1 (a) See lecture notes

$T_0 = \text{const}$

(b)

$P_2 = 118\ \text{kPa}$

$P_{02} = 140\ \text{kPa}$

(i) There must be a shock because P_0 drops

The nozzle must be choked.

(ii) $\therefore \quad \dfrac{\dot{m}\sqrt{c_p T_0}}{A^* P_{01}} = 1.281$

$\Rightarrow \boxed{\dot{m} = 4.286\ \text{kg/s}}$ (constant through the nozzle)

At the exit, $P_2/P_{02} = \dfrac{118}{140} = 0.843$

$\Rightarrow \boxed{M_2 = 0.5}$

$$\frac{\dot{m}\sqrt{c_p T_0}}{A_2 P_{02}} = 0.956 = \frac{1.281 \cdot A^* P_{01}}{A_2 P_{02}}$$

$$\Rightarrow A_2 = \frac{1.281 \times 0.01 \times 180}{140 \times 0.956} = \underline{\underline{0.017\ m^2}}$$

(iii) Across the shock,

$$\frac{P_{0s}}{P_{01}} = \frac{P_{02}}{P_{01}} = \frac{140}{180} = .778$$

$\Rightarrow \boxed{M_s = 1.86}$

$$\frac{\dot{m}\sqrt{c_p T_0}}{A_s P_{01}} = 0.837$$

$$\therefore \quad \frac{A_s}{A^*} = \frac{1.281}{0.837}$$

$\boxed{A_s = 0.0153\ m^2}$

(iv) For M_2 to just be subsonic, the nozzle would be just at choking

$$\therefore \frac{\dot{m}\sqrt{C_pT_0}}{A^*P_0} = 1.281 \quad \text{and} \quad \underline{P_0 = \text{const.}}$$

$$\therefore P_{0_2} = P_{0_1} = 180\ \text{kPa}$$

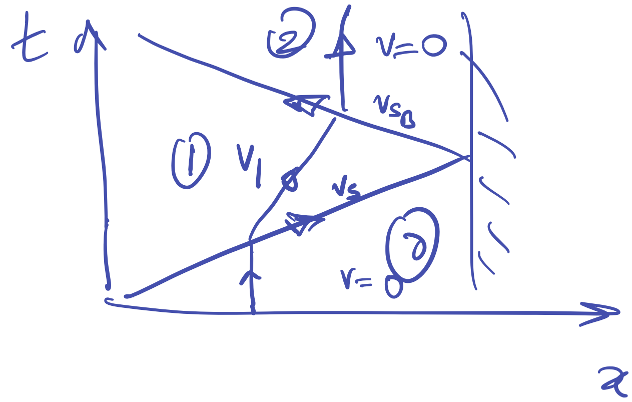
$$\frac{\dot{m}\sqrt{C_pT_0}}{A_2P_0} = 1.281 \times \frac{A^*}{A_2} = \frac{1.281 \times 0.01}{0.017} = 0.754$$

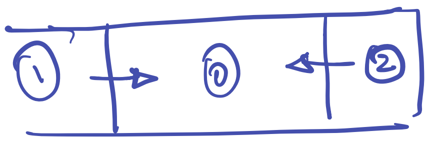
$$\therefore \boxed{M_2 = 0.37}$$

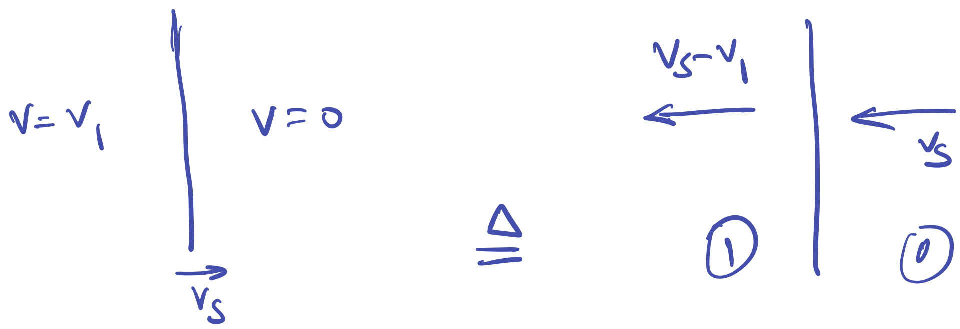
$$\frac{P_2}{P_0} = 0.91$$

$$\Rightarrow \boxed{P_2 = 163.8\ \text{kPa}}$$

2 $\quad \frac{p_s}{p} = 4 \Rightarrow M = \underline{1.89}$ from CUED tables







$$\frac{V_s}{a_0} = 1.89$$

From CUED tables,

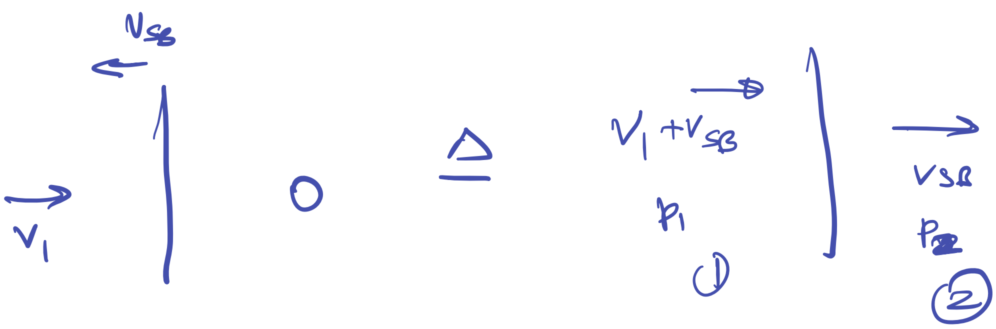
$$M_1 = 0.598 = \frac{V_s - V_1}{a_1}$$

$$\frac{T_1}{T_0} = 1.6 \qquad a^2 = \gamma R T$$

$$\therefore \frac{a_1^2}{a_0^2} = 1.6 \Rightarrow \boxed{a_1 = 1.265\, a_0}$$

$$\therefore 0.598 = \frac{1.89\, a_0 - V_1}{1.265\, a_0}$$

$$\boxed{V_1 = 1.13\, a_0}$$



$$\frac{\rho_1}{\rho_2} = \frac{2}{(\gamma+1)\left(\frac{V_1 + V_{SB}}{a_1}\right)^2} + \frac{\gamma-1}{\gamma+1}$$

