"MASTER"

GRID # 1.

[worked by
WND]

31/1/14 – 1/2/14.

Pages -i- to -xvii-.

3A1 2014

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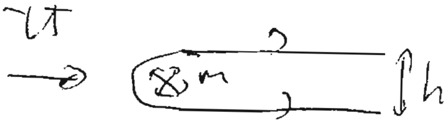
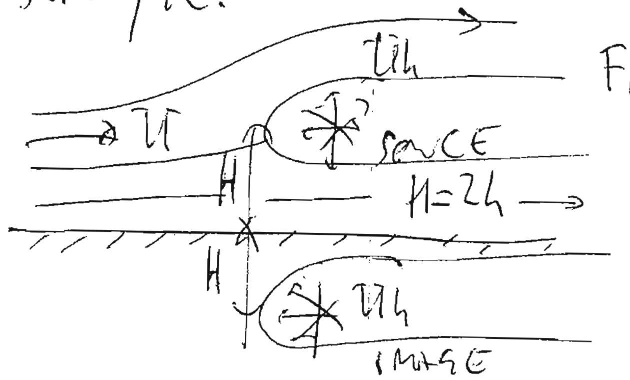
(a) The body is represented by the streamline which separates the source flow and the free-stream flow. Far downstream the vel.



flow rate between the s/lines is $\underline{Uh = m}$, source strength.

(b)



$$F(z) = Uz + \frac{m}{2\pi}\log(z-iH) + \frac{m}{2\pi}\log(z+iH)$$
$$= Uz + \frac{Uh}{2\pi}\log(z^2+H^2)$$

So (i) $u - iv = \frac{dF}{dz} = U\left[1 + \frac{h}{\pi}\frac{z}{z^2+H^2}\right] = 0$ @ stagnation point

$$\therefore z^2 + \frac{h}{\pi}z + H^2 = 0 \Rightarrow z = -\frac{h}{2\pi} \pm \sqrt{\left(\frac{h}{\pi}\right)^2 - 4H^2}\Big/2$$

Now, $H = 2h$ so $z = -\frac{h}{2\pi} \pm \sqrt{\frac{h^2}{\pi^2} - 16h^2}\Big/2$

$$= -\frac{h}{2\pi} \pm ih\sqrt{4 - \frac{1}{4\pi^2}} = -0.16h \pm i\,1.99h$$

$\therefore$ stagnation point is 0.16h upstream of source and 1.99h above the surface

and (ii) the stagnation point streamline is therefore

$$\psi = \text{Im}\left\{Uz + \frac{Uh}{2\pi}\log(z^2+H^2)\right\} \text{ for } z = -0.164 + i1.99h$$
$$= 1.76Uh \text{ (after some algebra ⊛)}$$

Far upstream and downstream $\psi = Uy$ so the streamline starts and finishes at $\Psi = 1.76 h$ ($\Psi/\pi h$)

$\therefore$ the distance between the start & the surface $= \underline{\underline{1.76 h}}$

(*) $\frac{\Psi}{\pi h} = \mathrm{Im}\left\{ \cancel{-0.16} + i1.99 + \frac{1}{2\pi}\log\left(4h^2 + (-0.16h + i1.99h)^2\right)\right\}$

$i1.99 + \frac{1}{2\pi}\log\left(4.026h^2 - 3.9601h^2 - i0.637h^2\right)$

$\frac{1}{2\pi}\log(0.066 - i0.637)$

.004 .402

$\frac{1}{2\pi}\left(e^{i\theta} - 0.451\ldots\right)$

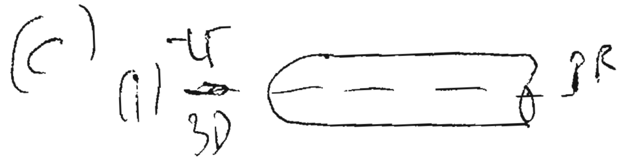
$-84.1^\circ = -1.467$

$\log(a+ib) = \log\sqrt{a^2+b^2}\, e^{i\theta}$ ...

