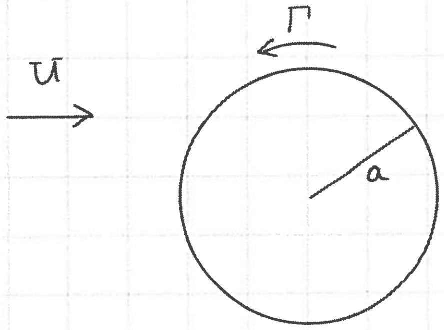
1. (a)



The flow is a sum of:

i) a uniform flow $F(z) = Uz$

ii) a x-wise doublet $F(z) = -\dfrac{\mu}{2\pi z}$

iii) a line vortex $F(z) = -\dfrac{i\Gamma}{2\pi}\ln z$

To find μ, use fact that $u_r = 0$ on $r = ae^{i\theta}$ for all θ. There are several ways to do this. One is to write $\psi = \mathrm{Im}\{F\}$ on $r=a$. Note that we can set $\Gamma = 0$ because Γ does not affect u_r:

$$F(z) = Uz - \frac{\mu}{2\pi z} = Uae^{i\theta} - \frac{\mu}{2\pi a}e^{-i\theta} \quad \text{on } r=a$$

$$= \left(Ua - \frac{\mu}{2\pi a}\right)\cos\theta + i\underbrace{\left(Ua + \frac{\mu}{2\pi a}\right)\sin\theta}_{\psi}$$

For ψ to be independent of θ we require $\dfrac{\mu}{2\pi} = -Ua^2$

The complex potential for this flow is therefore:

$$F(z) = Uz + \frac{Ua^2}{z} - \frac{i\Gamma}{2\pi}\ln z$$

Most students wrote this down correctly. A common mistake was to omit the doublet. 20%

(b) There are many ways to find the stagnation points. One is to set $\dfrac{dF}{dz} = U - U\dfrac{a^2}{z^2} - \dfrac{i\Gamma}{2\pi z} = 0$ and to use knowledge that $r = a$:

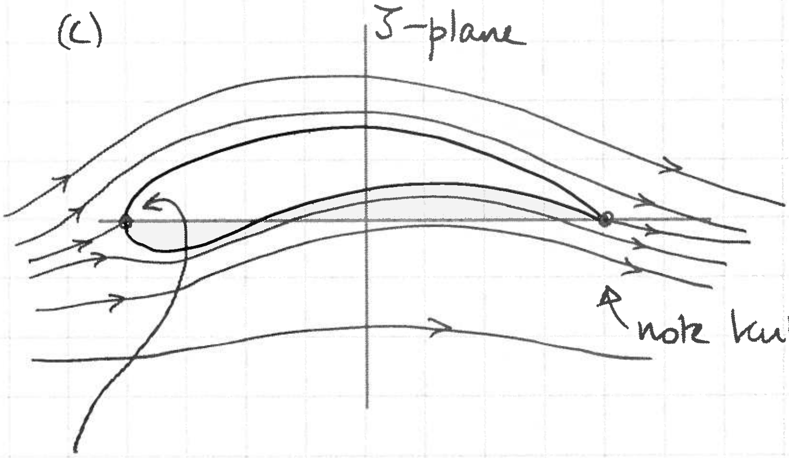
$$= U\left(1 - e^{-2i\theta}\right) - \frac{i\Gamma}{2\pi a}e^{-i\theta} = 0$$

$$\Rightarrow \quad U\left(e^{i\theta} - e^{-i\theta}\right) = 2Ui\sin\theta = \frac{i\Gamma}{2\pi a}$$

$$\Rightarrow \quad \sin\theta = \frac{\Gamma}{4\pi a U}$$

Most students answered this well in a variety of different ways. 20%

(c)



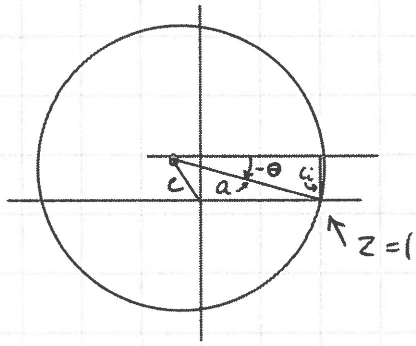
Some students drew excellent sketches but many did not. Students needed to show the stagnation points, the correct directions of the flow into/out of the stagnation points. Written notes on the sketches were helpful. 20%

$\Rightarrow$ angle of attack $= 0$.