<u>Continuity:</u>

$$\rho_1 (V_1 + V_{SB}) = \rho_2 V_{SB}$$

$$\frac{\rho_1}{\rho_2} = \frac{V_{SB}}{V_1 + V_{SB}}$$

$$\frac{V_1 + V_{SB}}{a_1} = M_1$$

$$\therefore \frac{V_{SB}}{a_1 M_1} = \frac{2}{\gamma+1 \, M_1^2} + \frac{\gamma-1}{\gamma+1}$$

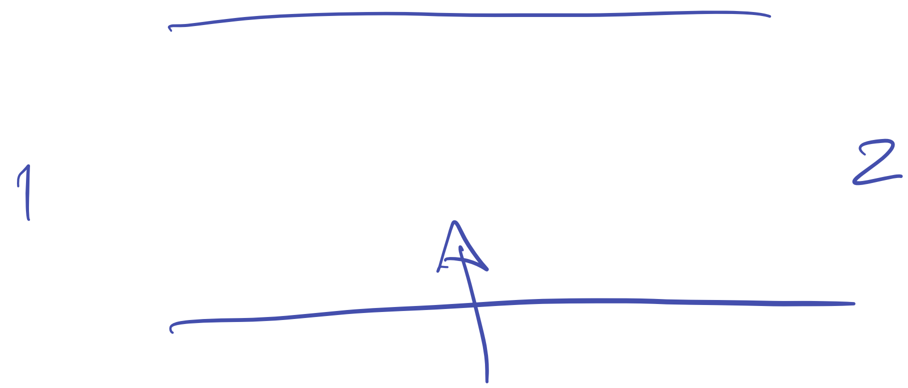
$$\left(M_1 - \frac{V_1}{a_1}\right)\frac{1}{M_1} = \frac{2}{(\gamma+1) M_1^2} + \frac{\gamma-1}{\gamma+1}$$

$$\frac{V_1}{a_1} = \frac{1.13}{1.265}$$

$$\Rightarrow M_1 = \underline{1.67}$$

Strength of the shock

3.



(a) $T_2 = 660\ K$

$u_2 = 330\ m/s$

$a_2 = \sqrt{\gamma R T_2} = 514.96\ m/s$

$M_2 = 0.64$

$\therefore \dfrac{T_2}{T_{02}} = 0.9286$ (GIVEN tables)

$\Rightarrow T_{02} = 710.75\ K$

$\dot{Q} = \dot{m} c_p (T_{02} - T_{01})$

$\therefore 2.5 \times 10^6 = 20 \times 1005 (710.75 - T_{01})$

(i) $\Rightarrow \underline{T_{01} = 586.4\ K.}$

As there is no friction, $F =$ const.

$$\therefore \frac{F_1}{\dot{m}\sqrt{c_p T_{01}}} \Big/ \frac{F_2}{\dot{m}\sqrt{c_p T_{02}}} = \sqrt{\frac{T_{02}}{T_{01}}}$$

From CUBD tables.

$$\frac{F_2}{\dot{m}\sqrt{c_p T_{02}}} = 1.068$$

$$\therefore \cancel{E_2}\frac{F_1}{\dot{m}\sqrt{c_p T_{01}}} = 1.068\sqrt{\frac{710.75}{586.4}}$$

$$= 1.175$$

$\Rightarrow M_1 = 0.513$ (tables)