$\theta = \tan^{-1}\frac{b}{a}$

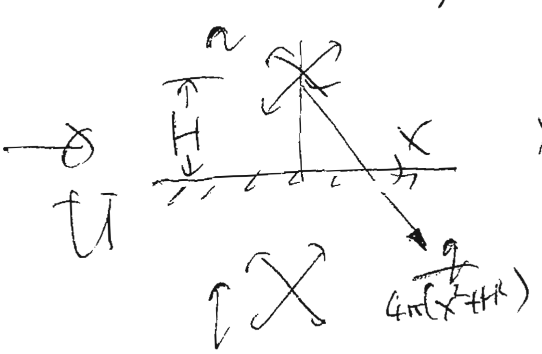
$= \mathrm{Im}\left\{\ldots i1.99 - i\frac{1.467}{2\pi}\ldots\right\} = 1.76$ then ... (*)

0.23

(c) (i) 

As before, volume flow $\underline{\underline{q = \pi R^2 U}}$

(ii) Image systems also in 3D, so:



resolve & add contributions to the x-wise velocity

$u = U + 2\cdot\frac{q}{4\pi(x^2+H^2)}\cdot\frac{x}{\sqrt{x^2+H^2}}$

$\frac{du}{dx} = 0$ when $(x^2+H^2)^{3/2} - x\frac{3}{2}(x^2+H^2)^{1/2}2x = 0$

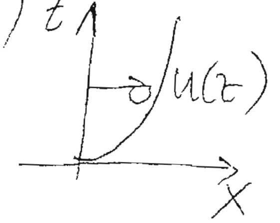
ie $x = \pm H/\sqrt{2}$

Separation is likely in regions where the flow is diffusing, ie. u decreasing in the flow direction, ie. $\underline{\underline{x \le -H/\sqrt{2} \ \& \ x \ge +H/\sqrt{2}}}$

2 (a) $\boxed{\bar{u}\cdot\nabla\bar{\omega}}$ represents convection of vorticity by the flow; with the time derivative $\partial\bar{\omega}/\partial t$ it forms the "derivative following the motion" $D\bar{\omega}/Dt$. (this represents the rate of change of vorticity as seen by an observer moving with the flow).

$\boxed{\bar{\omega}\cdot\nabla\bar{u}}$ represents stretching & tilting of vorticity

$\boxed{\nu\nabla^2\bar{\omega}}$ represents viscous diffusion of vorticity.

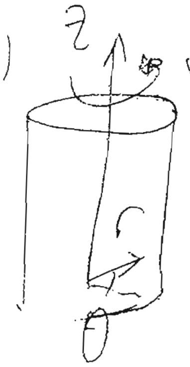
(b) 

$\bar{\omega}=\nabla\times\bar{u}$; in 2D $\omega=\frac{\partial u}{\partial z}-\frac{\partial w}{\partial x}$

(u,v,w)

B'layer assumptions: $w \ll u$; $\frac{\partial}{\partial x} \ll \frac{\partial}{\partial z}$ $\Rightarrow$ $\omega \sim \frac{\partial u}{\partial z}$

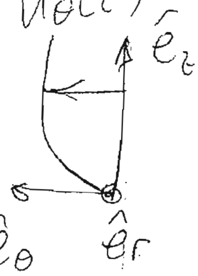
$\perp^{r}$ to streamwise velocity and surface normal.

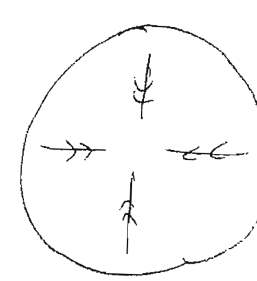
(c) (i) 

in cylindrical polar, $\bar{u}=\Omega r\hat{e}_\theta$

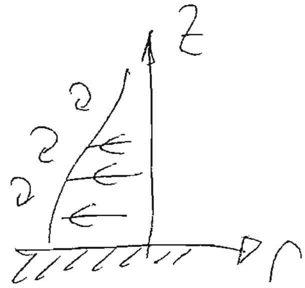
so $\bar{\omega}=\nabla\times\bar{u}=\frac{\partial}{\partial r}(r u_\theta)\frac{\hat{e}_z}{r}=2\Omega\hat{e}_z$

[DATA BOOK] in axial dir^n

(ii) base b'layer 

hence vorticity vector is in radial dir^n (ie. inwards) $\Rightarrow$ 

(iii) Velocity profile of radial base flow (ignoring no-slip condition at wall) from debris drift:

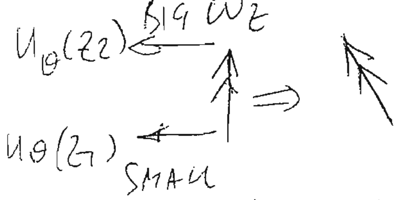


Direction of associated vorticity indicated by ⊘

⇒ this component is in the <u>azimuthal</u>, θ direction.