↑ note kutta condition at trailing edge ($\zeta = 2$)

In a frame of ref. moving with the aerofoil, the free stream velocity is horizontal. Therefore the free stream velocity in the z-plane is also horizontal. Therefore the front stagnation point must be on the x axis in the z-plane and on the ζ_r axis in the ζ plane

(d) Lift $= -\rho U \Gamma$ so we need to work out Γ. Γ is conserved in the transformation so we need to work out Γ in the z-plane:



The radius of the cylinder is $a = 1 - c$

The stagnation point is at $\sin\theta = -\dfrac{c_i}{a}$

From (b), $\sin\theta = \dfrac{\Gamma}{4\pi a U} = -\dfrac{c_i}{a}$

$$\Rightarrow \Gamma = -4\pi U c_i$$

$$\Rightarrow \text{Lift} = -\rho U \Gamma = 4\pi \rho U^2 c_i$$

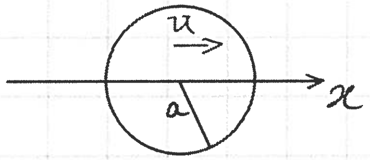
Only a few students answered this well, but those that did answer it well used a short method such as this one. Some students launched into pages of algebra without thinking, which didn't end well. 40%

This question showed that most students could model a flow with a complex potential and manipulate that model to obtain useful information about the flow. Many students need to improve their sketches of the flow (via streamlines). Students needed to take a moment to think before starting part (d) but many did not, leading to some poor attempts. Nevertheless, the overall performance was good.

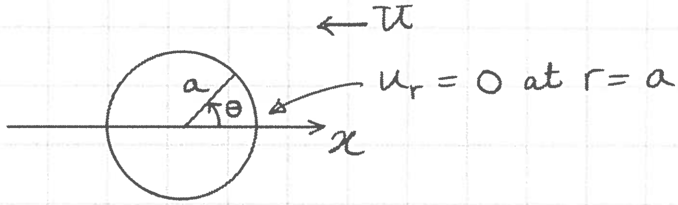
Matthew Juniper, April 2023

2

(a)



stationary frame



frame moving with sphere

The doublet centred on the sphere needs strength such that:

$$u_r\Big|_{(a,0)} = \underbrace{-U}_{\text{free stream}} + \underbrace{\frac{\mu\cos\theta}{2\pi r^3}}_{\text{doublet}}\Bigg|_{(a,0)} = -U + \frac{\mu}{2\pi a^3} \Rightarrow \frac{\mu}{2\pi} = Ua^3$$

Therefore the velocity potential in the stationary frame is:

$$\phi = -\frac{Ua^3}{2r^2}\cos\theta$$

Most students answered this well 20%

(b) Bernoulli's equation for inviscid flow is: $\frac{p}{\rho} + \frac{1}{2}|u|^2 + gz + \frac{\partial\phi}{\partial t} = 0$

$|u|^2 = u_r^2 + u_\theta^2$ (note that $u_r \neq 0$ in this frame)

$$u_\theta = \frac{1}{r}\frac{\partial\phi}{\partial\theta} = \frac{Ua^3}{2r^3}\sin\theta \quad ; \quad u_r = \frac{\partial\phi}{\partial r} = \frac{Ua^3}{r^3}\cos\theta$$

$$\Rightarrow \frac{1}{2}|u|^2 \text{ at } r=a = \frac{1}{2}U^2\left\{\cos^2\theta + \frac{1}{4}\sin^2\theta\right\}$$

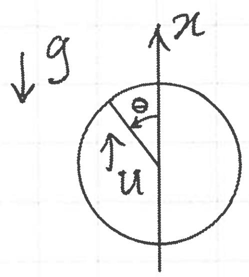
$$\Rightarrow \frac{p}{\rho} = k - \frac{1}{2}U^2\left\{\cos^2\theta + \frac{1}{4}\sin^2\theta\right\} + \frac{\dot{U}a}{2}\cos\theta$$

30%

Many students answered this well. A common mistake was to apply the steady Bernoulli equation, which then misses out the final term.

This question showed that most students could model a 3D flow with a velocity potential. Many (but not all) knew that the unsteady Bernoulli equation could and should be used for this unsteady problem. Even those who used steady Bernoulli were, however, able to obtain 75% of a correct answer and to continue. As expected, the last part was challenging but it was encouraging to see some excellent analysis from the top students.

(c)



Assume that the bubble remains spherical and that the flow around it can be modelled as inviscid. Driven by buoyancy the bubble accelerates upwards. We need to include g in the unsteady Bernoulli equation but, because the bubble starts from rest, we can assume that $u^2 << \dot{u}a$. Bernoulli's equation for the pressure in the surrounding liquid is:

Many students stated good assumptions

$$p_L = \rho_L\left\{k_L + \frac{1}{2}\dot{u}a\cos\theta - gx\right\}$$

check: if $\dot{u}=0$, p_L decreases as x increases ✓

The quick way to answer this question is to notice that, if the bubble remains spherical, then $p_L = p_g$ around the edge of the bubble. Further, p_g can be assumed to be uniform. We see this by assuming that $u_{\theta,liq} = u_{\theta,gas}$ at the bubble edge, so the flow inside the bubble obeys the same Bernoulli equation as that in the liquid

Nobody used this argument but a few students simply stated, correctly, that p = uniform inside the bubble.

