Version MPJ/3

EGT2
ENGINEERING TRIPOS PART IIA

Monday 24 April 2023 9.30 to 12.40

Module 3A1

FLUID MECHANICS I

*Answer not more than **five** questions.*

All questions carry the same number of marks.

*The **approximate** percentage of marks allocated to each part of a question is indicated in the right margin.*

*Write your candidate number **not** your name on the cover sheet.*

STATIONERY REQUIREMENTS
Single-sided script paper

SPECIAL REQUIREMENTS TO BE SUPPLIED FOR THIS EXAM
CUED approved calculator allowed
Engineering Data Book
Attachments:

- Incompressible Flow Data Card (2 pages);
- Boundary Layer Theory Data Card (1 page);
- 3A1 Data Sheet for Applications to External Flows (2 pages).

10 minutes reading time is allowed for this paper at the start of the exam.

You may not start to read the questions printed on the subsequent pages of this question paper until instructed to do so.

You may not remove any stationery from the Examination Room.

1 (a) A cylinder with radius a is placed in a flow at speed U in the positive x-direction. The cylinder encloses circulation Γ that induces flow in the anticlockwise θ-direction, where θ is the angle made with the x-axis. Derive an expression for the complex potential for this flow. [20%]

(b) Hence, or otherwise, show that stagnation points are situated at

$$\sin\theta = \frac{\Gamma}{4\pi a U}$$

[20%]

(c) The 2D cylinder in Fig. 1(a) is centred in the complex z-plane at $z = c = c_r + \mathrm{i}c_i$ and passes through the point $z = 1$. Under the transformation $\zeta = z + z^{-1}$, this cylinder maps to the Joukowski aerofoil shown in Fig. 1(b). Sketch the streamlines that would be observed around the Joukowski aerofoil if it were moved leftwards horizontally at speed U through a stationary fluid. [20%]

(d) Derive an expression for the lift of this aerofoil in terms of c_i, ρ, and U, where ρ is the density of the fluid. [40%]

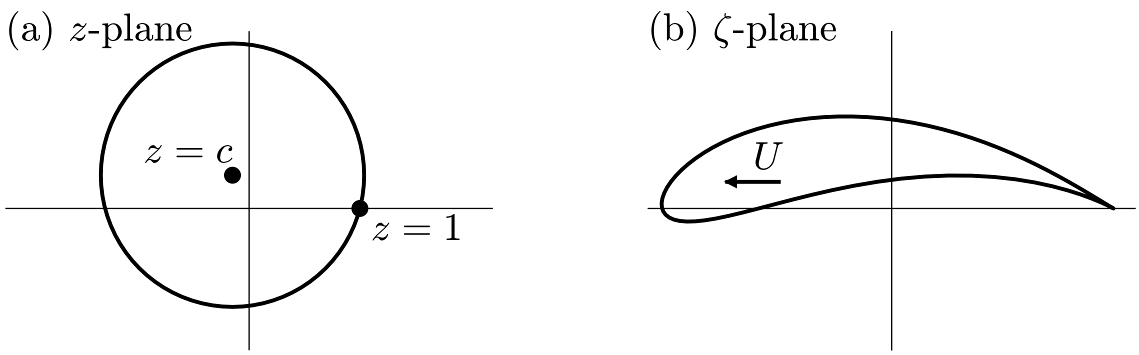

Fig. 1

2 (a) A 3D sphere of radius a moves in the positive x-direction at speed U through a stationary inviscid fluid with density ρ. Show that the velocity potential in the stationary frame is:

$$\phi = -U\frac{a^3}{2r^2}\cos\theta$$

where the origin is located at the instantaneous centre of the sphere, the radial co-ordinate, r, is the distance to this origin, and θ is the angle made with the x-axis. [20%]

(b) If U varies with time, show that the pressure on the surface of the sphere is given by:

$$\frac{p}{\rho} = k - \frac{1}{2}U^2\left(\cos^2\theta + \frac{1}{4}\sin^2\theta\right) + \frac{1}{2}\dot{U}a\cos\theta$$

where k is a constant. [30%]

(c) Stating all your assumptions, calculate the acceleration of a spherical air bubble starting from rest in water in the presence of gravity. [50%]

3 An incompressible fluid between two infinite plates is rotating around an axis perpendicular to the plates, as shown in Fig 2. The plates move apart, creating a steady axisymmetric velocity field (u_r, u_θ, u_z) with $u_z = 2\alpha z$, where z is the distance from the mid-point between the plates and α is a positive constant. The continuity equation in radial polar coordinates is:

$$\frac{1}{r}\frac{\partial(r u_r)}{\partial r} + \frac{1}{r}\frac{\partial u_\theta}{\partial \theta} + \frac{\partial u_z}{\partial z} = 0$$

(a) Derive an expression for the radial velocity, u_r. [20%]

(b) The vorticity equation is

$$\frac{D\boldsymbol{\omega}}{Dt} = (\boldsymbol{\omega} \cdot \nabla)\mathbf{u} + \nu \nabla^2 \boldsymbol{\omega}$$