(iv) when container is stopped vorticity is initially all in axial & radial components. Hence the azimuthal component must have arisen due to "<u>tilting</u>".

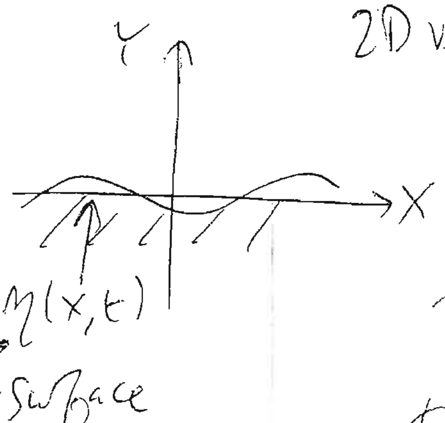
<u>Mechanism 1</u>: effect of base bulge profile on axial velocity component:



(vel. profile "tilts" the vorticity vector)

<u>Mechanism 2</u>: as friction at the sides takes effect the azimuthal velocity will lose its proportionality to r and this will also cause tilting.

3 (a) 2D velocity potential ϕ ...



$$\frac{d}{dx}\left[e^{i(\omega t-kx)}e^{\pm kY}\right] = -ik\,e^{i(\omega t-kx)}e^{\pm kY}$$

$$\therefore \frac{d^2}{dx^2}[\sim] = -k^2 e^{i(\omega t-kx)}e^{\pm kY}$$

$$\& \quad \frac{\partial^2}{\partial Y^2}[\sim] = +k^2 e^{i(\omega t-kx)}e^{\pm kY}$$

$\therefore \frac{d^2}{dx^2}[\sim] + \frac{\partial^2}{\partial Y^2}[\sim] = 0$, satisfying Laplace.

(b) As $y \to -\infty$ $e^{+kY} \to 0$, physically realistic

$e^{-kY} \to \infty$, not ——"——.

(c) Given $\eta(x,t) = \mathrm{Re}(y_0 e^{i(\omega t-kx)})$ then $\frac{d\eta}{dt} = i\omega y_0 e^{i(\omega t-kx)}$

← surface

This is equal to the Y-wise velocity component at $Y=0$, which is $\left.\frac{\partial\phi}{\partial Y}\right|_{Y=0} = \pm Ak\,e^{i(\omega t-kx)}e^{0}$

$(\bar{u} = \nabla\phi)$

where the potential matching the flow is $\phi = A e^{i(\omega t-kx)} e^{\pm kY}$

↑ arbitrary constant.

Hence, $A = i\omega y_0/k$

Therefore the velocity potential which matches the water boundary condition, η, is:

$$\phi = \frac{i\omega y_0}{k} e^{i(\omega t-kx)} e^{+kY}$$

(d) $u = \frac{\partial \phi}{\partial x} = \frac{i\omega y_0}{k} \cdot -ik\, e^{i(\omega t - kx)} \cdot e^{+ky} = \omega y_0 e^{i(-)} e^{+ky}$

$v = \frac{\partial \phi}{\partial y} = \frac{i\omega y_0}{k} \cdot k\, e^{i(-)} e^{ky} = i\omega y_0 e^{i(\cdot\cdot)} e^{+ky}$

since $e^{i\theta} = \cos\theta + i\sin\theta$

Hence $u \sim Re\{e^{i(\omega t - kx)} e^{ky}\} \sim e^{ky}\cos(\omega t - kx)$

& $v \sim Re\{\underline{i}\, e^{i} \text{———}\} \sim -e^{ky}\sin(\omega t - kx)$

Hence the fluid trajectories are elliptical ("sin" & "cos") with amplitude reducing with depth ("e^{ky}", $y<0$).

