Version WND/1.9

EGT2
ENGINEERING TRIPOS PART IIA

Friday 25 April 2014 9.30 to 12.30

Module 3A1

FLUID MECHANICS I

*Answer not more than **five** questions.*

All questions carry the same number of marks.

*The **approximate** percentage of marks allocated to each part of a question is indicated in the right margin.*

*Write your candidate number **not** your name on the cover sheet.*

STATIONERY REQUIREMENTS
Single-sided script paper

SPECIAL REQUIREMENTS TO BE SUPPLIED FOR THIS EXAM
CUED approved calculator allowed
Attachment: 3A1 Data Sheet for Applications to External Flows (2 pages);
Boundary Layer Data Card (1 page);
Incompressible Flow Data Card (2 pages).
Engineering Data Book

You may not start to read the questions printed on the subsequent pages of this question paper until instructed to do so.

1 (a) Two-dimensional flow past the semi-infinite strut shown in Fig. 1 is to be represented by superposition of the free stream, U, and a line source of strength m. Find m in terms of U and h.

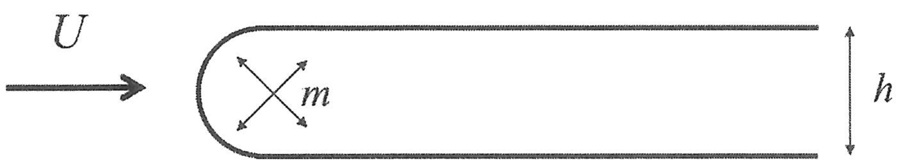


Fig. 1

[20%]

(b) The strut is now placed near a plane surface which is parallel to the oncoming flow. If the image source is at a distance $2h$ from the surface, find (in terms of h):

(i) the stagnation point location;

(ii) the distance between the strut and the surface. [40%]

(c) A long cylindrical shape with a streamlined nose has its axis aligned with an oncoming flow of speed U. Its effect is to be represented by a point source of strength q.

(i) If the cylinder radius is R, what is q ?

(ii) The body is now placed near a plane surface which is parallel to the oncoming flow. The image source is at a distance H from the surface, on which a boundary layer forms. Find the flow speed outside the boundary layer on the streamwise line that passes directly under the source, and identify the regions where separation might occur. (You may assume the boundary-layer thickness to be negligible.) [40%]

2 (a) The equation governing vorticity, $\overline{\omega}$, in a fluid with kinematic viscosity ν, is

$$\frac{\partial \overline{\omega}}{\partial t} + \overline{u} \, . \, \nabla \overline{\omega} = \overline{\omega} \, . \, \nabla \overline{u} + \nu \nabla^2 \overline{\omega}$$

where $\overline{u}$ is the fluid velocity vector. State the physical meanings of the component terms in the above equation. [20%]

(b) Explain how the classical boundary-layer approximations can be applied to simplify the expression for vorticity in a two-dimensional flow. What is the direction of the vorticity vector, relative to the streamwise direction and the surface normal? [40%]

(c) An open-topped cylindrical vessel filled with fluid is spun steadily about its axis until the contents are in a state of rigid-body rotation. The vessel is then stopped.

(i) What is the direction of the vorticity vectors in the rigid-body fluid motion?

(ii) Indicate, using sketches where appropriate, the direction of the vorticity vectors in the boundary layer that starts to form on the base of the container once it is stopped.

(iii) It is subsequently observed that debris on the base of the container drifts radially inwards, implying corresponding fluid motion in that region. What is the direction of the vorticity component associated with this motion? (You can assume that the no-slip boundary condition has not yet had a significant effect on the radial flow.)

(iv) Discuss qualitatively how the vorticity component identified in (iii) arises. [40%]

3 An oceanographer analyses wave motion shown in Fig. 2 by assuming that the flow is both incompressible and irrotational, and that perturbations associated with the waves are small. The mean position of the ocean surface is at y = 0, and the water, whose depth is effectively infinite, occupies $y < 0$.