The vorticity is unidirectional, $\boldsymbol{\omega} = (0, 0, \omega_z)$, and depends only on r. Show that the vorticity equation becomes

$$\alpha r \frac{\mathrm{d}\omega_z}{\mathrm{d}r} + 2\alpha\omega_z + \frac{\nu}{r}\frac{\mathrm{d}}{\mathrm{d}r}\left(r\frac{\mathrm{d}\omega_z}{\mathrm{d}r}\right) = 0$$

Given that the steady solution is $\omega_z = \omega_0 \exp(-\alpha r^2/(2\nu))$, identify each term physically and describe how the competing phenomena in the flow keep the flow steady. [50%]

(c) The plates are abruptly brought to rest. By considering the radial pressure gradient in the flow, describe the flow in the boundary layers if the plates do not rotate. [30%]

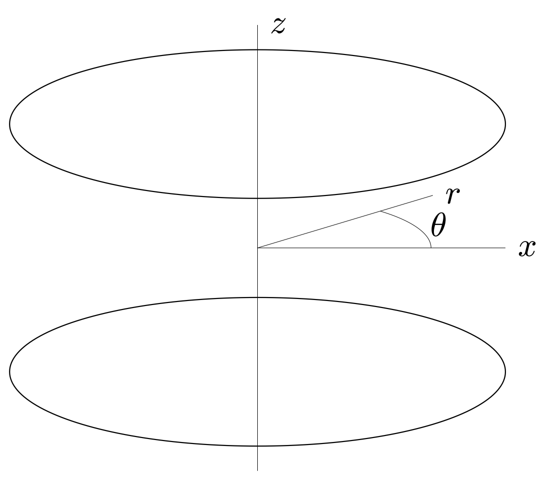

Fig. 2

4 Consider fully developed flow between parallel plates distance $2h$ apart, as sketched in Fig. 3. The fluid has density ρ and kinematic viscosity ν. The reduced pressure gradient, $\mathrm{d}(p/\rho)/\mathrm{d}x = G$, is constant. The mean velocity across the channel is V and the wall shear stress is τ_w.

(a) For a laminar flow, express the velocity profile $u(y)$ in term of G and ν. [20%]

(b) Suppose the flow is turbulent.

(i) Assume that the logarithmic-law correlates the local mean velocity $u(y)$ across the whole channel. Deduce a relation between V and the friction velocity $u^* = \sqrt{\tau_w/\rho}$. [30%]

(ii) The friction factor f is defined such that $V/u^* = (8/f)^{1/2}$. Establish a relation between f and the Reynolds number $\mathrm{Re} = 2hV/\nu$. [20%]

(iii) Express the ratio between the mean velocity, V, and the maximum velocity, u_{max}, in term of f. [20%]

(c) Sketch the velocity profiles of the laminar and turbulent flows, and discuss the main differences between these profiles. [10%]

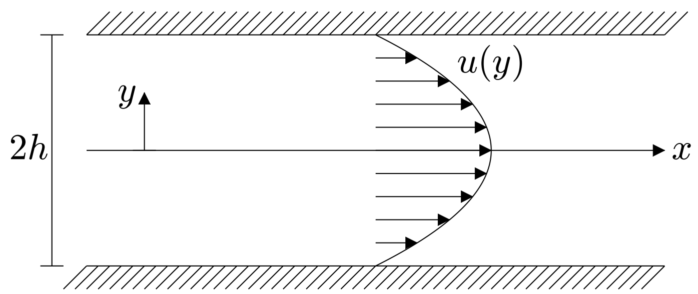

Fig. 3

5 Consider a turbulent boundary layer in a zero pressure gradient flow as sketched in Fig. 4. Let u be the mean velocity parallel to the wall, τ_w the wall shear stress and U the velocity at the edge of the outer layer ($y = \delta$). The friction velocity u^* is defined as $u^* = \sqrt{\tau_w/\rho}$, where ρ is the fluid density.

(a) Sketch the profiles of the laminar and turbulent shear stresses and discuss their main characteristics. [20%]

(b) In a region quite close to the wall, it can be assumed that the local flow has no information about the free-stream velocity, U, and the boundary layer thickness, δ.

(i) Use dimensional analysis to explain why the velocity profiles collapse according to the "law of the wall" scaling of the form

$$\frac{u}{u^*} = f\left(\frac{yu^*}{\nu}\right)$$

where f is some function. [10%]

(ii) Very close to the wall, the turbulent shear stress is negligible. Deduce the law of the wall in this region. What name is given to this region? [20%]

(iii) Further away from the wall, the velocity profile is dominated by the turbulent shear stress. Use dimensional analysis to simplify the law of the wall in this region. [20%]

(c) In Prandtl's mixing-length model, the turbulent shear stress is modelled via the turbulent viscosity $\nu_T(y)$:

$$-\overline{u'v'} = \nu_T \frac{\mathrm{d}\bar{u}}{\mathrm{d}y}$$

and ν_T is expressed as the product of the friction velocity u^* and a lengthscale l_m:

$$\nu_T = u^* l_m$$

where l_m varies linearly with y. Assuming that the turbulent stress is dominant and the shear stress is nearly constant, deduce a law of wall and discuss its relation with (b) (iii). [30%]