(e) Unsteady Bernoulli at surface of water:

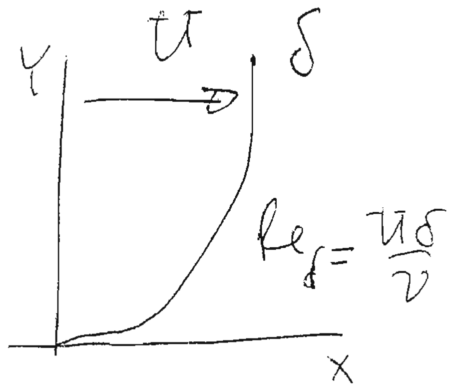
$$\cancel{\frac{1}{\rho} p_a} + \underbrace{\tfrac{1}{2}(u^2+v^2)}_{O(\omega y_0)^2 \text{ neglect}} + \underbrace{g\eta}_{g y_0 e^{i(-)} e^{ky}} + \underbrace{\frac{\partial \phi}{\partial t}\Big|_{y=0}}_{-\frac{\omega^2}{k} y_0 e^{i(\cdot\cdot)} e^{ky}} = \cancel{\frac{1}{\rho} p_a}$$

$y=0$ — y_0 small

$\therefore\ g y_0 = \omega^2 y_0 / k$

$\therefore$ wave speed, $\omega = \sqrt{g/k}$.

(4) (a)



Given: $\frac{u}{u^*} = \frac{1}{k}\log\left(\frac{u^* y}{\nu}\right) + B$

where $u^* = \sqrt{\tau_w/\rho}$

$k = 0.41$, $B = 5$

von-Karman constant.

Hence: $\left(\frac{u^2}{\tau_w/\rho}\right)^{1/2} = \frac{1}{k}\log\left(\frac{uy}{\nu}\cdot\frac{\sqrt{\tau_w/\rho}}{u}\right) + B$

$C_f = \frac{\tau_w}{\frac{1}{2}\rho U^2}$ and $u = U$ @ $y = \delta$, the b'layer edge

$$\therefore \sqrt{\frac{2}{C_f}} = \frac{1}{k}\log\left(Re_\delta\sqrt{\frac{C_f}{2}}\right) + B$$

$\frac{1}{8}\cdot\frac{72}{7}$

(b) $\frac{u}{U} = \left(\frac{y}{\delta}\right)^{1/7}$ $\delta^* = \int_0^\delta\left(1 - \frac{u}{U}\right)dy = \left[y - \frac{7}{8}\frac{1}{\delta^{1/7}}y^{8/7}\right]_0^\delta = \frac{1}{8}\delta$.

$$\theta = \int_0^\delta\left(1 - \frac{u}{U}\right)\frac{u}{U}dy = \left[\frac{7}{8}\frac{y^{8/7}}{\delta^{1/7}} - \frac{7}{9}\frac{y^{9/7}}{\delta^{2/7}}\right]_0^\delta = \frac{7}{72}\delta.$$

So $H = \delta^*/\theta = 1.29$.

(c) Given $C_f = 0.02 Re_\delta^{-1/6}$ then from the momentum integral equation [DATA SHEET]

$$\frac{d\theta}{dx} = \frac{C_f}{2}$$ [zero pressure gradient!!!]

$$\therefore \quad 0.01\left(\frac{U\delta}{\nu}\right)^{-1/6} = \frac{7}{72}\frac{d\delta}{dx}$$

Integrate: $\int_0^x 0.01\left(\frac{U}{\nu}\right)^{-1/6} dx = \int_0^\delta \frac{7}{72}\,\delta^{1/6}\, d\delta$

$$\therefore 0.01\left(\frac{U}{\nu}\right)^{-1/6} X = \cancel{\frac{7}{72}} \cdot \frac{6}{\cancel{7}} \cdot \delta^{7/6}$$

$$\therefore 0.01 \cdot \frac{72}{6}\left(\frac{Ux}{\nu}\right)^{-1/6} x^{7/6} = \delta^{7/6}$$

$$\therefore \frac{\delta}{x} = 0.16\left(\frac{Ux}{\nu}\right)^{-1/7}.$$

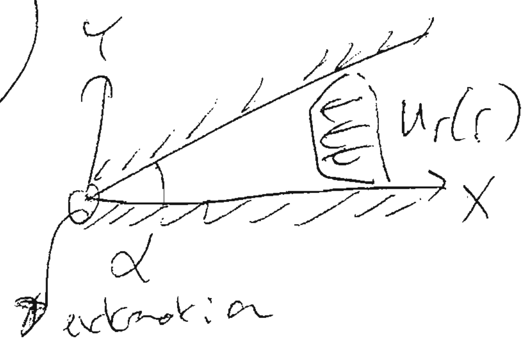
Whence: $\frac{\delta^*}{x} = 0.02\, Re_x^{-1/7}$ & $\frac{\theta}{x} = 0.016\, Re_x^{-1/7}$ $\Big\}$ $H = 1.25$.