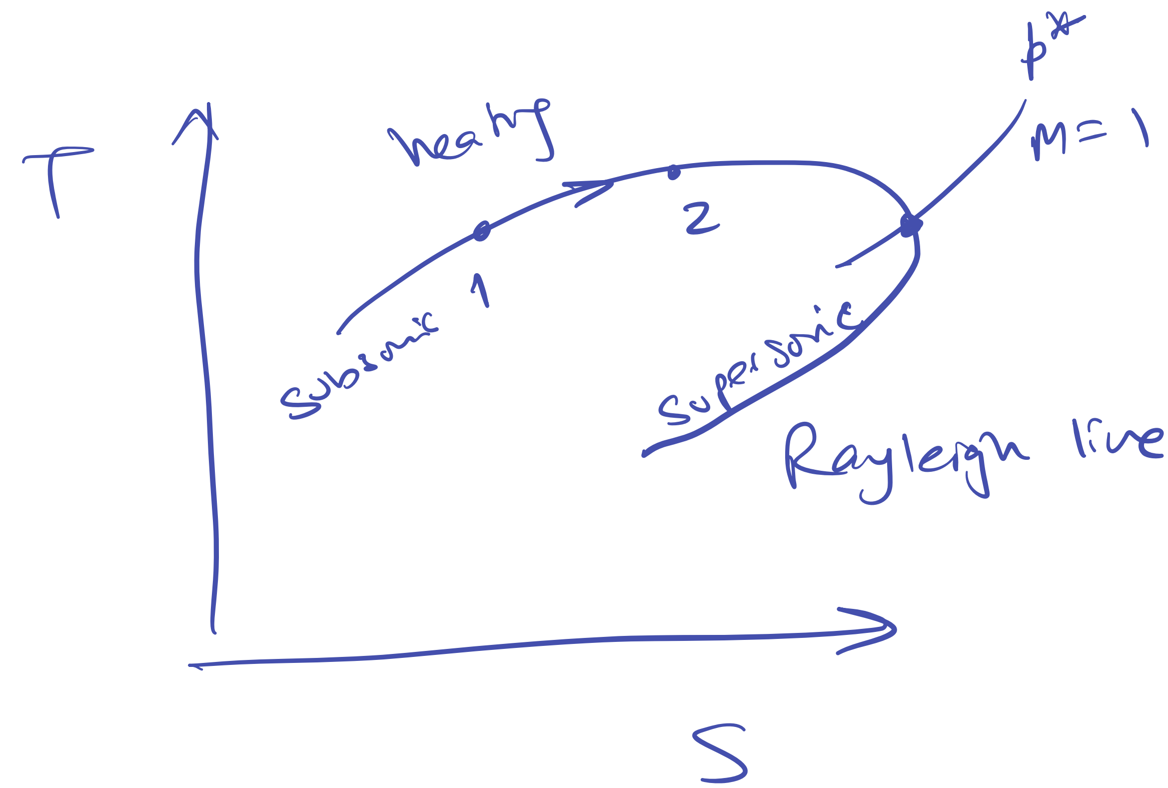
$$\frac{T_1}{T_{01}} = 0.953$$

$$T_1 = 558.84 \text{ K}.$$

$$a_1 = 473.86 \text{ m/s}.$$

(ii) $\Rightarrow$ ~~Δ~~ $V_1 = 243.6$ m/s

(b)



(c) Choked exit $\Rightarrow$ $M_2^* = 1$.

$$\frac{F_2}{\dot{m}\sqrt{c_p T_{02}^*}} = 0.9897$$

$$\therefore \frac{F_1}{\dot{m}\sqrt{C_p T_{o1}}} \Bigg/ \frac{F_2}{\dot{m}\sqrt{C_p T_{o2}^*}} = \sqrt{\frac{T_{o2}^*}{T_{o1}}}$$

$$\therefore \frac{1.196}{0.9897} = \sqrt{\frac{T_{o2}^*}{586.4}}$$

$$\Rightarrow T_{o2}^* = 856.35 \text{ K.}$$

$$\Rightarrow \dot{Q} = \dot{m} C_p (T_{o2}^* - T_{o1})$$

$$= 20 \times 1005 (856.35 - 586.4)$$

$$= 5.43 \text{ MW.}$$

$\Rightarrow$ Excess heat input $= 5.43 - 2.5$

$= \underline{2.93 \text{ MW}}$

d) If more heat is added the exit will remain choked and pressure wave will travel upstream to change ~~the~~ inlet conditions.

4 a) Estimate optimum angle α (in terms of pressure recovery) for n=2 mode (i.e. 1x oblique + 1x normal) for M=3.00.

M=3. δ		22	24	26	28	20
oblique	$\frac{p_{02}}{p_{01}}$	0.751	0.703	0.655	0.606	0.796
	M_2	1.89	1.77	1.66	1.54	1.99
normal	$\frac{p_{03}}{p_{02}}$	0.772	0.826	0.872	0.917	0.726
	p_{03}/p_{01}	0.580	0.581	0.571	0.556	0.578

↑

b) now n=3 with two obliques $\delta=16°$

δ	16	16	N	
$\frac{p_{02}}{p_{01}}$	0.877	0.934	0.903	$\Rightarrow$ 0.740
M_2	2.20	1.58		

c) Calculate the total flow turning angle for 100% pressure recovery.

Reversed Prandtl-Meyer expansion $\Rightarrow$ $\nu = 49.8°$

d) comment on trade-off

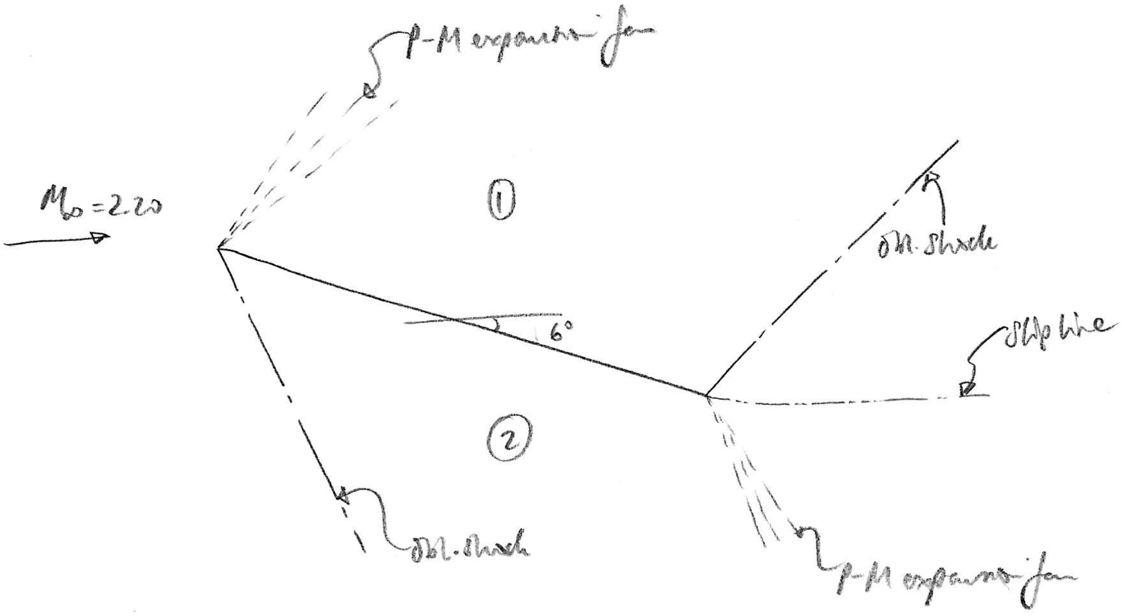
i.e. pressure recovery vs. cowl drag.

