Version CAH/5

EGT2
ENGINEERING TRIPOS PART IIA

Wednesday 7 May 2025   9.30 to 12.40

Module 3A3

FLUID MECHANICS II

*Answer not more than **five** questions.*

All questions carry the same number of marks.

*The **approximate** percentage of marks allocated to each part of a question is indicated in the right margin.*

*Write your candidate number **not** your name on the cover sheet.*

STATIONERY REQUIREMENTS
Single-sided script paper

SPECIAL REQUIREMENTS TO BE SUPPLIED FOR THIS EXAM
CUED approved calculator allowed
Engineering Data Book
Attachments:
Compressible Flow Data Book (38 pages).

10 minutes reading time is allowed for this paper at the start of the exam.

You may not start to read the questions printed on the subsequent pages of this question paper until instructed to do so.

You may not remove any stationery from the Examination Room.

1 (a) (i) Describe, with the aid of a diagram, the operation of a closed–circuit supersonic wind tunnel. [20%]

(ii) Explain the main sources of loss in the practical operation of a closed–circuit supersonic wind tunnel. [10%]

(b) A closed–circuit supersonic wind tunnel is designed to operate at a Mach number of 1.8 in a square test section of side 30 cm. The conditions upstream of the inlet nozzle are a stagnation temperature of 293 K and a stagnation pressure of 100 kPa.

(i) What is the area of the inlet nozzle throat? [15%]

(ii) What is the minimum area of the downstream throat? [20%]

(c) Describe the procedure for starting the tunnel and calculate the minimum power required. [20%]

(d) Calculate the minimum power required for continuous operation of the tunnel. [15%]

2 (a) An ideal gas flows with friction at velocity V in a duct with constant cross–sectional area A. The static pressure is p and the Mach number of the compressible flow is M. Show that the variation of the impulse function F is given by

$$\delta\left(\frac{F}{A}\right) = p(M^2 - 1)\frac{\delta V}{V}$$ [30%]

(b) Air flows from a large reservoir through a convergent–divergent nozzle into a pipe of inside diameter 0.5 m. The Mach number at the exit from the nozzle is 1.3. The temperature in the reservoir is 293 K and the pressure is 1.5 bar. The nozzle may be considered frictionless but the pipe has a skin friction coefficient of 0.0025. The pipe exit is found to be choked.

(i) What is the length of the pipe? [20%]

(ii) What is the mass flow rate through the pipe? [20%]

(c) The length of the pipe is increased to 4 m. A shock wave is formed in the pipe with a Mach number of 1.26. What is the location of the shock wave? [30%]

3 (a) A shock tube with both ends closed is fitted with a diaphragm in the middle. The tube is filled with air at a temperature of 293 K. The pressure to the left of the diaphragm is 1.8 bar while the pressure to the right of the diaphragm is 1 bar. At a certain instant the diaphragm is ruptured.

(i) Draw a space–time diagram to illustrate the development of the flow around the location of the diaphragm. [20%]

(ii) What is the Mach number of the initial shock wave? [10%]

(iii) What is the speed of sound behind the initial shock wave? [10%]

(iv) What is the velocity of the air behind the initial shock wave? [10%]

(b) After some time the initial shock wave is reflected from the end of the tube.

(i) Draw a space–time diagram to illustrate the reflection and its effect on a nearby fluid particle. [20%]

(ii) What is the Mach number of the reflected shock wave? [15%]

(iii) What is the speed of sound behind the reflected shock wave? [15%]

4 An irrotational, isentropic and two-dimensional flow may be described in a Cartesian coordinate system (x, y) by a flow potential ϕ and velocity $\mathbf{V}$.

(a) By considering the conservation of mass, momentum and energy, derive the equation

$$a^2\nabla \cdot \mathbf{V} - \mathbf{V} \cdot (\mathbf{V} \cdot \nabla\mathbf{V}) = 0$$

where a is the local speed of sound. [40%]

(b) Using the result of part (a) applied to an appropriate perturbation to the flow, derive the relationship

$$(1 - M_\infty^2)\frac{\partial^2\phi}{\partial x^2} + \frac{\partial^2\phi}{\partial y^2} = 0$$

for *small* disturbances, where M_∞ is the free stream Mach number. [40%]

