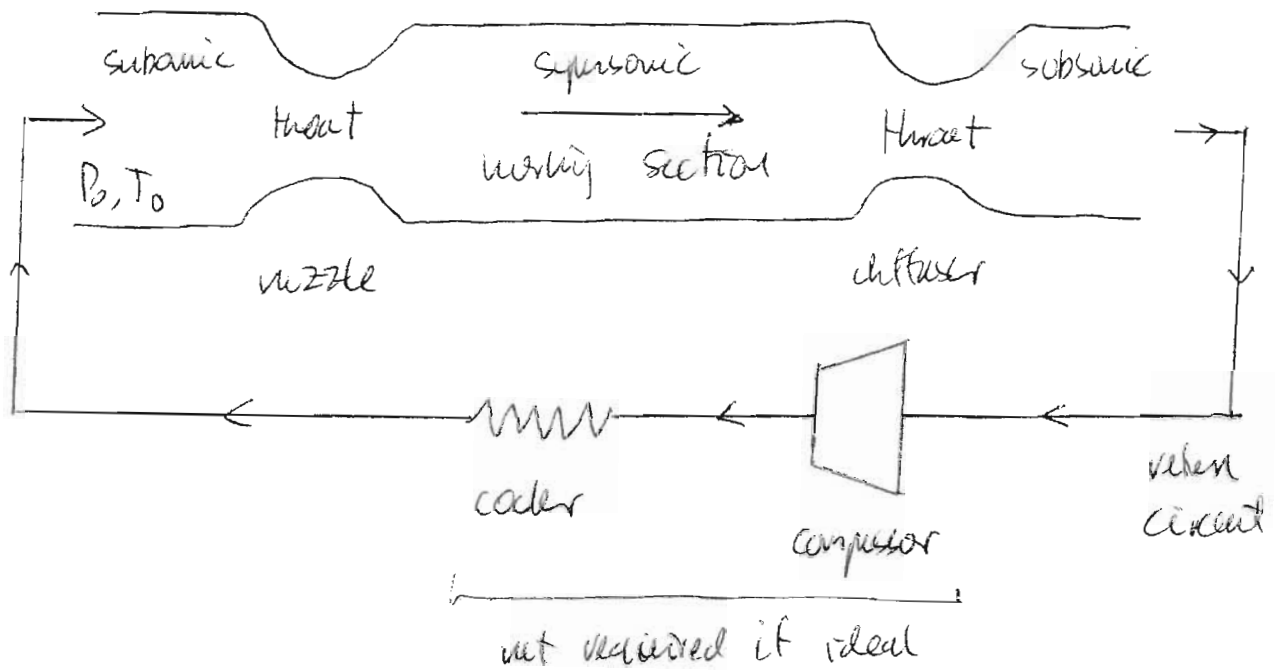


1.a)



In isentropic operation, the flow accelerates through $M=1$ at the nozzle throat to supersonic ($M=2-2$) in the working section, then decelerates isentropically through $M=1$ at the diffuser throat to subsonic in the return circuit.

There are no losses and no need to compensate for any drop in stagnation pressure using a compressor + cooler.

This is not possible in practice since there will be (at least) friction, and a shock wave will form in the working section on start-up. The shock must be repositioned downstream of the diffuser throat in normal operation, causing a further loss of stagnation pressure and requiring a compressor + cooler to restore P_0 and T_0 .

b) (i) at the nozzle throat $\frac{m \sqrt{4pT_0}}{A_n P_0} = 1.281$

at the entry to the W/S $\frac{m \sqrt{4pT_0}}{A_w P_0} = 0.6389$ [tables at $M=2.2$]

$$A_w = 0.2^2 = 0.04 \text{ m}^2$$

$$A_n = \frac{0.6389}{1.281} A_w = 0.499 \times 0.04 = \underline{0.01996 \text{ m}^2}$$

(ii) At startup have a $M=2.2$ shock in the W/S.

tables at $M=2.2$: $\frac{P_{0s}}{P_0} = 0.6281$

Diffuser throat $\frac{m \sqrt{4pT_0}}{A_D P_{0s}} \leq 1.281$ to swallow the shock.

$$\frac{A_D}{A_n} \geq \frac{P_0}{P_{0s}} ; A_D \geq \frac{0.01996}{0.6281} = \underline{0.0318 \text{ m}^2}$$

c) $P_{05} = 59 \text{ kPa}$ at exit ; $T_0 = 288 \text{ K}$ unchanged.