5 - a) $h = 18\,km$ (~FL590) $\frac{T}{T_{SL}} = 0.7519$ from datasheet, $\frac{p}{p_{SL}} = 0.0747$

$\frac{a}{a_{SL}} = \sqrt{\frac{T}{T_{SL}}}$ $a_{SL} = 340\,m/s$ for standard atmosphere $\Rightarrow a = 295\,m/s$

$p_\infty = 7570\,Pa$

$\Rightarrow 650\,m/s \Rightarrow M = 2.20.$



b) $M_0 = 2.20 \therefore \nu_\infty = 31.73 \Rightarrow \nu_1 = 37.73$, $M_1 = 2.44$, $\frac{p_1}{p_{0\infty}} = 0.0643$

$\frac{p_\infty}{p_{0\infty}} = 0.0935$, $\frac{p_2}{p_\infty} = 1.417$ $p_{0\infty} = 81\,kPa$ $\Rightarrow p_1 = 5210\,Pa$

$p_2 = 10700\,Pa$

Consider unit chord

$\therefore$ Force on plate per unit span ; $\frac{F}{W} = (p_2 - p_1) \cdot 1 = 5490\,kN$

Lift, $L = F\cos\alpha = 5460\,kN$ (since chord = 1 and span = 1)

$C_L = \frac{L}{\frac{1}{2}\rho_\infty u_\infty^2 \cdot A}$ (where A = chord × span). From tables $\frac{\frac{1}{2}\rho_\infty V_\infty^2}{p_{0\infty}} = 0.317$

$\therefore C_L = \frac{5460}{0.317 \times 81 \times 10^3} = 0.21$ $\frac{L}{D} = \frac{1}{\tan^{-1} 6°} = 9.5$

c) c.f. transonic transports $C_L \sim 0.5$ $\frac{L}{D} \sim 18$

6)

$$\frac{\partial u}{\partial t} - 2\frac{\partial u}{\partial x} = 0$$

is to be solved over the domain $0 < x < 1$, with an initial condition of

$$u(x,0) = \begin{cases} 0.5, & 0 \le x < 0.5 \\ 1, & 0.5 \le x < 1 \end{cases}$$

$$u(1,t) = 0.75$$

a) **State why the boundary condition need only be specified at $x = 1$**

The convection diffusion equation above has a negative velocity. All the characteristics are moving from right to left, i.e. from the x=1 boundary into the domain. Hence only the x=1 boundary can affect the solution

b) **The domain is discretised and the solution is approximated using finite differences. Derive the approximate difference equation using a first order upwind estimate for the spatial derivative, and a first order forward difference for the time derivative, that advances the solution forward by a time step of Δt.**

The trick here is to note that upwinding is now in the opposite direction to the examples given in lectures, i.e. an upwind difference is now the forward difference.

$$\frac{\partial u}{\partial x} = \frac{u_{i+1}^N - u_i^N}{\Delta x}$$

$$\frac{\partial u}{\partial t} = \frac{u_i^{N+1} - u_i^N}{\Delta t}$$

So

$$\frac{\partial u}{\partial t} - 2\frac{\partial u}{\partial x} = 0$$

becomes

$$\frac{u_i^{N+1} - u_i^N}{\Delta t} = 2\frac{(u_{i+1}^N - u_i^N)}{\Delta x}$$

$$u_i^{N+1} = u_i^N + 2\frac{(u_{i+1}^N - u_i^N)\Delta t}{\Delta x}$$

c) **Show that for a specific value of Δt the solution can be both stable and have an accuracy greater than first order in both space and time**

Find the effective pde being solved.

$$u_i^{N+1} = u_i^N + 2\frac{(u_{i+1}^N - u_i^N)\Delta t}{\Delta x}$$

$$u_{i+1}^{n} = u_i^n + \left.\frac{\partial u}{\partial x}\right|_{x,t} \Delta x + \frac{1}{2!}\left.\frac{\partial^2 u}{\partial x^2}\right|_{x,\,t} \Delta x^2 + \frac{1}{3!}\left.\frac{\partial^3 u}{\partial x^3}\right|_{x,t} \Delta x^3 \; + \cdots$$

And

$$u_i^{n+1} = u_i^n + \left.\frac{\partial u}{\partial t}\right|_{x,t} \Delta t + \frac{1}{2!}\left.\frac{\partial^2 u}{\partial t^2}\right|_{x,\,t} \Delta t^2 + \frac{1}{3!}\left.\frac{\partial^3 u}{\partial t^3}\right|_{x,t} \Delta t^3 \; + \cdots$$

Then

$$u_i^{N+1} = u_i^N + 2\frac{(u_{i+1}^N - u_i^N)\Delta t}{\Delta x}$$

Becomes

$$\left.\frac{\partial u}{\partial t}\right|_{x,t} \Delta t + \frac{1}{2!}\left.\frac{\partial^2 u}{\partial t^2}\right|_{x,\,t} \Delta t^2 + \frac{1}{3!}\left.\frac{\partial^3 u}{\partial t^3}\right|_{x,t} \Delta t^3 \; = \left(\left.\frac{\partial u}{\partial x}\right|_{x,t} \Delta x + \frac{1}{2!}\left.\frac{\partial^2 u}{\partial x^2}\right|_{x,\,t} \Delta x^2 + \frac{1}{3!}\left.\frac{\partial^3 u}{\partial x^3}\right|_{x,t} \Delta x^3 \;\right) 2\frac{\Delta t}{\Delta x}$$

$$\left.\frac{\partial u}{\partial t}\right|_{x,t} + \frac{1}{2!}\left.\frac{\partial^2 u}{\partial t^2}\right|_{x,\,t} \Delta t + \frac{1}{3!}\left.\frac{\partial^3 u}{\partial t^3}\right|_{x,t} \Delta t^2 \; = 2\left.\frac{\partial u}{\partial x}\right|_{x,t} + \frac{2}{2!}\left.\frac{\partial^2 u}{\partial x^2}\right|_{x,\,t} \Delta x + \frac{2}{3!}\left.\frac{\partial^3 u}{\partial x^3}\right|_{x,t} \Delta x^2$$

$$\left.\frac{\partial u}{\partial t}\right|_{x,t} - 2\left.\frac{\partial u}{\partial x}\right|_{x,t} = \frac{2}{2!}\left.\frac{\partial^2 u}{\partial x^2}\right|_{x,\,t} \Delta x - \frac{1}{2!}\left.\frac{\partial^2 u}{\partial t^2}\right|_{x,\,t} \Delta t + \frac{2}{3!}\left.\frac{\partial^3 u}{\partial x^3}\right|_{x,t} \Delta x^2 \; - \frac{1}{3!}\left.\frac{\partial^3 u}{\partial t^3}\right|_{x,t} \Delta t^2$$

i.e.