$$\Rightarrow \quad p_g = \rho_g\left\{k_g + \frac{1}{2}\dot{u}a\cos\theta - gx\right\} = \rho_L\left\{k_L + \frac{1}{2}\dot{u}a\cos\theta - gx\right\} = p_L$$

But, because $\rho_g << \rho_L$, the L.H.S. can safely be set to zero.

$$\Rightarrow \quad k_L + \frac{1}{2}\dot{u}a\cos\theta - gx = 0 \qquad \text{where } x = a\cos\theta$$

$$\Rightarrow \quad k_L + \left\{\frac{1}{2}\dot{u}a - ga\right\}\cos\theta = 0$$

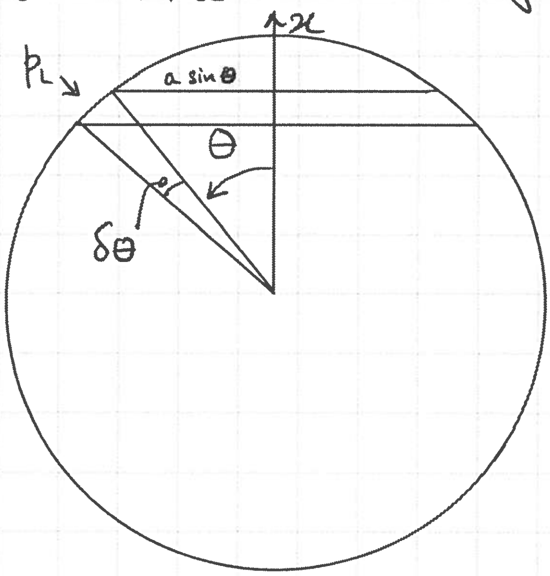
k_L is a constant so the term in brackets must be zero to avoid θ-dependence

$$\Rightarrow \quad \dot{u} = 2g$$

Around three students used this approach to achieve this result.

The alternative solution method is to consider separately the buoyancy force and the pressure force due to the flow around the bubble. The buoyancy force is $(\rho_L - \rho_g)\frac{4}{3}\pi a^3 g$.

The pressure force due to the flow can be found by integrating p_L around the bubble's surface:



Consider a ring element on the sphere. It has area:

$$\delta a = 2\pi a \sin\theta \, a\,\delta\theta$$

The pressure force resolved in the x-direction is:

$$\delta F = -p \cos\theta \, \delta a$$

where $p = \rho_L \left\{ k + \frac{1}{2}\dot{U} a \cos\theta \right\}$

will integrate to zero.

$$\Rightarrow F = \int \delta F = \int_{\theta=0}^{\pi} -\frac{1}{2}\rho_L \dot{U} a \cos^2\theta \,(2\pi a^2)\sin\theta \, d\theta$$

$$= \rho_L \pi a^3 \dot{U} \int_0^{\pi} \cos^2\theta \sin\theta \, d\theta = \rho_L \pi a^3 \dot{U} \left[\frac{\cos^3\theta}{3} \right]_0^{\pi} = -\frac{2}{3}\rho_L \pi a^3 \dot{U}$$

$$= -\frac{2}{3}\rho_L \pi a^3 \dot{U}$$

check: it is in the $-$ve x-direction

note: $\frac{2}{3}\rho_L \pi a^3$ is known as "added mass"

Around ten students started this correctly and a few completed the algebra successfully.

This must balance the buoyancy force up

$$\Rightarrow (\rho_L - \rho_g)\frac{4}{3}\pi a^3 g = \frac{2}{3}\rho_L \pi a^3 \dot{U}$$

50%

$$\Rightarrow \dot{U} = \frac{\rho_L - \rho_g}{\rho_L} 2g \simeq 2g \quad \text{for } \rho_L >> \rho_g$$

Matthew Juniper, April 2023

3 (a) From continuity, $\nabla \cdot \underset{\sim}{u} = 0 \Rightarrow \frac{1}{r}\frac{\partial(ru_r)}{\partial r} + \frac{1}{r}\frac{\partial u_\theta}{\partial \theta} + \frac{\partial u_z}{\partial z} = 0$ §12 of maths d-book

The flow is axisymmetric so all $\partial/\partial\theta$ terms are zero.

$$\Rightarrow \frac{1}{r}\frac{\partial(ru_r)}{\partial r} = -\frac{\partial u_z}{\partial z} = -2\alpha$$ because $u_z = 2\alpha z$

$$\Rightarrow \frac{d(ru_r)}{dr} = -2\alpha r$$ $\frac{\partial}{\partial r}$ becomes $\frac{d}{dr}$ because u_r is a function only of r

$$\Rightarrow \int_0^{ru_r} d(ru_r) = -\alpha\int_0^r 2r\,dr$$

$$\Rightarrow ru_r = -\alpha r^2$$

Almost every student answered this well.

$$\Rightarrow u_r = -\alpha r$$

20%.