Isentropic compression for minimum work input :

$$\frac{T_{05}}{T_0} = \left(\frac{100}{59} \right)^{\frac{\gamma-1}{\gamma}} \quad ; \quad \gamma = 1.4$$

$$= 1.1627$$

$$\therefore T_{05} = 334.8 \text{ K}$$

\therefore Compressor work = $C_p \Delta T$ per kg flow

$$= 1005 \times (334.8 - 288)$$

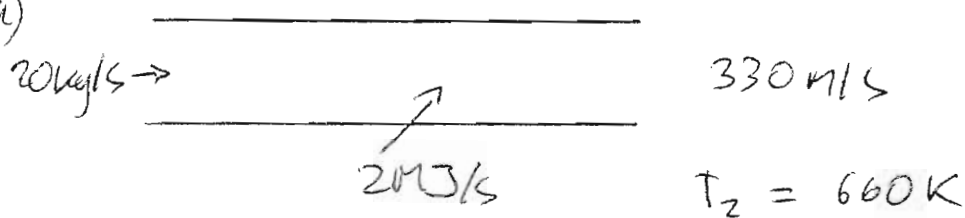
$$= 47034 \text{ J/kg}$$

In the nozzle $\frac{m_i \sqrt{C_p T_0}}{A_n P_0} = 1.281$

$$m_i = 1.281 \times \frac{0.01996 \times 1 \times 10^5}{\sqrt{1005 \times 288}} = 4.689 \text{ kg/s}$$

\therefore Compressor work is $4.689 \times 47034 = \underline{\underline{211 \text{ kW}}}$

2 a)



exit stagnation temperature : $\frac{T_{02}}{T_2} = \left(1 + \frac{\gamma-1}{2} M_2^2\right)$

$$T_2 = 660 \text{ K}; \quad a_2 = \sqrt{\gamma R T_2} = \sqrt{1.4 \times 287 \times 660}$$

$$= 515 \text{ m/s}$$

$$\therefore M_2 = \frac{330}{515} = 0.6407$$

$$\therefore \frac{T_{02}}{T_2} = 1 + \frac{1.4-1}{2} (0.6407)^2 = 1.0821$$

$$\therefore T_{02} = 714 \text{ K} \quad (\text{or use tables})$$

$$(i) \quad \dot{Q} = \dot{m} c_p (T_{02} - T_{01}) \quad \therefore T_{01} = 714 - \frac{2 \times 10^6}{20 \times 1005}$$

$$\therefore \underline{T_{01} = 614.5 \text{ K}}$$

(ii) impulse function unchanged (no friction)

$$\frac{F_2}{\dot{m} \sqrt{c_p T_{02}}} = \sqrt{\frac{T_{01}}{T_{02}}} = \sqrt{\frac{614.5}{714}}$$

$$= 0.9277$$

At exit $M_2 = 0.6407$: tables (0.64) $\frac{F_2}{\dot{m} \sqrt{c_p T_{02}}} = 1.0678$

$$\text{At entry } \frac{F_1}{\sqrt{\rho T_01}} = \frac{1.0678}{0.9277} = 1.151$$

$$\text{tables give } \eta_1 = 0.535 \text{ (approx)}$$

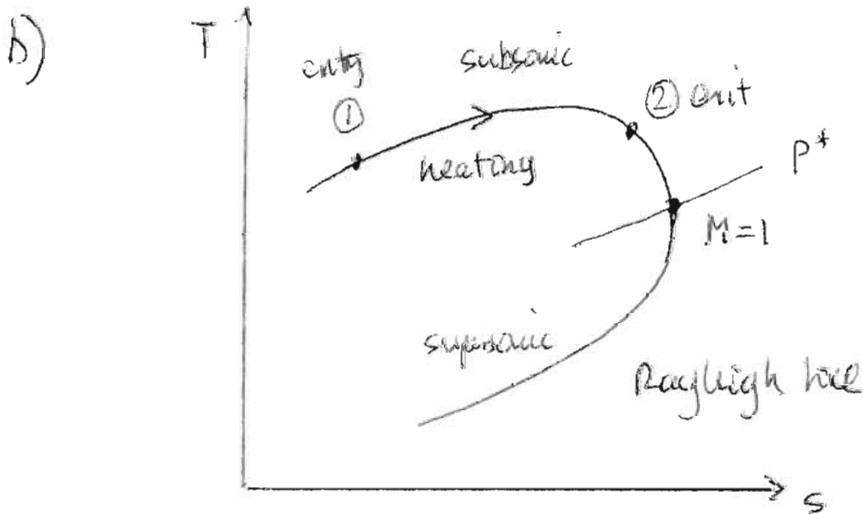
$$\text{and } T_1/T_01 = 0.946 \text{ (approx)}$$