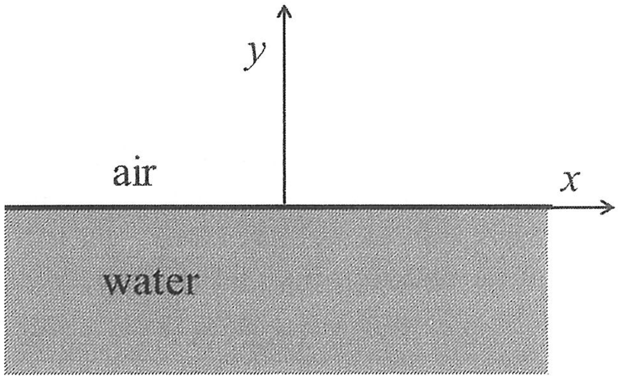


Fig. 2

(a) Show that the two-dimensional velocity potentials given by

$$\phi = \mathrm{Re}\{e^{i(\omega t - kx)}e^{ky}\} \quad \text{and} \quad \phi = \mathrm{Re}\{e^{i(\omega t - kx)}e^{-ky}\}$$

with ω and k positive real, are valid solutions of Laplace's equation. [20%]

(b) Identify which of the two solutions in (a) is the physically relevant one. Justify your answer. [20%]

(c) The surface displacement in the y direction is given by

$$\eta(x,t) = \mathrm{Re}\{y_0 e^{i(\omega t - kx)}\}$$

where y_0 is a constant. Find the velocity potential describing this fluid motion. (You may assume that $\partial \eta / \partial t$ is equal to the vertical fluid velocity component at $y = 0$.) [20%]

(d) Find expressions for the horizontal and vertical components of velocity in the water. Comment qualitatively on the form of the trajectories of the fluid particles in the wave motion. [20%]

(e) The pressure at the water surface must be constant, at the atmospheric value. Use this to find an expression for the wave speed. (Note: the small-perturbation assumption implies that $k y_0 \ll 1$, and that $\phi|_{y=\eta}$ can be approximated as $\phi|_{y=0}$.) [20%]

4 Consider a turbulent flow of an incompressible fluid of kinematic viscosity ν past a flat plate. The boundary layer thickness is δ. At the outer edge of the boundary layer, $y=\delta$ and velocity $u=U$.

(a) Assume that the logarithmic law

$$\frac{u}{u^*} = \frac{1}{k}\ln\left(\frac{u^* y}{\nu}\right) + B$$

holds all the way across the boundary layer with $k = 0.41$ and $B = 5.0$. Here u^* is the friction velocity. Deduce a skin friction law for this flow relating the local skin friction coefficient C'_f and the Reynolds number $Re_\delta = U\delta/\nu$.

[30%]

(b) To avoid the algebra of the logarithmic law, assume now that the boundary layer velocity profile is represented by $u/U = (y/\delta)^{1/7}$ for $\eta = y/\delta \leq 1$ and $u = U$ for $\eta > 1$. Express the displacement thickness δ^*, the momentum thickness θ and the shape factor H in terms of δ. [30%]

(c) Assume further that the skin friction coefficient agrees with the experimentally determined formula:

$$C'_f = 0.02\, Re_\delta^{-1/6}$$

Given also that $\delta=0$ at $x=0$, where x is the distance from the leading edge, and that the laminar boundary layer is over a negligible portion of the plate, determine the boundary layer thicknesses δ, δ^*, θ and the wall shear stress τ_w, as a function of x. [40%]

(TURN OVER

5 Consider a two-dimensional high Reynolds number flow between two plane walls, $y = 0$ and $y = x \tan\alpha$, there being a narrow slit at the origin through which fluid is extracted. We assume that the flow divides into an essentially inviscid mainstream with thin viscous boundary layers on the walls (see Fig. 3).