Now, $\frac{C_f}{2} = \frac{d\theta}{dx} = \frac{d}{dx}\left(0.016\, x\left(\frac{Ux}{\nu}\right)^{-1/7}\right)$

$$= 0.016\left(\frac{U}{\nu}\right)^{-1/7} \underbrace{\frac{d}{dx}\left(x^{6/7}\right)}_{\frac{6}{7}x^{-1/7}}$$

Hence, $\tau_w = \frac{1}{2}\rho U^2 \cdot 2 \cdot 0.016\, \frac{6}{7}\left(\frac{Ux}{\nu}\right)^{-1/7}$

$$\therefore \tau_w/\rho U^2 = 0.014\,(Re_x)^{-1/7}$$

(5) 

inward flow..

(a) Mass conservation: $u_r r$ = constant (2D)

$\therefore$ $\bar{u} = -\frac{Q}{r}\hat{e}_r$ where Q = constant > 0

(b) Consider the boundary $y=0$, the "external flow" is therefore: $\underline{U(x) = -Q/x}$.

Given $\psi = F(x) f(\eta)$ where $\eta = y/g(x)$, a similarity solution, then,

$$u = \frac{\partial \psi}{\partial y} = F(x) \underbrace{f'(\eta)}_{df/d\eta} \cdot \underbrace{\frac{1}{g(x)}}_{d\eta/dy}$$

$\rightarrow U(x)$ as $\eta \rightarrow \infty$ (i.e. out of b'layer);

$$\therefore U(x) = \frac{F(x)}{g(x)} \cdot \underbrace{f'(\infty)}_{\text{choose} = 1}$$

$$\therefore F(x) = g(x) U(x) = -g(x) Q/X.$$

(c) The b'layer equation is:

$$u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} = U\frac{dU}{dx} + \nu\frac{\partial^2 u}{\partial y^2}$$

$$u = \frac{\partial\psi}{\partial y} = \frac{\partial}{\partial y}[g(x)\overbrace{U(x)}^{-Q/x}f(\underbrace{\eta}_{y/g(x)})] = U(x)\,\cancel{g(x)}\,\frac{df}{d\eta}\cdot\underbrace{\frac{d\eta}{dy}}_{1/g(x)} = U(x)f'$$

$$v = -\frac{\partial\psi}{\partial x} = -g'Uf - gU'f + gUf'\,\frac{g'y}{g}$$

$$\frac{\partial u}{\partial x} = U'f' - Uf''\,yg'/g$$

$$\frac{\partial u}{\partial y} = Uf''/g \quad ; \quad \frac{\partial^2 u}{\partial y^2} = Uf'''/g^2$$

(d) Substitute into the b'layer equation (c)

$$\underbrace{Uf'(U'f' - Uf''yg'/g)}_{u\frac{\partial u}{\partial x}} + \underbrace{(-g'Uf - gU'f + gUf'\frac{g'y}{g})Uf''/g}_{v\frac{\partial u}{\partial y}}$$

$$= \underbrace{UU'}_{-\frac{1}{\rho}\frac{dp}{dx}} + \underbrace{\nu Uf'''/g^2}_{\nu\frac{\partial^2 u}{\partial y^2}}$$

$$UU'f'^2 - \cancel{U^2f'f''yg'/g} - U^2ff''g'/g - UU'ff'' + \cancel{U^2f'f''g'y/g}$$

$$= UU' + \nu Uf'''/g^2$$

$\div\, UU'$ $$f'^2 - ff''\left(1 + \frac{U}{U'}\cdot\frac{g'}{g}\right) = 1 + \nu f'''/U'g^2$$

from (b) $U = -Q/x$ so $U' = +Q/x^2$

$$\therefore\quad f'^2 - ff'(1 - xg'/g) = 1 + \nu\frac{x^2}{Q}\cdot\frac{1}{g^2}f'''$$

(e) The following conditions must therefore be met for a "similarity" solution to exist:

$$\left.\begin{array}{l} (1 - x g'/g) = \text{const} \\ \dfrac{\nu x^2}{Q g^2} = \text{const} \end{array}\right\} \text{eg. } g \sim x$$