$$\therefore T_1 = 0.946 \times 614.5 = 581.3 \text{ K}$$

$$\therefore a_1 = \sqrt{1.4 \times 287 \times 581.3} = 483.3 \text{ m/s}$$

$$\therefore u_1 = 0.535 \times 483.3 = \underline{258.5 \text{ m/s}}$$

($\eta_1 = 0.535$ dia)



c) choked exit $M_2 = 1$; $\frac{F_2}{\sqrt{\rho T_02}} = 0.9897$

$$\frac{T_02}{T_01} = \left(\frac{\frac{F_1}{\sqrt{\rho T_01}}}{\frac{F_2}{\sqrt{\rho T_02}}} \right)^2 = \left(\frac{1.15}{0.9897} \right)^2 = 1.35$$

(tables at $\eta_1 = 0.535$)

$$T_{02} = 1.35 \times T_{01} = 1.35 \times 614.5 = 829.6 \text{ K}$$

$$\therefore \dot{Q} = 20 \times 1005 \times (829.6 - 614.5) = 4323510 \text{ W}$$

i.e. 2.16 times more heat required to chill.

3 (a) Mention: equivalence of $\left\{ \begin{array}{l} \text{speed of sound } a = \sqrt{\gamma p / \rho} \\ \text{wave speed } c = \sqrt{g h} \end{array} \right.$

$\left\{ \begin{array}{l} \text{Mach number } Ma \\ \text{Froude number } Fr \end{array} \right.$

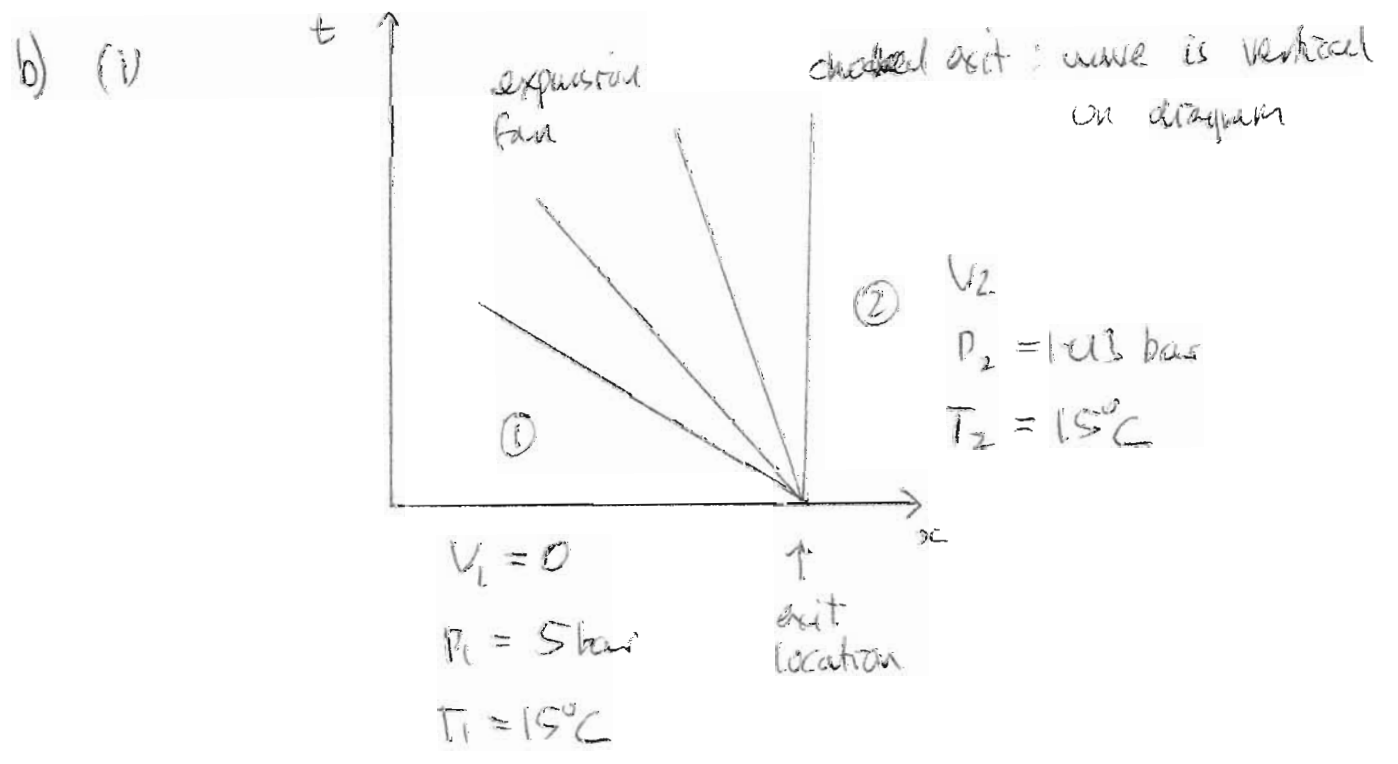
Riemann invariants $V \pm \frac{2a}{\gamma-1} \Leftrightarrow V \pm 2c$