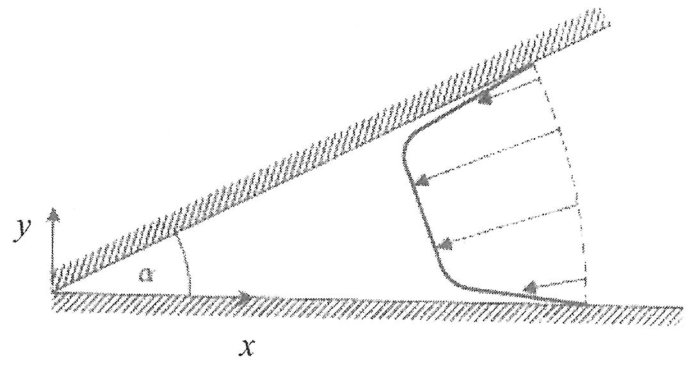


Fig. 3

(a) Assume that the mainstream flow is purely radial, i.e., $\boldsymbol{u} = u_r(r)\boldsymbol{e}_r$. Show that the inviscid equations of motion then demand that

$$\boldsymbol{u} = -\frac{Q}{r}\,\boldsymbol{e}_r$$

where Q is a positive constant. [10%]

(b) On the boundary $y = 0$, the external flow is $U(x) = -Q/x$. Consider a similarity solution of the form

$$\Psi = F(x)f(\eta), \quad \eta = y/g(x)$$

to the boundary layer equations, where Ψ denotes the stream function. Show that the mainstream boundary condition demands that $F(x) = -cQg(x)/x$, where c is a constant, and then, for convenience, choose c to be 1. [20%]

(c) Write down the boundary layer equation and calculate expressions for u, v, $\partial u/\partial x$, $\partial u/\partial y$ and $\partial^2 u/\partial y^2$. [20%]

(d) Substitute the above expressions into the boundary layer equations to deduce a differential equation relating f and g. [20%]

(e) Find the necessary conditions for the existence of a similarity solution and hence deduce the differential equation for f. What are the boundary conditions for f? [30%]

6 Figure 4 represents the horseshoe vortex structures, W1 and W2, of two elliptically loaded airplanes flying in formation. Each airplane has wing span $2s$ and is separated and offset by αs and βs as shown.

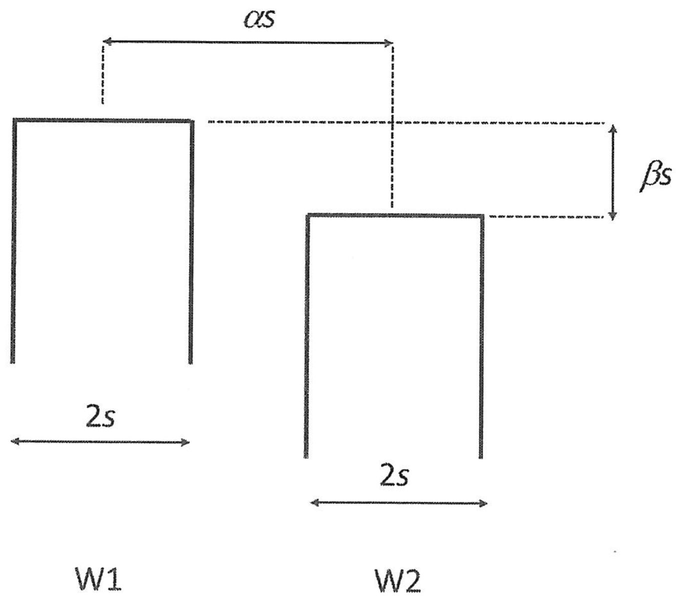


Fig. 4

(a) Using simple 3D lumped parameter modelling estimate the change in the downwash on W2 due to the presence of W1 [40%]

(b) Estimate the change in induced drag on W2 due to the presence of W1. [40%]

(c) Comment on the result. [20%]

7 Consider classical 2D thin aerofoil theory.

(a) The two velocity components are expressed in general using

$$u = U\cos\alpha + u' \ ; v = U\sin\alpha + v'$$

where U is the free stream velocity, u' and v' perturbation velocities and α the angle of attack. Show how these expressions are modified for the case of small perturbations and hence derive the following expression for the local pressure coefficient:

$$c_p = -2\frac{u'}{U}$$

State clearly your approximations. [40%]