(b) The z-component of the vorticity equation is:

$$\frac{\partial \omega_z}{\partial t} + \underbrace{\underset{\sim}{u} \cdot \nabla \omega_z}_{1} = \underbrace{(\underset{\sim}{\omega} \cdot \nabla) u_z}_{2} + \underbrace{\nu \nabla^2 \omega_z}_{3}$$

1 $\underset{\sim}{u} \cdot \nabla \omega_z = (u_r \underset{\sim}{e_r} + u_\theta \underset{\sim}{e_\theta} + u_z \underset{\sim}{e_z}) \cdot \left(e_r \frac{\partial \omega_z}{\partial r} + \underbrace{\frac{e_\theta}{r}\cancel{\frac{\partial \omega_z}{\partial \theta}}^{0} + e_z \cancel{\frac{\partial \omega_z}{\partial z}}^{0}}_{\omega_z \text{ depends only on } r}\right)$

$$= u_r \frac{\partial \omega_z}{\partial r} = -\alpha r \frac{\partial \omega_z}{\partial r}$$

2 $(\underset{\sim}{\omega} \cdot \nabla) u_z = \omega_z \frac{\partial u_z}{\partial z} = 2\alpha \omega_z$

3 $\nu \nabla^2 \omega_z = \frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial \omega_z}{\partial r}\right) + \frac{1}{r^2}\cancel{\frac{\partial^2 \omega_z}{\partial \theta^2}}^{0} + \cancel{\frac{\partial^2 \omega_z}{\partial z^2}}^{0}$

$\partial/\partial r \to \frac{d}{dr}$ because ω_z is a function only of r.

$$\frac{\partial \omega_z}{\partial t} = \underset{1}{\alpha r \frac{d\omega_z}{dr}} + \underset{2}{2\alpha \omega_z} + \underset{3}{\frac{\nu}{r}\frac{d}{dr}\left(r\frac{d\omega_z}{dr}\right)} = 0$$

because flow is steady

Term ① is convection of vorticity.
Term ② is creation of vorticity due to vortex stretching.
Term ③ is diffusion of vorticity.

① $\frac{d\omega_z}{dr} = -\frac{2r\alpha}{2\nu}\omega_z = -\frac{r\alpha}{\nu}\omega_z$, which is always the opposite sign to ω_z. Convection is bringing low vorticity fluid in from the outside.

② $2\alpha\omega_z$, which is always the same sign as ω_z. Vortex stretching is always creating more vorticity everywhere.

③ diffusion is continually transporting vorticity from regions of high vorticity to regions of low vorticity.

② Creates vorticity everywhere in proportion to the amount of vorticity already there – ie. higher vorticity creation near $r=0$. ③ diffuses this vorticity outwards radially and ① brings in low vorticity fluid from higher radii. The convection and diffusion of vorticity balance the creation.

Most answers were reasonable.
Many students identified vortex stretching, convection, and diffusion correctly but did not explain how they balance each other.

50%

(c) momentum equation in the radial direction if $u_r = -\alpha r$:

$$\frac{1}{\rho}\frac{\partial p}{\partial r} = -u_r(-\alpha) + \frac{u_\theta^2}{r} + \frac{\nu}{r}\left[\frac{\partial}{\partial r}(-r\alpha) + \alpha\right]$$

$$= -\alpha^2 r - \frac{u_\theta^2}{r} + \frac{\nu}{r}\left[-\alpha + \alpha\right] = -\alpha^2 r + \frac{u_\theta^2}{r} \simeq \frac{u_\theta^2}{r}$$

if $u_\theta^2 \gg (\alpha r)^2$

$$\Rightarrow \frac{1}{\rho}\frac{\partial p}{\partial r} \simeq \frac{u_\theta^2}{r}$$ i.e. positive pressure gradient.

Away from the plates, the positive radial pressure gradient supplies the centripetal force required to keep the flow in circular motion. At the plates, there is a boundary layer, where the fluid drops to zero velocity because the plates do not rotate. The pressure gradient within the boundary layer is the same as that in the bulk fluid but the fluid is not rotating. Therefore the pressure gradient drives the flui radially inwards at the top and bottom plates.

This was answered well by around 1/4 of the students. A few students applied Bernoulli's equation, which is invalid because (i) the flow is rotating $\nabla \times \underline{u} \neq 0$ and (ii) viscous effects are not neglected. This was disappointing.

30%

This question contained some rather basic algebraic manipulation, which was performed well, but was primarily designed to test students' physical understanding of the physical mechanisms active in this flow. There were some excellent answers from a few students but many gave rather short answers and some seemed not to answer the question being asked. It was disappointing to see students attempting to use Bernoulli's equation in a viscous rotating flow.