\Rightarrow analogy best if $\gamma=2$

Depth \Leftrightarrow temperature ; Shock wave \Leftrightarrow hydraulic jump.

Usefulness includes ease of testing esp. for unsteady flows

Limitations include 2D restriction.



(ii) Pressure ratio $P_1/P_2 = 4.936$

temperature ratio $T_1/T_2 = (4.936)^{\frac{\gamma-1}{\gamma}}$ isentropic waves
 $= 1.578$

$\therefore T_2 = \frac{288.15}{1.578} = 182.6K$

$a_1 = \sqrt{1.4 \times 287 \times 288.15} = 340.3 \text{ m/s}$

$a_2 = \sqrt{1.4 \times 287 \times 182.6} = 270.9 \text{ m/s}$

Riemann (Lr waves) $V_1 + \frac{2a_1}{\gamma-1} = V_2 + \frac{2a_2}{\gamma-1}$

$V_2 = \frac{2}{0.4} (340.3 - 270.9) = 347 \text{ m/s}$

$V_2 > a_2 \therefore$ exit is choked (confirmed).

Since exit is choked, $a^* = \frac{2a_1}{\gamma+1} = \frac{2}{2.4} \times 340.3 = 283.6 \text{ m/s}$

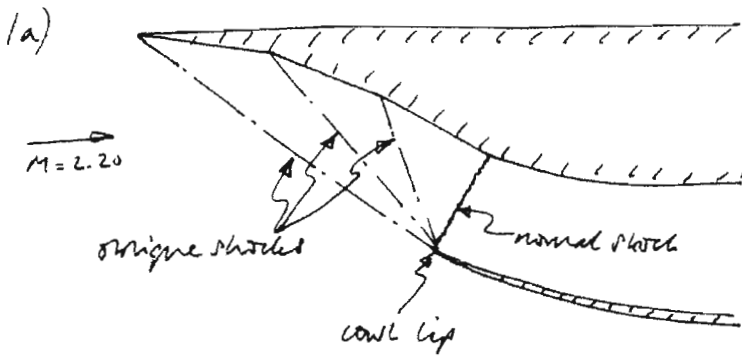
Density at exit is $\frac{\rho_2}{\rho_1} = \left(\frac{T_2}{T_1}\right)^{\frac{1}{\gamma-1}} = \left(\frac{a^*}{a_1}\right)^{\frac{2}{\gamma-1}} = \left(\frac{283.6}{340.3}\right)^{\frac{2}{\gamma-1}} = 0.402$

$\rho_1 = \frac{P_1}{RT_1} = \frac{5 \times 10^5}{287 \times 288.15} = 6.05 \text{ kg/m}^3$

$\therefore \rho^* = 0.402 \times 6.05 = 2.43 \text{ kg/m}^3$

sq

$$\dot{m} = \rho VA \quad ; \quad A = \frac{\dot{m}}{\rho V} = \frac{200}{243 \times 283.6} = \frac{0.29 \text{ m}^3}{\text{(Square)}} \text{ m}^2$$