(b) Describe the physical basis used for modelling *camber* making clear, with appropriate equations, how the camber slope is used, where the boundary conditions come from and how the local aerofoil loading, Δc_p, is derived. [30%]

(c) Describe the physical basis used for modelling *thickness* making clear, with appropriate equations, how the thickness slope is used, where the boundary conditions come from and how the local aerofoil pressure coefficient, c_p, is derived. [30%]

8 (a) What is the 'wind-averaged drag coefficient' and why is this of particular significance for long-haul heavy goods vehicles operating in the UK? [25%]

(b) On a typical heavy goods configuration, consisting of a towing vehicle and a trailer, identify the 3 or 4 main sources of aerodynamic drag. [30%]

(c) Wherever possible, give an example of how aerodynamic drag is minimised in modern haulage vehicle design for each of the problem areas identified in (b). [25%]

(d) Give an example of an aerodynamic drag reduction strategy that is highly successful in passenger cars but hardly used in heavy goods vehicles. Give reasons for this discrepancy. [20%]

END of PAPER

3A1 Data Sheet for Applications to External Flows

Aerodynamic Coefficients

For a flow with free-stream density, ρ, velocity U and pressure p_∞:

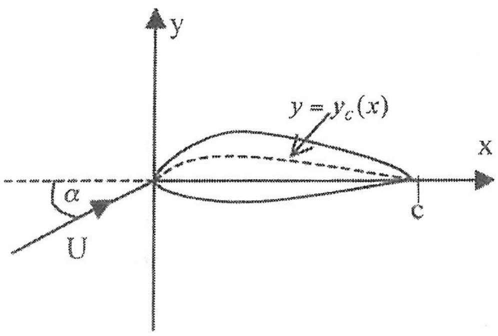
Pressure coefficient: $$c_p = \frac{p - p_\infty}{\frac{1}{2}\rho U^2}$$

Section lift and drag coefficients: $$c_l = \frac{\text{lift } (N/m)}{\frac{1}{2}\rho U^2 c}, \quad c_d = \frac{\text{drag } (N/m)}{\frac{1}{2}\rho U^2 c}$$ (section chord c)

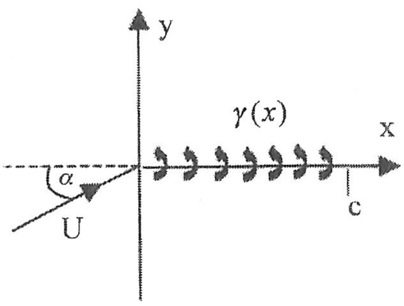
Wing lift and drag coefficients: $$C_L = \frac{\text{lift } (N)}{\frac{1}{2}\rho U^2 S}, \quad C_D = \frac{\text{drag } (N)}{\frac{1}{2}\rho U^2 S}$$ (wing area S)

Thin Aerofoil Theory

Geometry



Approximate representation



Pressure coefficient: $c_p = \pm\gamma / U$

Pitching moment coefficient: $c_m = (\text{moment about } x = 0) / \frac{1}{2}\rho U^2 c^2$

Coordinate transformation: $x = c(1 + \cos\theta)/2 = c\cos^2(\theta/2)$

Incidence solution: $$\gamma = -2U\alpha \frac{1-\cos\theta}{\sin\theta}, \quad c_l = 2\pi\alpha, \quad c_m = c_l / 4$$

Camber solution: $$\gamma = -U\left[g_0 \frac{1-\cos\theta}{\sin\theta} + \sum_{n=1}^{\infty} g_n \sin n\theta\right], \text{ where}$$