Replace the time second derivative in time with a space difference.

$$\left.\frac{\partial u}{\partial t}\right|_{x,t} - 2\left.\frac{\partial u}{\partial x}\right|_{x,t} = O(\Delta t, \Delta x)$$

$$\left.\frac{\partial^2 u}{\partial t^2}\right|_{x,t} - 2\left.\frac{\partial^2 u}{\partial t \partial x}\right|_{x,t} = O(\Delta t, \Delta x)$$

$$\left.\frac{\partial^2 u}{\partial x \partial t}\right|_{x,t} - 2\left.\frac{\partial^2 u}{\partial x^2}\right|_{x,t} = O(\Delta t, \Delta x)$$

$$\left.\frac{\partial^2 u}{\partial t^2}\right|_{x,t} - 4\left.\frac{\partial^2 u}{\partial x^2}\right|_{x,t} = O(\Delta t, \Delta x)$$

$$\left.\frac{\partial u}{\partial t}\right|_{x,t} - 2\left.\frac{\partial u}{\partial x}\right|_{x,t} = \frac{2}{2!}\left.\frac{\partial^2 u}{\partial x^2}\right|_{x,\,t} \Delta x - \frac{1}{2!}4\left.\frac{\partial^2 u}{\partial x^2}\right|_{x,t} \Delta t + O(\Delta t^2, \Delta t \Delta x) + \frac{2}{3!}\left.\frac{\partial^3 u}{\partial x^3}\right|_{x,t} \Delta x^2 \; - \frac{1}{3!}\left.\frac{\partial^3 u}{\partial t^3}\right|_{x,t} \Delta t^2$$

$$\left.\frac{\partial u}{\partial t}\right|_{x,t} - 2\left.\frac{\partial u}{\partial x}\right|_{x,t} = \frac{2}{2!}\left.\frac{\partial^2 u}{\partial x^2}\right|_{x,\,t} \Delta x - \frac{1}{2!}4\left.\frac{\partial^2 u}{\partial x^2}\right|_{x,t} \Delta t + O(\Delta t^2, \Delta t \Delta x, \Delta x^2\,)$$

$$\left.\frac{\partial u}{\partial t}\right|_{x,t} - 2\left.\frac{\partial u}{\partial x}\right|_{x,t} = \Delta x\left.\frac{\partial^2 u}{\partial x^2}\right|_{x,\,t} \left(1 - 2\frac{\Delta t}{\Delta x}\right) + O(\Delta t^2, \Delta t \Delta x, \Delta x^2\,)$$

Require

$$1 - 2\frac{\Delta t}{\Delta x} > 0$$

$$2\frac{\Delta t}{\Delta x} < 1$$

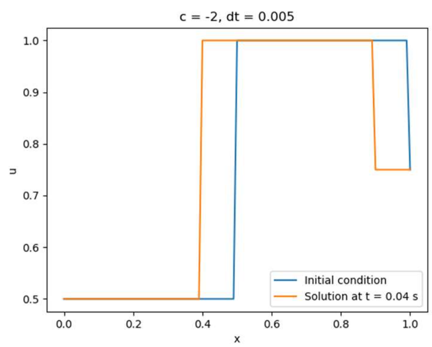
d) If the domain is discretised with 101 equally spaced nodes, calculate the required value of Δt in this particular case.

This is the CFL condition!

101 equally spaced points means 100 intervals so $\Delta x = 0.01$. Therefore when $\Delta t = 0.005$ there is no additional viscosity and the solution becomes second order accurate, i.e. $O(\Delta t^2, \Delta t\Delta x, \Delta x^2)$

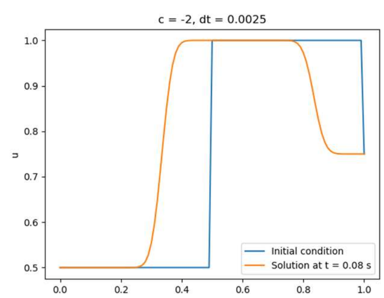
e) Sketch the evolution of the solution when

i. The value of Δt from part (c) is used.



There is no artificial viscosity so profile is convected perfectly right to left.

ii. Half the value of Δt from part (c) is used.



The artificial viscosity term is now important and the profiles are smudged as they are convected.

