are considered and show how the pressure distribution is affected. [20%]

(c) The drag of the object can be reduced by releasing gas through its rear-facing surface. This is known as *base bleed*. Without adding any extra elements to the model, how could the above model of the Rankine body be altered such that the width of released gas far downstream is w? [20%]

(d) Sketch the streamlines for this flow and sketch the pressure distribution over the object. Explain how base bleed would reduce drag on the object. [20%]

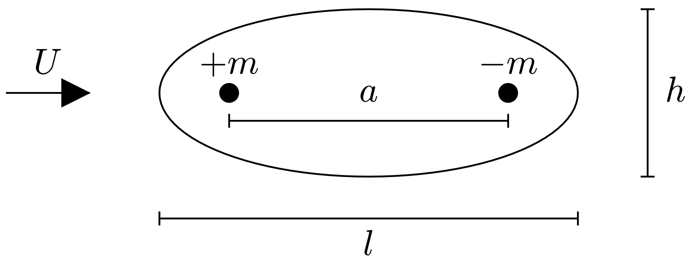

Fig. 1

2 (a) Consider an infinite row of identical vortices, each with circulation Γ, placed distance d apart, as shown in Fig 2.

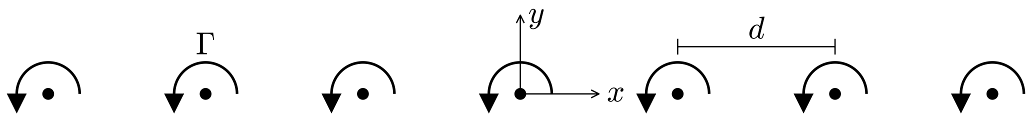

Fig. 2

(i) Aligning the origin with one of the vortices, write down an expression for the complex potential, $F(z)$, where $z = x + \mathrm{i}y$. [10%]

(ii) Defining n to be integer and using the identity

$$\sum_{n=-\infty}^{+\infty} \frac{1}{z+nd} = \frac{\pi}{d}\cot\left(\frac{\pi z}{d}\right) \quad , \quad \text{where } \cot\theta = \frac{\cos\theta}{\sin\theta}$$

or otherwise, show that stagnation points lie at $x/d = n + 1/2$. [20%]

(iii) Find the velocities as $y/d \to +\infty$ and $y/d \to -\infty$. [10%]

(iv) Sketch the streamlines for this flow, indicating the flow direction. [20%]

(b) Now consider an infinite square grid of vortices, each distance d from its nearest neighbours. In odd rows, all vortices have circulation Γ. In even rows, all vortices have circulation $-\Gamma$ as shown in Fig. 3(a). Without further calculations, sketch the streamlines within a 4×4 grid of vortices, indicating the flow direction. [10%]

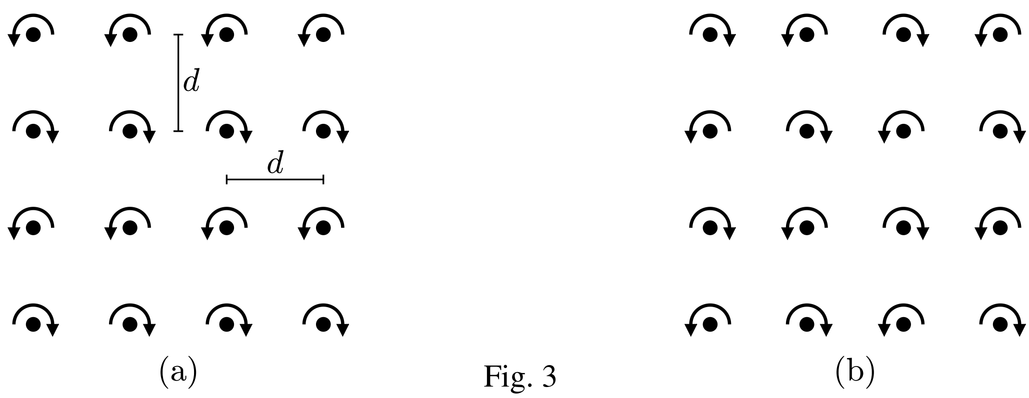

Fig. 3

(c) The circulation is reversed in every even column of the infinite grid as shown in Fig. 3(b). Sketch the streamlines within a 3×3 grid of vortices. [10%]