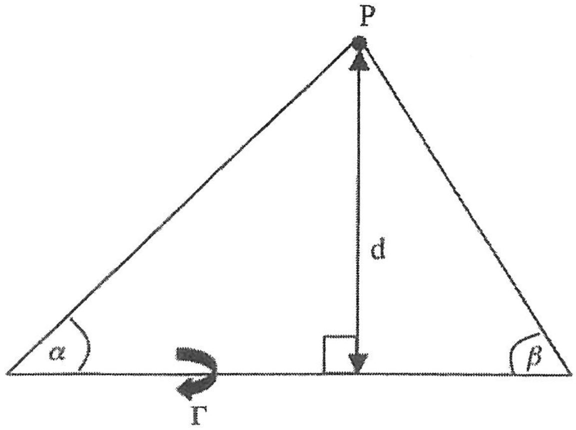
$$g_0 = \frac{1}{\pi}\int_0^\pi \left(-2\frac{dy_c}{dx}\right) d\theta, \quad g_n = \frac{2}{\pi}\int_0^\pi \left(-2\frac{dy_c}{dx}\right)\cos n\theta d\theta$$

$$c_l = \pi\left(g_0 + \frac{g_1}{2}\right), \quad c_m = \frac{\pi}{4}\left(g_0 + g_1 + \frac{g_2}{2}\right) = \frac{c_l}{4} + \frac{\pi}{8}(g_1 + g_2)$$

Glauert Integral

$$\int_{0}^{\pi} \frac{\cos n\phi}{\cos\phi - \cos\theta}\, d\phi = \pi \frac{\sin n\theta}{\sin\theta}$$

Line Vortices



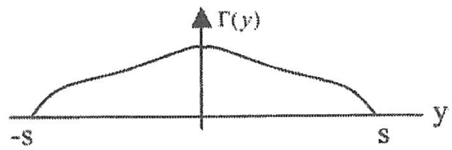
A straight element of circulation Γ induces a velocity at P of

$$\frac{\Gamma}{4\pi d}(\cos\alpha + \cos\beta)$$

perpendicular to the plane containing P and the element.

Lifting-Line Theory

Spanwise circulation distribution:



Aspect ratio: $A_R = 4s^2 / S$

Wing lift: $L = \rho U \int_{-s}^{s} \Gamma(y) dy$

Downwash angle: $\alpha_d(y) = \frac{1}{4\pi U} \int_{-s}^{s} \frac{d\Gamma(\eta)/d\eta}{y - \eta}\, d\eta$

Induced drag: $D_i = \rho U \int_{-s}^{s} \Gamma(y) \alpha_d(y) dy$

Fourier series for circulation: $\Gamma(y) = Us \sum_{n\ \mathrm{odd}} G_n \sin n\theta$, with $y = -s\cos\theta$

Relation between lift and induced drag:

$$C_{Di} = (1+\delta)\frac{C_l^2}{\pi A_R}, \text{ where } \delta = 3\left(\frac{G_3}{G_1}\right)^2 + 5\left(\frac{G_5}{G_1}\right)^2 + \ldots$$

Module 3A1
Boundary Layer Theory Data Card

Displacement thickness;

$$\delta^* = \int_0^\infty \left(1 - \frac{u}{U_1}\right) dy$$

Momentum thickness;

$$\theta = \int_0^\infty \frac{(U_1 - u)u}{U_1^2} dy = \int_0^\infty \left(1 - \frac{u}{U_1}\right) \frac{u}{U_1} dy$$

Energy thickness;

$$\delta_E = \int_0^\infty \frac{(U_1^2 - u^2)u}{U_1^3} dy = \int_0^\infty \left(1 - \left(\frac{u}{U_1}\right)^2\right) \frac{u}{U_1} dy$$

$$H = \frac{\delta^*}{\theta}$$

Prandtl's boundary layer equations (laminar flow);

$$\begin{aligned} u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} &= -\frac{1}{\rho}\frac{dp_1}{dx} + \nu\frac{\partial^2 u}{\partial y^2} \\ \frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} &= 0 \end{aligned}$$

von Karman momentum integral equation;

$$\frac{d\theta}{dx} + \frac{H+2}{U_1}\theta\frac{dU_1}{dx} = \frac{\tau_o}{\rho U_1^2} = \frac{C_f'}{2}$$