7(a)

$$\frac{\partial \boldsymbol{u}}{\partial \boldsymbol{t}} = \boldsymbol{\alpha} \frac{\partial^2 \boldsymbol{u}}{\partial \boldsymbol{x}^2}$$

The following numerical scheme is used to produce a numerical solution,

$$\boldsymbol{u_i^{N+1} = u_i^N + (\, u_{i-2}^N - 2u_{i-1}^N + u_i^N) \frac{\alpha \delta t}{\delta x^2}}.$$

Identify the leading error terms in both space and time and hence the order of accuracy of this scheme.

$$u_{i-1}^n = u_i^n - \left.\frac{\partial u}{\partial x}\right|_{x,t} \Delta x + \frac{1}{2!}\left.\frac{\partial^2 u}{\partial x^2}\right|_{x,\,t} \Delta x^2 - \frac{1}{3!}\left.\frac{\partial^3 u}{\partial x^3}\right|_{x,t} \Delta x^3 + \cdots$$

$$u_{i-2}^n = u_i^n - 2\left.\frac{\partial u}{\partial x}\right|_{x,t} \Delta x + \frac{4}{2!}\left.\frac{\partial^2 u}{\partial x^2}\right|_{x,\,t} \Delta x^2 - \frac{8}{3!}\left.\frac{\partial^3 u}{\partial x^3}\right|_{x,t} \Delta x^3 + \cdots$$

$$u_{i-2}^n = u_i^n - 2\left.\frac{\partial u}{\partial x}\right|_{x,t} \Delta x + \frac{4}{2!}\left.\frac{\partial^2 u}{\partial x^2}\right|_{x,\,t} \Delta x^2 - \frac{8}{3!}\left.\frac{\partial^3 u}{\partial x^3}\right|_{x,t} \Delta x^3 + \cdots$$

$$2u_{i-1}^n = 2u_i^n - 2\left.\frac{\partial u}{\partial x}\right|_{x,t} \Delta x + \frac{2}{2!}\left.\frac{\partial^2 u}{\partial x^2}\right|_{x,\,t} \Delta x^2 - \frac{2}{3!}\left.\frac{\partial^3 u}{\partial x^3}\right|_{x,t} \Delta x^3 + \cdots$$

Subtracting

$$u_{i-2}^N - 2u_{i-1}^N + u_i^N = \frac{2}{2!}\left.\frac{\partial^2 u}{\partial x^2}\right|_{x,\,t} \Delta x^2 - \frac{6}{3!}\left.\frac{\partial^3 u}{\partial x^3}\right|_{x,t} \Delta x^3$$

$$\frac{u_{i-2}^N - 2u_{i-1}^N + u_i^N}{\Delta x^2} = \left.\frac{\partial^2 u}{\partial x^2}\right|_{x,\,t} - \frac{6}{3!}\left.\frac{\partial^3 u}{\partial x^3}\right|_{x,t} \Delta x$$

Similarly

$$\frac{u_i^{N+1} - u_i^N}{\Delta t} = \left.\frac{\partial u}{\partial t}\right|_{x,t} + \frac{1}{2!}\left.\frac{\partial^2 u}{\partial t^2}\right|_{x,\,t} \Delta t + \cdots$$

The leading error terms are highlighted in red.

Therefore the scheme is only first order accurate in space. It is also first order accurate in time.

$$u_i^{N+1} = u_i^N + (\, u_{i-2}^N - 2u_{i-1}^N + u_i^N) \frac{\alpha \delta t}{\delta x^2}$$

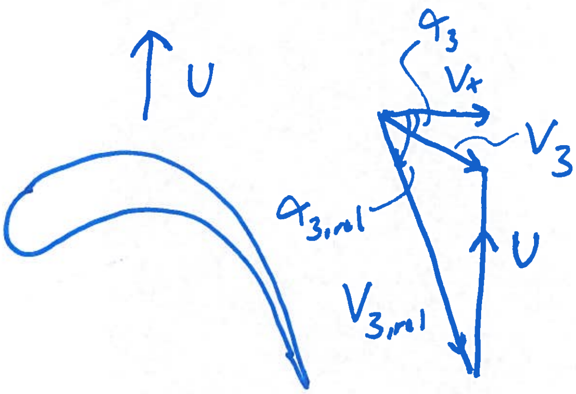
b) i) $\dot{m} = \rho A V_x$

$$V_x = \frac{4.31}{1.23 \times 0.1} = 35.0\ \mathrm{ms^{-1}}$$

$$\phi = \frac{V_x}{U} = \frac{35}{70} = 0.500$$

$$\psi = \frac{\Delta h_0}{U^2} = \frac{c_p \Delta T_0}{U^2} = \frac{1005 \times 7.31}{70^2} = 1.50$$

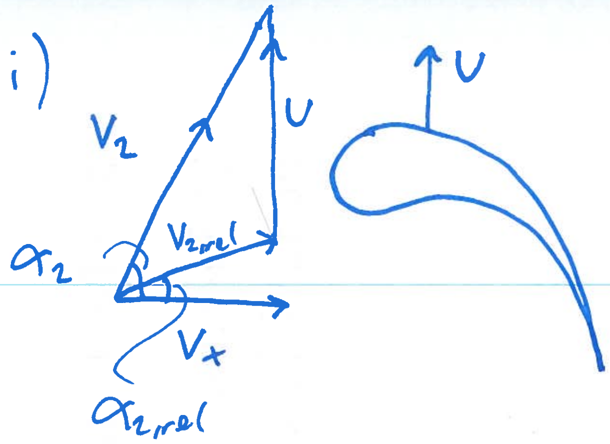
ii)



$$\tan \alpha_{3,rel} = \frac{V_x \tan \alpha_3 - U}{V_x}$$

$$\alpha_{3,rel} = \tan^{-1}\left(\frac{35 \tan(-10) - 70}{35}\right) = -65.3^\circ$$

iii)



$$\Delta h_0 = U \Delta V_\theta$$
$$= U(V_{\theta 2} - V_{\theta 3})$$

$$V_{\theta 3} = V_x \tan \alpha_3$$
$$= 35 \times \tan(-10) = -6.171$$

$$V_{\theta 2} = \frac{1005 \times 7.31}{70} - 6.171 = 98.78$$

$$\alpha_2 = \tan^{-1}\left(\frac{V_{\theta 2}}{V_x}\right) = \tan^{-1}\left(\frac{98.78}{35}\right) = 70.5^\circ$$