(d) Discuss how such an array of vortices could be used to precisely control mixing of fluids. Discuss any defects in this potential flow model when modelling real fluids. [20%]

3 The Helmholtz laws state that, for an inviscid fluid with uniform density, (i) vortex lines move with the fluid; (ii) the circulation is the same for all cross-sections of a vortex tube and is independent of time.

(a) Consider a vortex tube with length l and cross-sectional area S that contains uniform vorticity, ω. Using the Helmholtz laws show that ω/l remains constant when l changes. [20%]

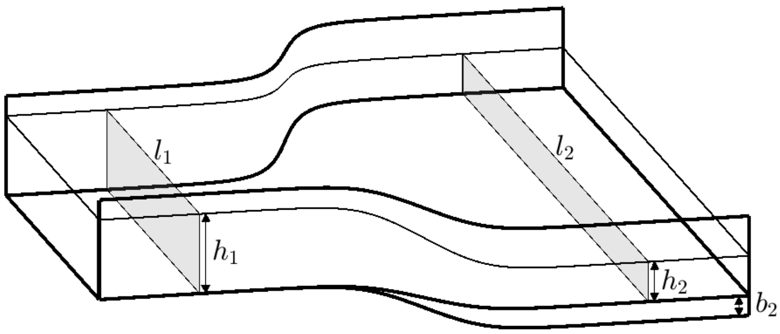

Fig. 4

(b) Figure 4 shows an open channel whose bottom rises from height zero to height b_2. At section 1 the channel has width l_1 and contains water with depth h_1 and uniform vorticity ω_1. At sections 1 and 2 the vorticity vector is aligned with the width of the channel and the velocity is zero at the channel bottom. Assuming that the flow is steady and inviscid, and that the vorticity vector remains aligned with the width of the channel, use a constraint on the vorticity to find the vorticity, ω_2, at section 2, where the width is $l_2 = 2l_1$. By considering the volumetric flowrate, calculate the water depth h_2. [40%]

(c) The flux of mechanical energy is defined as

$$\dot{E} = \int \left(p + \rho g z + \frac{1}{2}\rho u^2 \right) \mathrm{d}Q$$

where Q is the volumetric flowrate and u is the flow velocity. Find b_2 in terms of h_2 and h_1, such that the assumptions in part (b) are consistent with each other. [40%]

4 Figure 5 shows the development of a laminar boundary layer along a flat plate, where δ is the thickness of the boundary layer, L is the length of development and $U(x)$ is the freestream velocity. We consider here steady high Reynolds number flows. Assume that the flow has a third-order polynomial velocity profile u:

$$\frac{u}{U} = a_0 + a_1\left(\frac{y}{\delta}\right) + a_2\left(\frac{y}{\delta}\right)^2 + a_3\left(\frac{y}{\delta}\right)^3 \ ,$$

where a_0, a_1, a_2 and a_3 are constants.

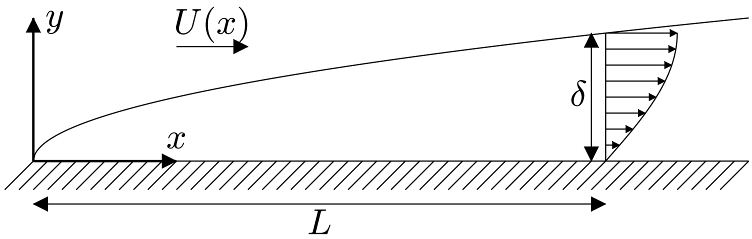

Fig. 5

(a) What is the precise order of magnitude of δ/L? Explain briefly why. [20%]

(b) What are the boundary conditions for u and $\partial^2 u/\partial y^2$ at $y = 0$? Explain the physical justification for these conditions. [15%]