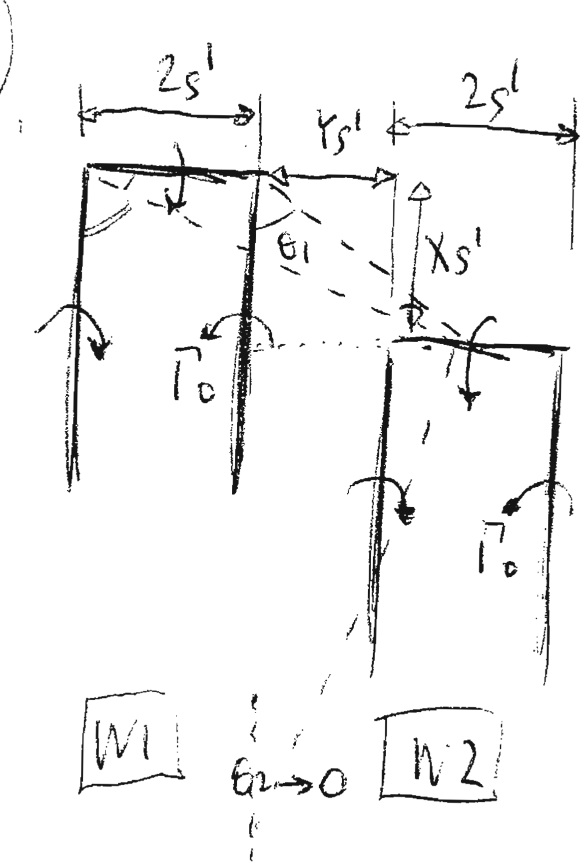
So, for convenience choose $g = x\sqrt{\frac{\nu}{Q}}$ so that $(1 - x g'/g) = 0$ and $\frac{\nu x^2}{Q g^2} = 1$!!!

Hence, the similarity equation to be solved is:

$$\underline{f'^2 = 1 + f'''}$$

with boundary conditions: $f(0) = f'(0) = 0$; $f(\infty) =$

⑥



Lumped parameter model, effective wing span (for elliptical loading) $s' = \frac{\pi}{4} s$

The downwash, w_d, at the centre of the span of W2:

$$w_d = \underbrace{\frac{\Gamma_0}{4s'}}_{\text{its own (elliptic) downwash, W2}} - \frac{\Gamma_0}{4\pi s'(Y+1)}\left[\frac{X}{\sqrt{X^2+(Y+1)^2}} + 1\right]$$

cos θ

$$+ \frac{\Gamma_0}{4\pi s'(Y+3)}\left[\frac{X}{\sqrt{X^2+(Y+3)^2}} + 1\right]$$

$$+ \frac{\Gamma_0}{4\pi s' X}\left[\frac{Y+2}{\sqrt{(Y+2)^2+X^2}} - \underbrace{\frac{Y+1}{\sqrt{(Y+1)^2+X^2}}}_{\cos(90^\circ+\theta_1) = -\sin\theta_1}\right]$$

$$u = \frac{\Gamma}{4\pi d}(\cos\theta_1 + \cos\theta_2)$$

Biot–Savart

Induced drag, $D_i \sim \rho \Gamma_0 w_d 2s'$

$$= \rho\Gamma_0 2\cancel{s'}\frac{\Gamma_0}{4\cancel{s'}}\left[1 - \frac{1}{\pi(Y+1)}\left(1+\frac{X}{\sqrt{X^2+(Y+1)^2}}\right) + \frac{1}{\pi(Y+3)}\left(1+\frac{X}{\sqrt{X^2+(Y+3)^2}}\right) + \frac{1}{\pi X}(\cdots)\right]$$

$$D_i = \underbrace{\tfrac{1}{2}\rho\Gamma_0^2}_{\text{"elliptic" drag}}\left[1 - \frac{1}{\pi}\underbrace{\left\{\frac{1+\frac{X}{\sqrt{X^2+(Y+1)^2}}}{Y+1} - \frac{1+\frac{X}{\sqrt{X^2+(Y+3)^2}}}{Y+3} - \frac{\frac{Y+2}{\sqrt{(Y+2)^2+X^2}} - \frac{Y+1}{\sqrt{(Y+1)^2+X^2}}}{X}\right\}}_{\text{excess/reduced } \Delta D_i}\right]$$

Hence: $\left.\begin{matrix} Y=1 \\ X=1 \end{matrix}\right\}$ $\Delta D_i = \dfrac{1}{\pi}\left\{ \dfrac{1+\frac{1}{\sqrt{5}}}{2} - \dfrac{1+\frac{1}{\sqrt{17}}}{4} - \dfrac{\frac{3}{\sqrt{10}} - \frac{2}{\sqrt{5}}}{1} \right\}$