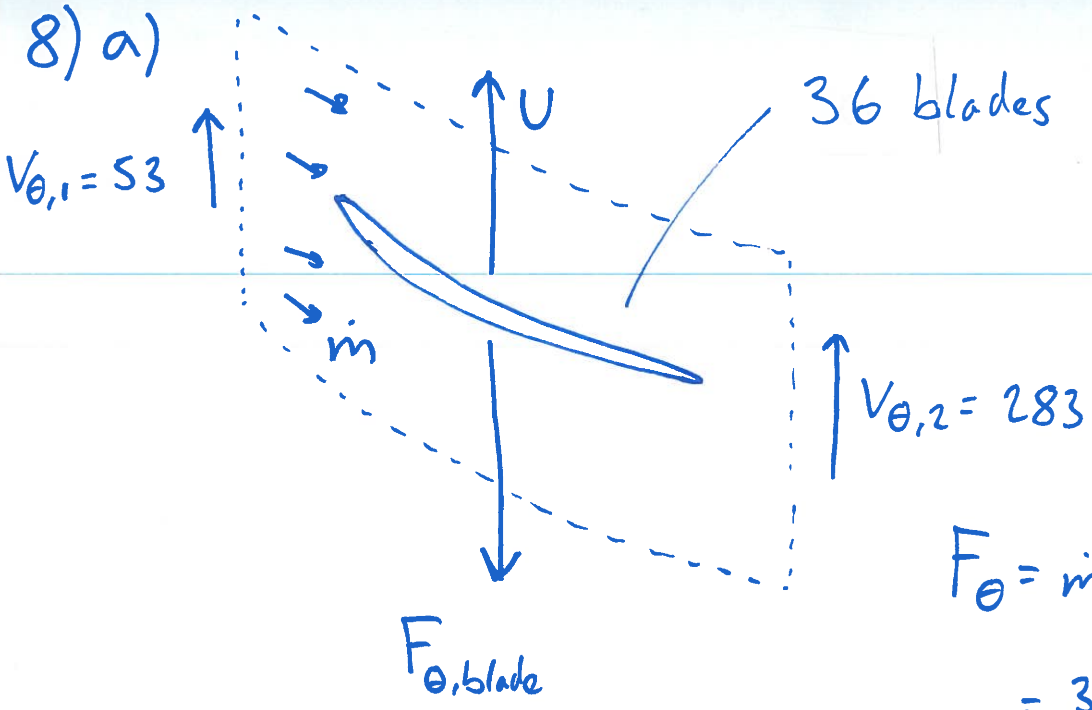
iv) Low speed $\rho = \text{const}$

$$T \Delta s = \Delta h_0 - \frac{\Delta P_0}{\rho} \rightarrow \Delta h_{0,ideal} = \frac{\Delta P_0}{\rho}$$

$$\eta = \frac{\Delta h_{0,actual}}{\Delta h_{0,ideal}} = \frac{\rho c_p \Delta T_0}{\Delta P_0} = \frac{1.23 \times 1005 \times 7.31}{0.1 \times 10^5}$$

$$= 90.4\%$$

8) a)



$$F_\theta = \dot{m}(V_{\theta 2} - V_{\theta 1})$$

$$= \frac{30}{36}(283 - 53)$$

$$= 192\,\text{N}$$

b) $\dot{Q} - \dot{W}_x = \dot{m}(h_2 + \tfrac{1}{2}V_2^2) - \dot{m}(h_1 + \tfrac{1}{2}V_1^2)$

$= \dot{m}(h_{02} - h_{01}) = -\dot{W}_x$ adiabatic

Power from blade torque

opposite direction

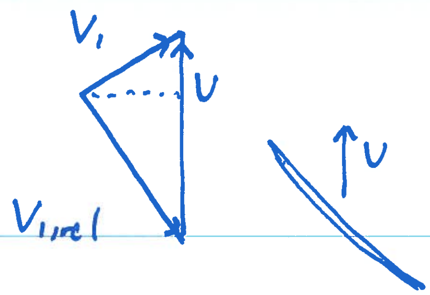
$$\dot{W}_x = U \times -F_\theta = -\dot{m}U(V_{\theta 2} - V_{\theta 1})$$

$$\cancel{\dot{m}}(h_{02} - h_{01}) = \cancel{\dot{m}}U(V_{\theta 2} - V_{\theta 1})$$

c) $U = \dfrac{r \times rpm \times 2\pi}{60} = 398\,\text{ms}^{-1}$

$\dot{W}_{x,\text{blade}} = F_\theta \times U = 76.3\,\text{kW}$

d) $V_1 = (V_x^2 + V_{\theta,1}^2)^{\frac{1}{2}} = 212\,ms^{-1}$



$V_{\theta 1,rel} = U - V_{\theta,1}$

$= 345\,ms^{-1}$

$V_{1,rel} = (V_x^2 + V_{\theta,1,rel}^2)^{\frac{1}{2}} = 401\,ms^{-1}$

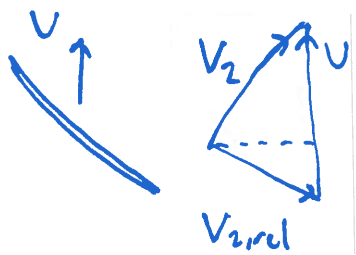
$$\frac{V_1}{\sqrt{c_p T_0}} = f(M_1) = 0.394 \rightarrow M_1 = 0.647$$

$$\frac{T_1}{T_{01}} = f(M_1) = 0.923 \qquad T_1 = 0.923 \times T_{01}$$

$$= 266\,K$$

$$M_{1,rel} = \frac{V_{1,rel}}{\sqrt{\gamma R T_1}} = 1.23$$

2)