(c) What are the boundary conditions for u and $\partial u/\partial y$ at $y = \delta$? Explain the physical justification for these conditions. [10%]

(d) Assume the free stream velocity U is constant. Use the above four boundary conditions to determine the four constants a_0, a_1, a_2, and a_3, and hence the velocity profile. [15%]

(e) Find the momentum thickness θ and the local skin friction coefficient C_f'. [20%]

(f) Find the variation of δ in terms of $Re_x = Ux/\nu$, where x is the distance from the leading edge of the plate (where $\delta = 0$) and ν is the kinematic viscosity. [20%]

5 Figure 6 shows the *inviscid axisymmetric* external flow towards a stagnation point defined by the linear velocity field $(U, V) = (Ar, -Bz)$ in $z \geq 0$, where U is the radial velocity, V is the axial velocity, and A and B are positive constants. The boundary-layer equations for the axisymmetric flow are

$$\frac{\partial (ru)}{\partial r} + r\frac{\partial v}{\partial z} = 0 \tag{1}$$

$$u\frac{\partial u}{\partial r} + v\frac{\partial u}{\partial z} = U\frac{\partial U}{\partial r} + \nu\frac{\partial^2 u}{\partial z^2} \tag{2}$$

where u is the radial velocity, v is the axial velocity, and ν is the kinematic viscosity. Note that Eq. (1) is the continuity equation and Eq. (2) is the momentum equation for axisymmetric flow. We look for the solution (u, v) of the above equations to match the external flow (U, V) with the no-slip condition at $z = 0$.

(a) Find the relationship between A and B. [10%]

(b) In axisymmetric flow, the streamfunction ψ is defined as

$$u = \frac{1}{r}\frac{\partial \psi}{\partial z}, \quad v = -\frac{1}{r}\frac{\partial \psi}{\partial r} .$$

Consider a streamfunction of the form $\psi(r, z) = F(r)f(\eta)$, where $\eta = z/g(r)$, and g is an appropriate length scale of the boundary layer thickness. Express F in terms of U, g, and r. [20%]

(c) Calculate u, v, $\partial u/\partial r$, $\partial u/\partial z$, and $\partial^2 u/\partial z^2$ in terms of A, r, ν, and f. [40%]

(d) Substitute the above expressions into the boundary-layer equation to deduce an ordinary differential equation for f. State the boundary conditions for f. [30%]

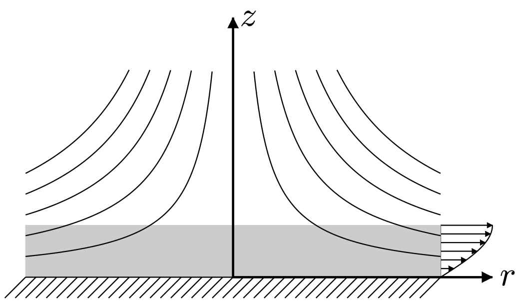

Fig. 6

6 A thin symmetric airfoil operating at an angle of attack α has a trailing-edge flap with a hinge line at 80% of chord.

(a) The flap is deflected downwards by an angle δ. Find an expression for the additional lift coefficient due to the flap deflection. You may apply the usual small-angle assumptions. [60%]

(b) If the flap is deflected by 10°, what is the magnitude of the additional lift coefficient? [10%]

(c) If the flap is not deflected, what change in angle of attack α is required to achieve the same lift as that in (b)? [10%]

(d) In order to achieve high lift, is it better to increase the angle of attack or deploy a flap? Explain your answer. [20%]

7 (a) A lifting aerofoil can be modelled simply by representing the overall circulation with a bound vortex located at the quarter-chord point of a flat plate at incidence ('lumped-parameter model'). Making the usual assumptions, show that there is exactly one location along the plate where the 'no-flow' boundary condition is fulfilled. What is the chord-wise location of this point? [30%]