$=$ 11% drag reduction

$\left.\begin{matrix} Y=1 \\ X=3 \end{matrix}\right\}$ $\Delta D_i = \dfrac{1}{\pi}\left\{ \dfrac{1+\frac{3}{\sqrt{13}}}{2} - \dfrac{1+\frac{3}{\sqrt{25}}}{4} - \dfrac{\frac{3}{\sqrt{18}} - \frac{2}{\sqrt{13}}}{3} \right\}$

$=$ 15% drag reduction

$\left.\begin{matrix} Y=3 \\ X=3 \end{matrix}\right\}$ $\Delta D_i = \dfrac{1}{\pi}\left\{ \dfrac{1+\frac{3}{\sqrt{25}}}{4} - \dfrac{1+\frac{3}{\sqrt{45}}}{6} - \dfrac{\frac{5}{\sqrt{34}} - \frac{4}{\sqrt{25}}}{3} \right\}$

$=$ 4% drag reduction

Therefore there would appear to be worthwhile drag reductions — but only by flying close together!!! Also there would appear to be an optimum X & Y for best result.

Don't try this at home ...

7

(a)

: thickness/chord ~ small

: α ~ small; $\sin\alpha \simeq \alpha$; $\cos\alpha \simeq$

velocity perturbations : $u', v' \ll U$

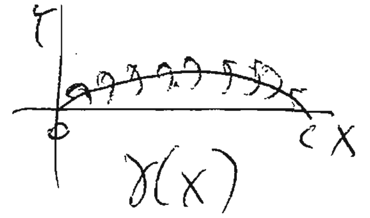
hence: $u = U\cos\alpha + u' \simeq U + u'$
$v = U\sin\alpha + v' \simeq U\alpha + v'$

Bernoulli: $p_\infty + \frac{1}{2}\rho U^2 = p + \frac{1}{2}\rho\left[(U+u')^2 + (U\alpha + v')^2\right]$

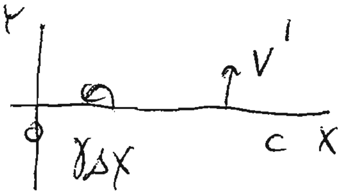
$U^2 + 2Uu' + \cancel{u'^2} \quad \cancel{U^2\alpha^2} + 2U\alpha v' + \cancel{v'^2}$

$\therefore \; C_p = \dfrac{p - p_\infty}{\frac{1}{2}\rho U^2} = -\dfrac{2u'}{U}$

(b)

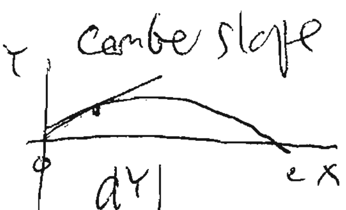


Camber represented by a vortex distribution, γ, circulation/length.



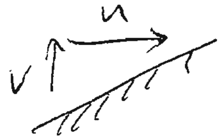
hence, $v'(x) = \displaystyle\int_0^c \frac{\gamma(X)\,dX}{2\pi(X - x)}$

(which will need the Glauert integral to evaluate in practice)



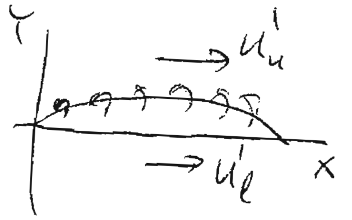
$\left.\dfrac{dY}{dX}\right|_c$

physically we match the "camber slope" (so airfoil a streamline)



$\dfrac{v}{u} = \dfrac{\cancel{U\alpha} + v'}{U + \cancel{u'}} \quad \therefore \; \dfrac{v'}{U} = \left.\dfrac{dY}{dX}\right|_c$

the "Incidence solution)

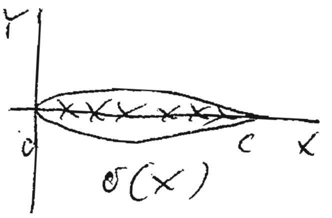


here the local airfoil loading

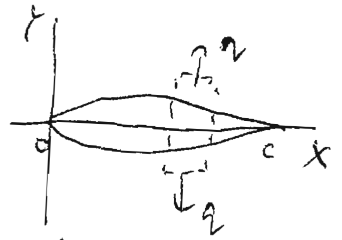
$$\Delta C_p = \frac{2}{U}(u_u' - u_l') = \frac{2\gamma}{U}$$

velocity "jump"