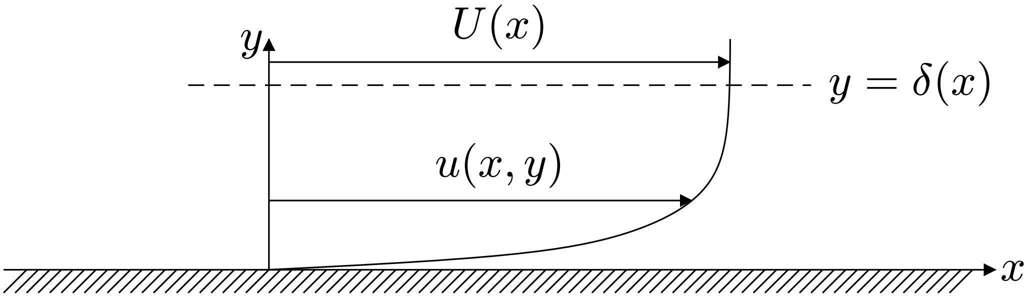

Fig. 4

6 An aircraft has high-lift devices deployed over part of its wing. The effect can be modelled by an extension of the effective chord, resulting in the wing planform shown in Fig. 5. The quarter-chord line is straight along the whole of the wing. The wing is to be modelled by two horseshoe vortices.

(a) Sketch a suitable horse-shoe vortex system. [20%]

(b) Assuming that the local lift coefficient is the same everywhere along the wing and that the aircraft is in steady flight, express the circulation of each horseshoe vortex as a function of the air density, ρ, aircraft weight, W, flight speed, U, and wing semi-span, b. [20%]

(c) Calculate the downwash at the tailplane (marked by P) located a distance b behind the quarter-chord line, as shown in Fig. 5. [20%]

(d) The horseshoe-vortex model assumes a constant local lift coefficient.

(i) Sketch the resulting wing loading (the local lift along the span), together with a more realistic wing-loading distribution. [20%]

(ii) Based on your sketch, where would you expect the wing to stall first? [10%]

(iii) What could be done to alleviate the danger of stall in this area? [10%]

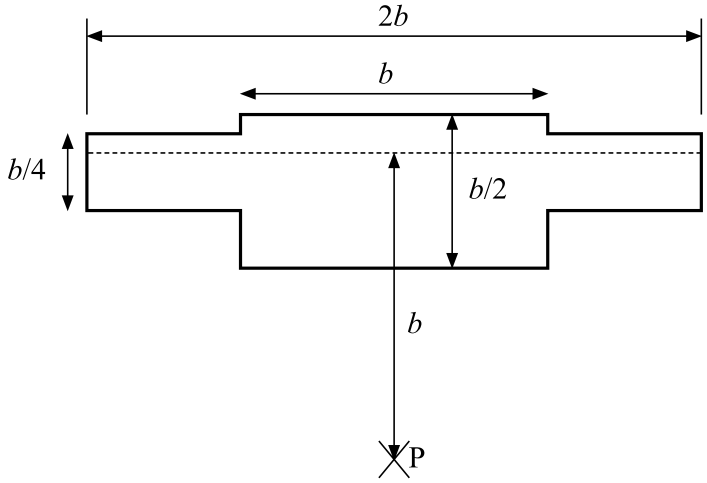

Fig. 5

7 A model gliding aircraft of mass 1 kg is in steady flight at a speed of 20 ms^{-1} in air with density 1.2 kgm^{-3}. It features an elliptic untwisted wing of 0.1 m chord and 0.8 m span. Ignoring any lift or downforce from the fuselage and tailplane, use lifting line theory to determine:

(a) the angle of attack; [30%]

(b) the viscous drag if the glide slope is 1/15; [40%]

(c) the maximum lift-to-drag ratio that can be achieved by the wing if it is assumed that the viscous drag contribution is independent of incidence. [30%]

8 (a) By referring to the main drag mechanisms, briefly explain what is meant by an aerodynamically ‘bluff body’. [10%]

(b) Sketch typical inviscid and viscous streamlines and surface pressures about a simple generic bluff body. [20%]

(c) Use the sketches from (b) to explain why bluff body drag is dominated by one particular mechanism. [25%]

(d) What are the main parameters affecting the drag coefficient of a bluff body? [10%]

(e) What are the options for reducing the drag coefficient of bluff bodies? [15%]

(f) Which measures for bluff body drag reduction are used in passenger cars? [20%]

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1.	(d)	$4\pi\rho U^2 c_i$
2.	(c)	$2g$
4.	(b) (iii)	$\frac{V}{U_{max}} = \left(1 + \frac{1}{\kappa}\left(\frac{f}{8}\right)^{\frac{1}{2}}\right)^{-1}$
5.	(b) (ii)	$\frac{u}{u^\star} = \frac{yu^\star}{\nu}$
	(b) (iii)	$\frac{u}{u^\star} = \frac{1}{\kappa}\ln\left(\frac{yu^\star}{\nu}\right) + B$
6.	(c)	$3.32\frac{\Gamma}{\pi b}$
7.	(a)	7.1°
	(b)	0.46 N
	(c)	16.3