(c) A small survey aircraft has a wing designed to operate at a lift coefficient $C_L = 0.2$ and a Mach number $M_\infty = 0.5$ with an ambient air temperature of 15°C. Its suitability for operation at the same true air speed and lift coefficient with an ambient air temperature of −40°C is to be assessed by wind tunnel testing. Estimate the lift coefficient the wing should be tested at in a low-speed wind tunnel. [20%]

5 A supersonic jet aircraft is designed for a cruise Mach number $M = 2.05$. Two designs of two-dimensional engine intake are under consideration. Both are external compression designs with a shock system focused on the cowl lip and both incorporate two fixed 10° ramps, as shown in Fig. 1a and Fig. 1b. The designs differ in the angle the inner face of the cowl lip makes to the freestream direction being reduced by 8° in the second design. While the shock system of the first design terminates in a normal shock, that in the second ends with a curved shock that makes an angle of 70° to the local flow direction at the cowl lip and 67° where it meets the other side of the passage. You may assume that the passage downstream of the cowl lip is shaped to prevent reflections.

(a) For each design, draw carefully labelled sketches of the shock system. [30%]

(b) For both designs, estimate the static pressure rise and pressure recovery (in terms of the ratio of exit to entry stagnation pressure) through the shock system. [30%]

(c) Describe two advantages of the second design. [20%]

(d) Describe two possible modifications to the rest of the engine/intake system (to compensate for the change in static pressure ratio of the shock system) that will allow the second design to be used. [20%]

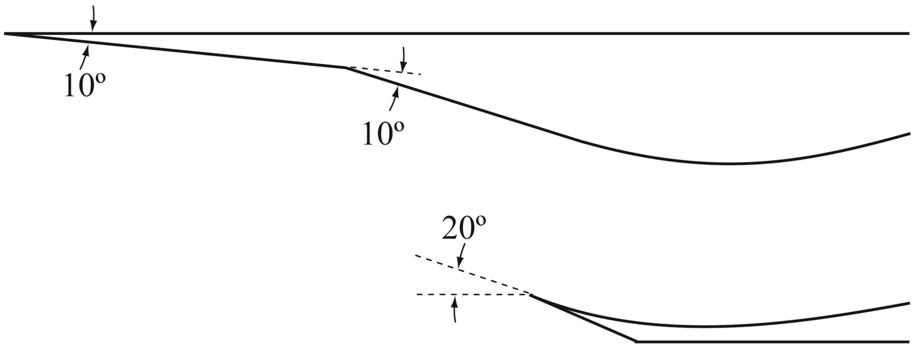

Fig. 1a. (not to scale)

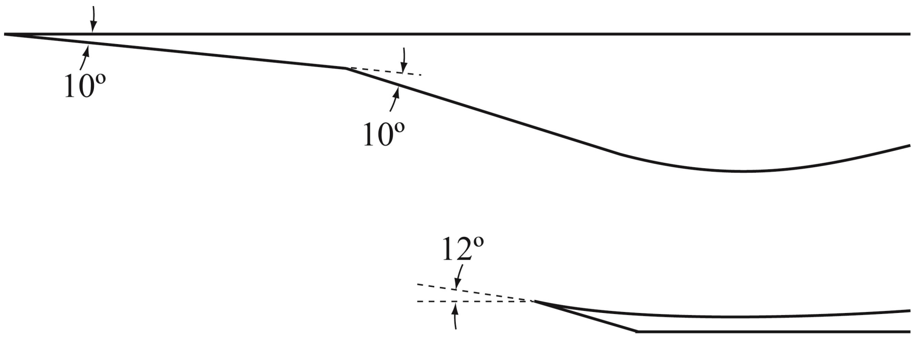

Fig. 1b. (not to scale)

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6 The advection equation

$$\frac{\partial u}{\partial t} + 2\frac{\partial u}{\partial x} = \frac{\partial^2 u}{\partial x^2}$$

is to be solved with initial conditions

$$u(x,0) = \begin{cases} 1, & 0 < x < 1, \\ 0, & 1 \le x \end{cases}$$

and an inlet boundary condition of $u(x = 0, t) = 1$.