(c)

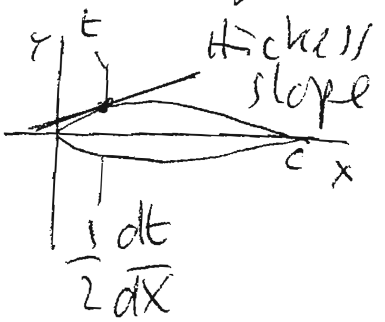


<u>thickness</u> represented by a source distribution, σ, source/length



volume flow for an element Δx is

$$2q\Delta x = \sigma \Delta x \quad \text{so} \quad v' = \frac{1}{2}\sigma$$



physically we match the local streamline to the "thickness slope"

$$\frac{v'}{U} = \frac{1}{2}\frac{dt}{dx} \quad \text{so} \quad \sigma(x) = \frac{dt}{dX}\Big|_{t=0}$$

hence the local u-velocity perturbation is

$$u'(X) = \int_0^c \frac{\sigma(x)\,dx}{2\pi(X-x)}$$

and the local pressure coefficient

$$C_p = -\frac{2u'}{U}.$$

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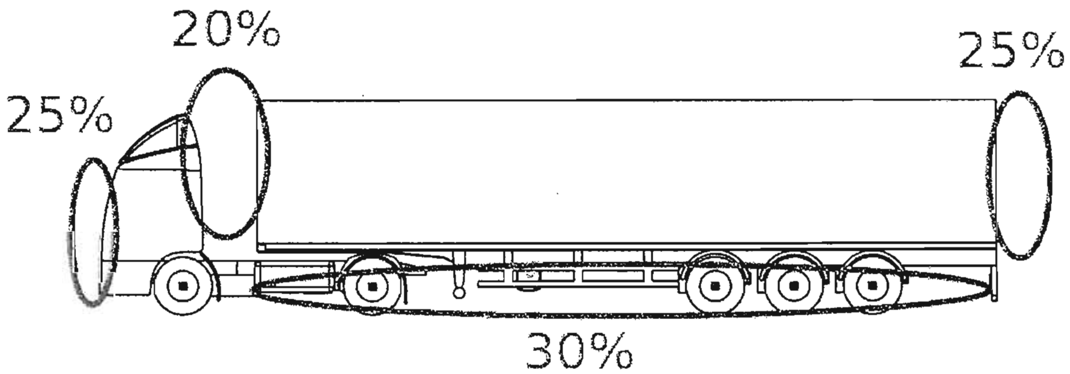
XVI

Answers

a) The wind-averaged drag coefficient is a method whereby the effect of cross-wind can be incorporated in a measurement of vehicle drag. Typically, drag measurements include various yaw angles to produce a 'wind- averaged' drag coefficient, which is a weighted average of the individual results. Ideally, this includes measurements at all possible angles of relative wind-speed, however, in practice it has been shown that a yaw angle of 5° is reasonably representative of most circumstances.
The effect of cross-wind on drag is of particular significance for road haulage vehicles because they drive at lower speeds (thus the relative speed to wind-induced cross-wind is significant) and because they are quite tall and bluff, thus more susceptible to cross-wind.
In the UK there is the added problem that many trunk roads (used by long-haul road transport) run north-south whereas the prevailing wind direction is mainly east-west. Thus cross-wind is statistically more significant here than in other countries.

b) (from the notes). Typically the largest drag contributions are
i: underbody
ii: cab/tractor unit
iii: rear /wake
iv: cab-gap (the gap between the tractor unit and the trailer)



c) following the nomenclature from above:
i: front spoilers and side-skirts are used to reduce the flow underneath the vehicle
ii: the tractor unit is carefully streamlined, featuring a variety of front spoilers, turning vanes and rounded corners
iii: currently there are no common treatments at the rear
iv: It is difficult to reduce the cab gap (turning capability and access) but deflectors on the roof of the tractor can reduce this drag contribution

d) The most successful drag reduction strategy on road cars is boat-tailing to reduce the size of the bluff tail and the associated wake. In road haulage vehicles this is pretty much impossible because doing so would reduce load capacity and hinder access for loading/unloading. Also, retro-fit boat-tails reduce the overall length of the vehicle which is illegal.

END