Matthew Juniper, April 2023

4. a) Navier - Stokes for uni-directional flow:

$$0 = -\frac{1}{\rho}\frac{dp}{dx} + \nu\frac{d^2u}{dy^2} \Rightarrow \frac{d^2u}{dy^2} = \frac{1}{\rho\nu}\frac{dp}{dx} = \frac{G}{\nu}$$ with boundary conds. $u(\pm h) = 0$

By inspection, or by integrating twice w.r.t. y this has solution:

$$u(y) = \frac{G}{2\nu}(y-h)(y+h)$$

Around $\frac{1}{3}$ of the students did this correctly – surprisingly few given that it is a 1B problem

20%

(b)

(i) Define the wall variable $Y = h - y$

The mean velocity across the channel, $V = \frac{1}{2h}\int_{-h}^{h} u\,dy = \frac{1}{h}\int_0^h u\,dy = \frac{1}{h}\int_0^h u\,dY$

The log law models u as: $\frac{u}{u^*} = \frac{1}{k}\ln\left(\frac{Yu^*}{\nu}\right) + B$ (u^* is not a function of Y)

Substitute one into the other: $V = \frac{u^*}{h}\int_0^h \frac{1}{k}\ln\left(\frac{Yu^*}{\nu}\right) + B \;\; dY$

Note that $\int \ln x\,dx = x(\ln x - 1) + \text{const}$ and sets $x = \frac{Yu^*}{\nu}$

$$V = \frac{u^*}{h}\int_0^{hu^*/\nu}\left\{\frac{1}{k}\ln x + B\right\}\frac{\nu}{u^*}\,dx = \frac{\nu}{h}\left[\frac{x}{k}(\ln x - 1) + Bx\right]_0^{hu^*/\nu}$$

$$= \frac{u^*}{k}\left(\ln\frac{hu^*}{\nu} - 1\right) + Bu^* = u^*\left(\frac{1}{k}\ln\frac{hu^*}{\nu} - \frac{1}{k} + B\right)$$

Around $\frac{1}{5}$ of the students managed this reasonably well. Most students didn't know where to start.

30%

(ii) Reynolds number $\equiv \frac{2hV}{\nu}$; friction factor $\equiv \frac{V}{u^*} = \left(\frac{8}{f}\right)^{1/2}$

From (i) we need $\frac{hu^*}{\nu}$, which is $\frac{hu^*}{\nu} = Re\frac{u^*}{2V} = \frac{Re}{2}\left(\frac{f}{8}\right)^{1/2}$

Most students who completed (i) also completed (ii) and (iii)

From (i): $\frac{V}{u^*} = \left(\frac{8}{f}\right)^{1/2} = \frac{1}{k}\ln\left\{\frac{Re}{2}\left(\frac{f}{8}\right)^{1/2}\right\} - \frac{1}{k} + B$

20%

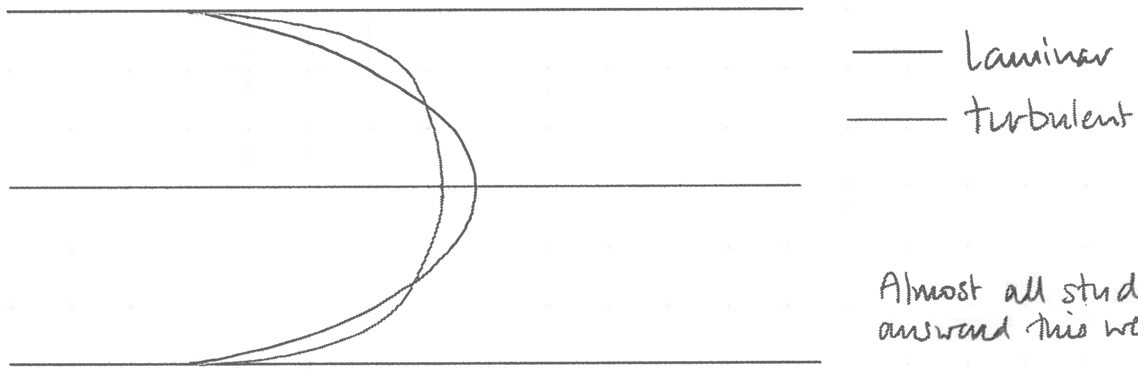
(iii) u_{max} occurs at the centreline where $Y = h$ in the log law expression.

$$\frac{u_{max}}{u^*} = \frac{1}{k}\ln\left(\frac{hu^*}{\nu}\right) + B = \frac{1}{k}\ln\frac{Re}{2}\left(\frac{f}{8}\right)^{1/2} + B = \frac{V}{u^*} + \frac{1}{k}$$

$$\Rightarrow \frac{V}{u_{max}} = \frac{V/u^*}{u_{max}/u^*} = \frac{V/u^*}{V/u^* + \frac{1}{k}} = \frac{(8/f)^{1/2}}{(8/f)^{1/2} + 1/k} = \left\{1 + \frac{1}{k}\left(\frac{f}{8}\right)^{1/2}\right\}^{-1}$$

20%

(c)