$$\frac{T_{01,rel}}{T_1} = \left(f(M_{1,rel})\right)^{-1}$$

$$T_{01,rel} = \frac{265.8}{0.7682} = 346\,K$$

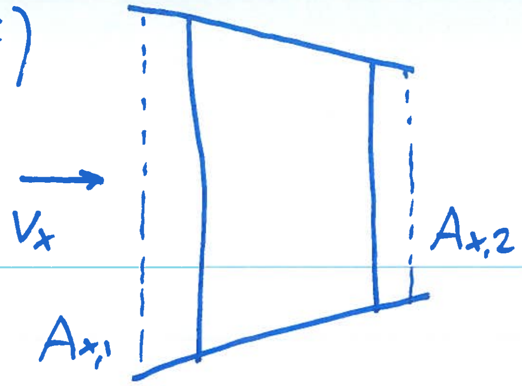
$$T_{02,rel} = T_{01,rel}$$

$V_{\theta,2,rel} = U - V_{\theta,2} = 114\,ms^{-1}$

$V_{2,rel} = (V_x^2 + V_{\theta 2,rel}^2)^{\frac{1}{2}} = 235\,ms^{-1}$

$$\frac{V_{2,rel}}{\sqrt{c_p T_{02,rel}}} = f(M_{2,rel}) = 0.399 \rightarrow M_{2,rel} = 0.656$$

F)



In a compressor the density is increased. In order to maintain a constant V_x the annular area must decrease according to continuity

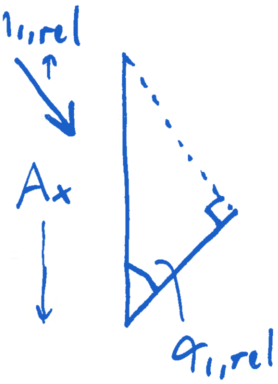
$$\dot{m} = \rho A_x V_x$$

$$\frac{\dot{m}\sqrt{c_p T_{01,rel}}}{A_{x,1}\cos\alpha_{1,rel}\, P_{01,rel}} = \frac{\dot{m}\sqrt{c_p T_{02,rel}}}{A_{x,2}\times\cos\alpha_{2,rel}\, P_{02,rel}} \; \frac{A_{x,2}\cos\alpha_{2,rel}}{A_{x,1}\cos\alpha_{1,rel}}$$

effective flow area

$f(M_{1,rel}) = 1.234$

$f(M_{2,rel}) = 1.134$



$$\alpha_{1,rel} = \tan^{-1}\left(\frac{V_{\theta 1,rel}}{V_x}\right) = 59.3^\circ$$

$$\alpha_{2,rel} = \tan^{-1}\left(\frac{V_{\theta 2,rel}}{V_x}\right) = 29.3^\circ$$

$$\frac{A_{x,2}}{A_{x,1}} = \frac{1.234\times\cos(29.3)}{1.134\times\cos(59.3)} = 0.637$$

3A3 Examiners' Report 2024

Question 1

Attempted by 99% of candidates. Average 16.4/20. Convergent-divergent nozzle, answered by all but one candidate. The standard of answers matched the question's popularity, demonstrating a sound grasp of con-di nozzle flows and the associated calculations. Marks were lost through slips and omissions only.

Question 2

Attempted by 45% of candidates. Average 12.2/20. Travelling shock waves. Space-time diagrams and changes of reference frame showed a good appreciation of the topic, and the characteristics of the initial wave were generally obtained successfully. The main exception here was illegitimate use of the Riemann invariants for isentropic waves. The calculation for the reflected wave required formulation and solution of a quadratic equation, and only a few candidates completed this part.

Question 3

Attempted by 93% of candidates. Average 15.1/20. Rayleigh flow, almost rivalling Qu 1 for popularity. The majority of candidates exhibited a good understanding of the topic, with only a few failing to apply the condition of constant impulse. Most also recognised the implication of choking, though fewer were able to explain specifically how it would apply in this context.

Question 4

Attempted by 53% of candidates. Average 15.5/20 with many perfect answers. Supersonic intakes. The most common mistake was in part (c) where the maximum theoretically achievable pressure recovery is 100% when the intake continuously turns the flow through an infinite number of isentropic compression waves. In part (d) many candidates missed the impact of the different intakes on the cowl drag, a larger quantity of turning requires larger intakes on the aircraft.

Question 5

Attempted by 81% of candidates. Average 14.5/20. A straightforward shock-expansion question, attempted by most candidates. Almost all grasped what was required and produced answers worth the majority of the marks. Some sketches of the flow field omitted the slip-line behind the plate, and some lift-coefficient calculations watered down the problem by employing the linearised method of characteristics. As this approach had not been 'authorised' by the question, it was penalised.

Question 6

Attempted by 33% of candidates. Average 11.6. Advection equation. Unpopular and poorly answered. The root of most candidates' issues was missing that the characteristic waves travel in the negative x direction in this question, thus the upwind estimate should use the i and i+1 indices. Many candidates struggled to find the equivalent PDE to show the order of accuracy although many could identify the correct CFL number to achieve greater than first order accuracy. Most candidates drew the final part incorrectly, if the correct CFL number is used then the solution is convected exactly in (e)(i), if a smaller value is used then diffusion errors smooth the solution in (e)(ii).

Question 7

Attempted by 74% of candidates. Average 14.2/20. Scheme accuracy and velocity triangles. The majority of candidates achieved perfect answers on the first part to find the leading error terms, the most common mistake here was to not carry enough Taylor series terms through to determine what the leading error terms actually are, only that they scale with $O(\Delta x)$. In the second part many mistakes come in the drawing of the velocity triangles and calculation of angles. Few candidates could do the final part using the Gibbs equation to get the efficiency of this incompressible machine.

Question 8

Attempted by 36% of candidates. Average 14.0/20. Compressible compressor rotor row. A question well answered in many cases, most had no issue with the derivation of Euler's work equation but many confused the direction of force on the flow, force on the blade, the direction of rotation of this compressor of the fact that it refers to a single blade only. The most common error in the calculation of the Mach numbers is to confuse static temperature with absolute stagnation temperature or relative stagnation temperature.

JVT & WG

May 24