Boundary layer equations for turbulent flow;

$$\begin{aligned} \overline{u}\frac{\partial \overline{u}}{\partial x} + \overline{v}\frac{\partial \overline{u}}{\partial y} &= \frac{-1}{\rho}\frac{d\overline{p}}{dx} - \frac{\partial \overline{u'v'}}{\partial y} + \nu\frac{\partial^2 \overline{u}}{\partial y^2} \\ \frac{\partial \overline{u}}{\partial x} + \frac{\partial \overline{v}}{\partial y} &= 0 \end{aligned}$$

Module 3A1: Fluid Mechanics I

INCOMPRESSIBLE FLOW DATA CARD

Continuity equation $\nabla \cdot \boldsymbol{u} = 0$

Momentum equation (inviscid) $\rho \dfrac{D\boldsymbol{u}}{Dt} = -\nabla p + \rho \boldsymbol{g}$

D/Dt denotes the material derivative, $\partial/\partial t + \boldsymbol{u} \cdot \nabla$

Vorticity $\boldsymbol{\omega} = \text{curl } \boldsymbol{u}$

Vorticity equation (inviscid) $\dfrac{D\boldsymbol{\omega}}{Dt} = \boldsymbol{\omega} \cdot \nabla \boldsymbol{u}$

Kelvin's circulation theorem (inviscid) $\dfrac{D\Gamma}{Dt} = 0, \quad \Gamma = \oint \boldsymbol{u} \cdot d\boldsymbol{l} = \int \boldsymbol{\omega} \cdot d\boldsymbol{S}$

For an irrotational flow

velocity potential ϕ $\boldsymbol{u} = \nabla \phi$ and $\nabla^2 \phi = 0$

Bernoulli's equation for inviscid flow: $\dfrac{p}{\rho} + \frac{1}{2}|\boldsymbol{u}|^2 + gz + \dfrac{\partial \phi}{\partial t} = \text{constant}$ throughout flow field

TWO-DIMENSIONAL FLOW

Streamfunction ψ

$$u = \frac{\partial \psi}{\partial y}, \qquad v = -\frac{\partial \psi}{\partial x}$$

$$u_r = \frac{1}{r}\frac{\partial \psi}{\partial \theta}, \qquad u_\theta = -\frac{\partial \psi}{\partial r}$$

Lift force Lift / unit length $= \rho U(-\Gamma)$

For an irrotational flow

complex potential $F(z)$ $F(z) = \phi + i\psi$ is a function of $z = x + iy$

$$\frac{dF}{dz} = u - iv$$

TWO-DIMENSIONAL FLOW (continued)

Summary of simple 2 - D flow fields

	ϕ	ψ	$F(z)$	$\boldsymbol{u}$
Uniform flow (x - wise)	Ux	Uy	Uz	$u = U,\ v = 0$
Source at origin	$\frac{m}{2\pi}\ln r$	$\frac{m}{2\pi}\theta$	$\frac{m}{2\pi}\ln z$	$u_r = \frac{m}{2\pi r},\ u_\theta = 0$
Doublet (x - wise) at origin	$-\frac{\mu\cos\theta}{2\pi r}$	$\frac{\mu\sin\theta}{2\pi r}$	$-\frac{\mu}{2\pi z}$	$u_r = \frac{\mu\cos\theta}{2\pi r^2},\ u_\theta = \frac{\mu\sin\theta}{2\pi r^2}$
Vortex at origin	$\frac{\Gamma}{2\pi}\theta$	$-\frac{\Gamma}{2\pi}\ln r$	$-\frac{i\Gamma}{2\pi}\ln z$	$u_r = 0,\ u_\theta = \frac{\Gamma}{2\pi r}$

THREE-DIMENSIONAL FLOW

Summary of simple 3 - D flow fields

	ϕ	$\boldsymbol{u}$
Source at origin	$-\frac{m}{4\pi r}$	$u_r = \frac{m}{4\pi r^2},\quad u_\theta = 0,\quad u_\psi = 0$
Doublet at origin (with θ the angle from the doublet axis)	$-\frac{\mu\cos\theta}{4\pi r^2}$	$u_r = \frac{\mu\cos\theta}{2\pi r^3},\quad u_\theta = \frac{\mu\sin\theta}{4\pi r^3},\quad u_\psi = 0$