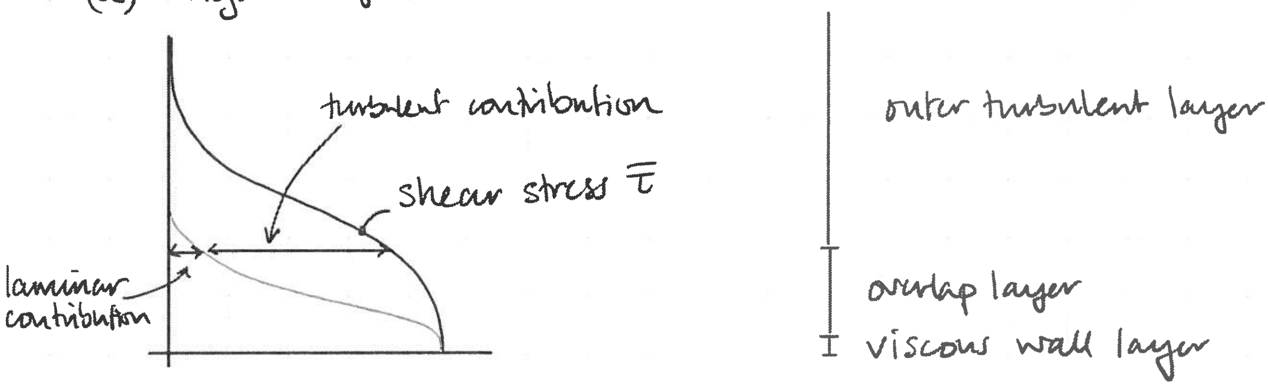
Almost all students answered this well.

The laminar velocity profile is parabolic.
The turbulent velocity profile is flatter in the middle and has a larger velocity profile near the wall. 10%

This question tested students' ability to model a flow between two flat plates (i) when laminar and (ii) when turbulent, given the instruction to assume a log law. Although there were two excellent answers to this question, most of the other students attempted this very badly, and it was surprising that so few students could do part (a). Most students who attempted this question scored low marks in the other questions too, so it seems that the better students avoided this question.

Jie Li & Matthew Juniper, April 2023

5. (a) Fig. 35 of the notes



The shear stress τ comprises $\tau_{laminar}$ and $\tau_{turbulent}$

$$\tau = \tau_{lam} + \tau_{turb}$$
$$\tau_{lam} \leftarrow \mu \frac{du}{dy} \qquad \tau_{turb} \leftarrow -\rho \overline{u'v'}$$

Most students answered this well. A common error was to sketch the velocity profiles instead of the shear stress profiles.

Main characteristics:

(i) The shear stress, τ, varies slowly close to the wall and can be treated as a constant : $\tau \simeq \tau_w$

(ii) Very close to the wall (viscous wall layer) the viscous stress is dominant and the turbulent stress is negligible

(iii) further away (overlap layer) the turbulent stresses dominate. 20%

(b) (i) Away from the outer edge, the velocity, u, is a function of u^*, ν, y. There are 4 quantities and 2 units so, by dimensional analysis, there are 2 non-dimensional groups : $\frac{u}{u^*}$ and $\frac{u^* y}{\nu}$

Hence the law of the wall: $\frac{u}{u^*} = f\left(\frac{y u^*}{\nu}\right)$

Almost all students answered this well.

10%

(ii) Very close to the wall, $\tau_{turb} \ll \tau_{lam}$, so $\tau \simeq \tau_w = \mu \frac{du}{dy} = \text{const}$

Now, $\tau_w \equiv \rho u^{*2} = \mu \frac{du}{dy} \Rightarrow u^{*2} = \nu \frac{du}{dy} = \frac{u}{y}$ if $\tau = \text{constant}$

Most answered this well

$\Rightarrow \frac{u}{u^*} = \frac{y u^*}{\nu}$ in this region, which is the viscous sublayer. 20%

(iii) Further away from the wall $\tau_{lam} \ll \tau_{turb}$ so ν is not relevant.

Dimensional analysis leads to : $\dfrac{du}{dy} = \dfrac{1}{\kappa}\dfrac{u^*}{y}$ (where $\kappa = 0.41$)

Integrating this gives $\dfrac{u}{u^*} = \dfrac{1}{\kappa}\ln\left(\dfrac{u^* y}{\nu}\right) + B$ ← another constant $\simeq 0.5$

Some students wrote down the answer without explaining their reasoning, which was insufficient.

20%

(c) The turbulent viscosity is modelled as $\nu_T = u^* l_m$ where $l_m = \alpha y$ ← constants.

$$u^{*2} \equiv \frac{\tau_w}{\rho} \simeq -\overline{u'v'} = \nu_T \frac{d\bar{u}}{dy} = u^* l_m \frac{d\bar{u}}{dy} = u^* \alpha y \frac{d\bar{u}}{dy}$$

$$\Rightarrow \quad \frac{d\bar{u}}{dy} = \frac{u^*}{\alpha}\frac{1}{y}$$

$$\Rightarrow \quad \frac{\bar{u}}{u^*} = \frac{1}{\alpha}\ln y + \underbrace{B}_{\text{constant}} = \frac{1}{\alpha}\left\{\ln\frac{u^* y}{\nu} - \ln\frac{u^*}{\nu}\right\} + B$$

Around half the students answered this well.