(b) A canard aircraft flying at an angle of attack α and velocity U has two identical high-aspect ratio wings, each of chord c, separated by a distance $4c$, as shown in Fig. 7. Using a two-dimensional lumped parameter model calculate the strengths of the two bound vortices (one on each wing). Express your answers in terms of $\Gamma_0 = \pi \alpha c U$. [40%]

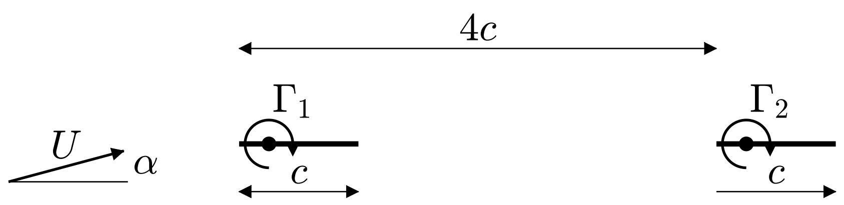

Fig. 7

(c) Comment on the result of part (b). What are the implications for the stall behaviour of canard aircraft? [30%]

8 Figure 8 shows a simple model of a generic commercial road vehicle.

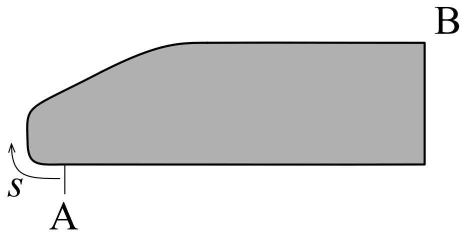

Fig. 8

(a) Assuming that this is to be tested without wheels in a wind tunnel, sketch the expected pressure distribution on the symmetry plane along the coordinate, s, from point (A) to the top rear corner (B). [20%]

(b) Indicate where you might expect flow separation to occur. [10%]

(c) Estimate the drag coefficient for a vehicle based on such a shape and explain on what basis you have arrived at this estimate. You are not required to make any calculations but may make reference to other shapes/vehicles. [20%]

(d) How could you improve the drag of such a vehicle? What are the potential drawbacks of any modifications? [20%]

(e) The wind tunnel model is now fitted with wheels. How might the drag compare between tests with stationary and rotating wheels? In each case a stationary ground plane is used. Explain your answer. [10%]

(f) The frontal area of the vehicle is 2.5 m^2 and the ambient air density is 1.24 kg m^{-3}. If the vehicle is fitted with a radiator requiring a mass flow rate of 0.2 kg s^{-1} at a driving speed of 30 m s^{-1}, what is the additional drag coefficient? To determine this value, derive an expression for the radiator drag using a simple control volume and the steady-flow momentum equation. [20%]

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1. (a) $l/a = \pm(1+\eta)^{1/2}$
   (b) $h/a = \eta \cot^{-1}(h/a)$
2. (a) $u \to -\Gamma/2d$ as $y/d \to \infty$ and $+\Gamma/2d$ as $y/d \to -\infty$; $v \to 0$
3. (b) $h_2 = h_1/2$
   (c) $b_2 = h_1 - h_2$
4. (a) $\delta/L \sim \mathrm{Re}_L^{-1/2}$
   (d) $a_1 = 3/2$; $a_3 = -1/2$
   (e) $\theta = (39/280)\delta$; $c'_f = 3\nu/(\delta U)$
   (f) $\delta/x = 4.64\mathrm{Re}_x^{-1/2}$
5. (a) $B = 2A$
   (b) $F(r) = rg(r)U(r)$
   (c) $u = Arf'$; $v = -2A(\nu/A)^{1/2} f$; $\partial u/\partial r = Af'$; $\partial u/\partial z = Arf''(A\nu)^{1/2}$
   (d) $f''' + 2ff'' + 1 - (f')^2 = 0$; $f(0) = f'(0) = 0$; $f'(\infty) = 1$
6. (a) 3.45δ
   (b) 0.6
   (c) 5.5°
7. (a) $X = 0.75c$
   (b) $\Gamma_1 = (9/8)\Gamma_0$; $\Gamma_2 = (7/8)\Gamma_0$
8. (f) $C_D \approx 0.004$