(a) Sketch the solution as time evolves, paying attention to the rough timescales over which the solution changes. [15%]

(b) Using first order in time step of Δt and a central difference for space with grid spacing Δx, discretise this equation and write the update formula in the form

$$u_i^{N+1} = f(u_i^N, u_{i-1}^N, u_{i+1}^N)$$

where i is the index of the grid point in space and N is the index of the time point. Determine the order of accuracy in space. [20%]

(c) For this update scheme, determine the effective PDE and the magnitude of ratio of numerical to physical dissipation. The contribution of fourth derivatives or higher to the dissipation can be ignored. [50%]

(d) The time step is set to be *half* that of the minimum required for CFL stability. What percentage reduction in dissipation does this numerical scheme cause compared to the true solution for $\Delta x = 0.01$? [15%]

7 (a) The following numerical scheme is used to update a numerical solution using time steps of Δt on a grid with spacing Δx

$$u_i^{N+1} = u_i^{N-1} + \left(u_{i+1}^N + u_i^N - 2u_{i-1}^N\right) \frac{\Delta t}{\Delta x}$$

where i is the index of the grid point in space and N is the index of the time point. Determine

(i) the equation being solved, and, [35%]

(ii) the order of accuracy of the numerical solution. [15%]

(b) Euler's work equation can be used to consider the off design performance of turbomachines, often represented as a characteristic curve of stage loading ψ against flow coefficient ϕ.

(i) Assuming constant radius and constant axial velocity, show that the characteristic of an axial turbine is given by

$$\psi = \phi[\tan(\alpha_2) - \tan(\alpha_{3,rel})] - 1$$

where α_2 is the absolute angle at stator exit and $\alpha_{3,rel}$ is the relative angle at rotor exit. Sketch the characteristic and identify the no-load point. [10%]

(ii) Derive the equivalent equation for the ideal characteristic of an axial compressor stage and identify which angles are assumed constant in this. [15%]

(iii) For incompressible machines it is often preferred to use the non-dimensional stagnation pressure rise coefficient

$$\frac{\Delta p_0}{\rho U^2}$$

Show how this is related to stage loading and sketch the ideal characteristic of stagnation pressure rise coefficient against ϕ for an axial compressor. [15%]

(iv) Add a sketch of a typical, non-ideal, characteristic of stagnation pressure rise against ϕ for an axial compressor, explaining the differences from the ideal case. [10%]

8 A single stage axial turbine is run on a test stand. Air enters the turbine with zero swirl, inlet stagnation temperature of 1200 K and stagnation pressure of 10 bar. The turbine consists of a row of stators followed by a rotor. The annulus is designed such that the axial velocity and mean radius are constant throughout the machine. During testing with a stage loading coefficient $\psi = 1.3$, the absolute stator inlet Mach number $M_1 = 0.2$, the absolute stator exit Mach number $M_2 = 0.9$ and there is zero exit swirl from the rotor. Use $c_p = 1.01\,\mathrm{kJ\,kg^{-1}K^{-1}}$ and $\gamma = 1.4$ for air throughout this question.

(a) Determine the absolute swirl angle at stator exit. [10%]

(b) Determine the flow coefficient at which the turbine was tested, the blade speed and the exit stagnation temperature. [25%]

(c) Determine the relative flow angles at rotor inlet and exit. [15%]

(d) Calculate the relative rotor exit Mach number. [20%]

(e) The stator losses are measured using a total pressure loss coefficient

$$Y_p = \frac{p_{01} - p_{02}}{p_{02} - p_2} = 0.08$$

Determine the absolute stagnation pressure at stator exit. [10%]

(f) During the test the exit absolute total pressure was measured to be $p_{03} = 4.2$ bar. Calculate the total-total efficiency of the rotor and comment on the value. [20%]

END OF PAPER

3A3 Answers 2025

1 (b) (i) 0.06254 m^2 (ii) 0.07696 m^2 (c) 265.4 kW (d) 127.6 kW

2 (b) (i) 3.24 m (ii) 65.2 kg/s (c) 0.66m

3 (a) (ii) 1.3 (iii) 374.4 m/s (b) (ii) 1.272 (iii) 405.5 m/s

4 (c) 0.166

5 (b) 5.56 and 0.94, 4.65 and 0.955

6 (b) 2nd order in space (c) $2\Delta t$ (d) 1%

7 (a) (ii) 2nd order in time, 1st order in space.

8 (a) 76.2^o (b) 0.32, 434.2 m/s, 957.3 K (c) 43.4^o -72.3^o (d) 0.739 (e) 9.68 bar (f) 92%

CAH & CJC

May 25