This is consistent with b(iii) with $\alpha = \kappa$.

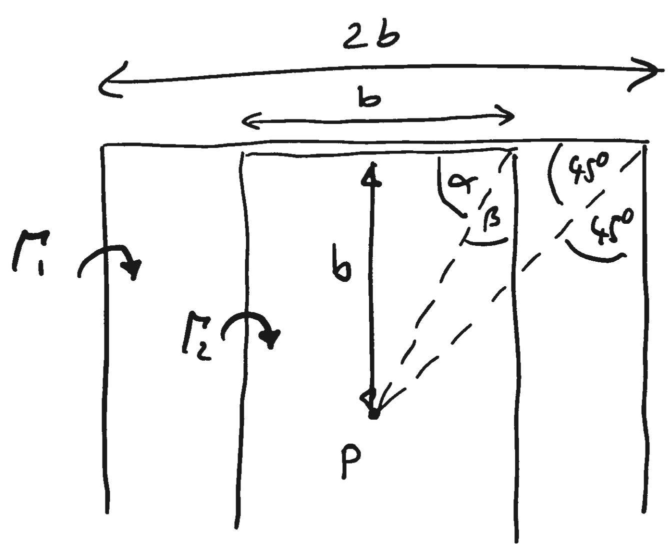
30%.

This question mainly tested students' ability to recall their boundary layer notes and to repeat some of the analysis within the notes. On the whole this question was answered well.

Jie Li & Matthew Juniper, April 2023

Q6

a)



$\alpha = 63.4°$

$\cos\alpha = \frac{1}{\sqrt{5}}$

$\cos\beta = \frac{2}{\sqrt{5}}$

$\cos 45° = \frac{1}{\sqrt{2}}$

b) Local c_l = const.

In centre, chord is doubled. Thus each horseshoe vortex has same strength $\Gamma_1 = \Gamma_2 = \Gamma$

$$L = W = \rho U \Gamma 2b + \rho U \Gamma b = 3\rho U \Gamma b$$

$$\Gamma = \frac{W}{3\rho U b}$$

c) Outer system:

$$w_1 = \underbrace{2 \cdot \frac{\Gamma}{4\pi b}(\cos 45° + 1)}_{\text{tip vortex}} + \frac{\Gamma}{4\pi b}(\cos 45° + \cos 45°) = \frac{\Gamma}{4\pi b}(2 + 2\sqrt{2})$$

$$= \frac{\Gamma}{\pi b} 1.21$$

Inner system:

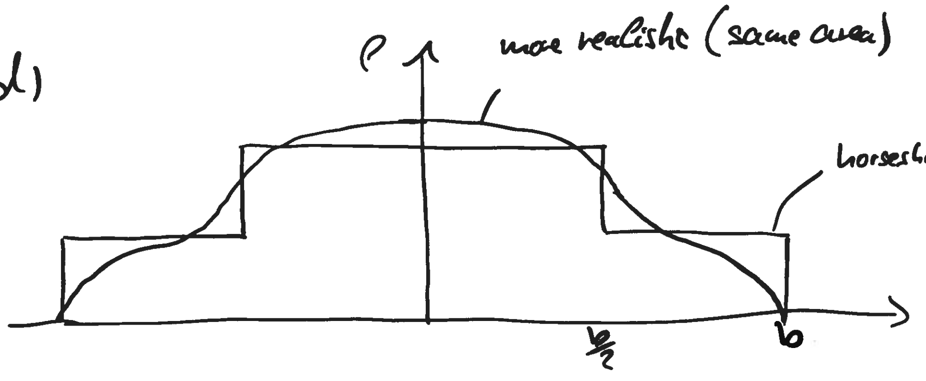
$$w_2 = 2 \cdot \frac{\Gamma}{4\pi \frac{b}{2}}(\cos\beta + 1) + \frac{\Gamma}{4\pi b}(\cos\alpha + \cos\alpha) = \frac{\Gamma}{4\pi b}\left(\frac{8}{\sqrt{5}} + 4 + \frac{2}{\sqrt{5}}\right) = \frac{\Gamma}{\pi b}\left(1 + \frac{\sqrt{5}}{2}\right)$$

$2\sqrt{5}+4$

2.11

$$w = w_1 + w_2 = \frac{\Gamma}{4\pi b}\left(6 + 2\sqrt{2} + 2\sqrt{5}\right) = \frac{\Gamma}{\pi b} 3.32$$

d)



e) The biggest danger area for stall is when $\frac{\ell}{c}$ is max. This is just outboard of the high lift device (where local chord is $\frac{b}{4}$)

f) Increase local chord in danger area,
extend high lift device outboard,
change local wing profile
or use suitable flow control (e.g. VGs, if TE stall).

Q67

For an elliptic wing with elliptic wing loading global and local lift coefficient are equal:
CL=cl=2 pi alpha_e

Hence:

a) $C_\ell = 2\pi\alpha_e = \dfrac{mg}{\frac{1}{2}\rho V^2 \cdot A}$ $\qquad A = \dfrac{\pi b c}{4}$ (ellipse)

$\therefore \alpha_e = \dfrac{4mg}{\pi^2 \rho V^2 bc} = 0.104 \text{ rad} \cong 5.9^0$

We need geometric angle of attack, calculate downwash

$w = \dfrac{\Gamma_0}{2b}$ (elliptic wing)

$mg = L = \frac{\pi}{4}\rho V b \Gamma_0$ (elliptic wing) $\quad \therefore \Gamma_0 = \dfrac{4mg}{\pi \rho V b}$

$w = \dfrac{2mg}{\pi \rho V b^2} = 0.41 \ \frac{m}{s}$

induced angle of attack:

$\alpha_i = \dfrac{w}{V} = 0.02 \cong 1.16^0$

Hence: geometric angle of attack $\alpha = 7.1^0$

b) Total drag: $C_D = C_{D_i} + C_{D_V}$

where $C_{D_i} = \dfrac{C_L^2}{\pi AR}$ for elliptic wing

Here: $AR = \dfrac{b^2}{A} = \dfrac{4b}{\pi c} = 10.2$ and $C_L = \dfrac{8mg}{\rho V^2 \pi bc} = 0.65$

$C_{D_i} = \dfrac{0.65^2}{\pi \cdot 10.2} = 0.013$

Glide slope $\dfrac{C_L}{C_D} = \dfrac{1}{15}$ hence $C_D = 0.043$

Thus: $C_{D_V} = 0.03$

and $D_V = 0.03 \cdot \frac{1}{2}\rho V^2 \cdot \frac{\pi}{4} bc = 0.46 N$

c) $C_{D_V} = $ const. $\quad C_D = 0.0 + \dfrac{C_L^2}{\pi AR}$

$\dfrac{C_L}{C_D} = \dfrac{C_L}{C_{D_V} + \frac{1}{\pi AR} C_L^2}$ $\quad$ easier: $\dfrac{C_D}{C_L} = \dfrac{C_{D_V}}{C_L} + \dfrac{C_L}{\pi AR}$

$\dfrac{\partial\left(\frac{C_D}{C_L}\right)}{\partial C_L} = -\dfrac{C_{DV}}{C_L^2} + \dfrac{1}{\pi AR} = 0 \qquad C_L = \sqrt{\pi AR\, C_V} = 0.98$

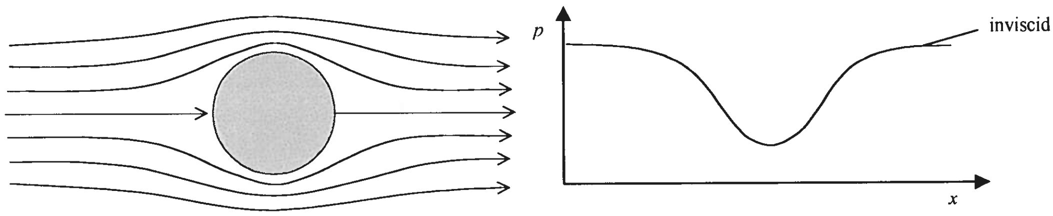
thus: $C_{D_V} = C_{D_i}$ so $\dfrac{C_L}{C_D} = \underline{\underline{16.3}}$

Q8

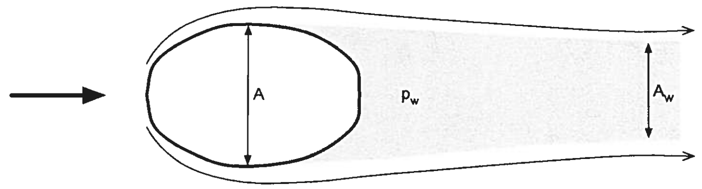
a) Bluff bodies have significantly greater pressure drag than skin friction drag. The pressure drag is caused by large separations at the rear

b) See lecture notes

Inviscid



Viscous



Viscous pressure distribution should show region of low near-constant pressure at back (the base pressure).

c) There is relatively little wetted area – hence low skin friction drag. The average pressure on the front half is not far off free-stream, so there is little contribution to pressure drag from the front. The average pressure across the rear is low – this generates considerable drag.

d) The pressure drag is affected by the average pressure in the separated wake – the base pressure – and the size (relative cross-section) of the separated region.

e) The drag can be reduced by reducing the size of the separation and increasing the pressure in the wake. This can be done by delaying separation (eg if the flow is laminar at the separation point, it can be tripped to become turbulent, or one might introduce vortex generators) or by re-shaping the body to reduce the adverse pressure gradients (= streamlining). One popular solution is part-streamlining by introducing a boat-tail.

f) boat-tailing is the most common technique because the boundary layer is already turbulent at the separation location. (Vortex generators are not used because they do not have a sufficiently strong effect, given the 'bluffness' of a typical car).