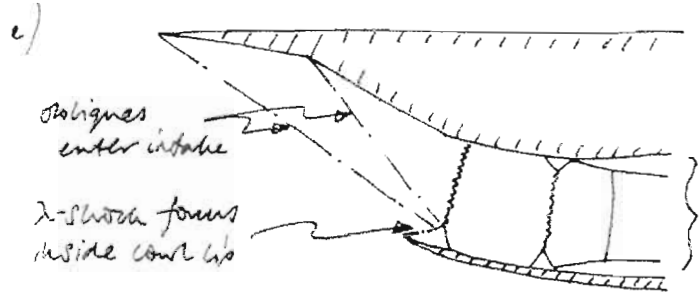
M_1	δ	P_{02}/P_{01}	M_2
2.20	8°	0.9902	1.899
1.90	6°	0.9968	1.690
1.69	6°	0.9972	1.485
1.49	N	0.934	

\therefore total pressure ratio = 0.919
 i.e. 8.1% loss
 The loss through the normal is far greater than the oblique as increase angle of both 6° ramps.

c) All other things equal, the cowl lip will move forwards and upwards to retain forces. Hence expect the intake to become shorter (\therefore lighter) and of reduced frontal area (\therefore potentially lower drag)

M_1	δ	P_{02}/P_{01}	M_1
2.05	8°	0.9915	1.761
1.76	13°	0.9716	1.289
1.29	N	0.981	

\therefore total pressure ratio = 0.945
 i.e. 5.5% loss
 This is a significant improvement on the initial design: the shock waves are far better matched and inlet speed lower.



possible shock formation in subsonic (as designed) diffuser. May lead to separation on endwalls. Resonance distortion may provoke engine surge.

5

- ii) Elliptic - a "jumpy" problem which must satisfy all boundary conditions
 ii) Hyperbolic - a "wave" problem: only some boundary conditions apply.

b) Introduce $\tilde{x} = \frac{x}{c}$, $\tilde{y} = \beta \frac{y}{c}$ \therefore b.c. $\rightarrow \frac{\partial \phi}{\partial \tilde{y}} = \frac{c}{\beta} \frac{\partial \phi}{\partial y} = \frac{\tau}{\beta} U_\infty g'(\tilde{x})$

So take $\phi = \frac{\tau}{\beta} U_\infty \tilde{\phi} \rightarrow \frac{\partial \tilde{\phi}}{\partial \tilde{y}} = g'(\tilde{x})$ on $\tilde{y} = 0$ for $0 < \tilde{x} < 1$

Hence $\beta^{-2} (1 - M_\infty^2) \frac{\partial^2 \tilde{\phi}}{\partial \tilde{x}^2} + \frac{\partial^2 \tilde{\phi}}{\partial \tilde{y}^2} = 0$ \therefore take $\beta = \sqrt{1 - M_\infty^2}$

$$\therefore \frac{\partial^2 \tilde{\phi}}{\partial \tilde{x}^2} + \frac{\partial^2 \tilde{\phi}}{\partial \tilde{y}^2} = 0$$

c) Consider x -momentum:

$$\frac{\partial p}{\partial x} = -\rho \left(u \frac{\partial u}{\partial x} + v \frac{\partial v}{\partial x} \right) \rightarrow \frac{\partial p}{\partial x} = -\rho_\infty U_\infty \frac{\partial u}{\partial x} = -\rho_\infty U_\infty \frac{\partial^2 \phi}{\partial x^2}$$

$$\Rightarrow p = -\rho_\infty U_\infty \frac{\partial \phi}{\partial x} + f(y) \quad \text{but far upstream } \frac{\partial \phi}{\partial x} = 0 \text{ as } p \text{ is uniform } \therefore f(y) = p_\infty$$

$$p - p_\infty = -\rho_\infty U_\infty \frac{\partial \phi}{\partial x} \quad \therefore C_p = \frac{p - p_\infty}{\frac{1}{2} \rho U_\infty^2} = \frac{-2}{U_\infty} \frac{\partial \phi}{\partial x} = \frac{-2}{U_\infty} \frac{\tau U_\infty}{\sqrt{1 - M_\infty^2}} \frac{1}{c} \frac{\partial \tilde{\phi}}{\partial \tilde{x}}$$

$$\therefore C_p = \frac{k \tau}{c \sqrt{1 - M_\infty^2}} \quad \text{where } k \text{ only depends on shape factor, } g.$$

$$\text{as: } C_{p2} = \left(\frac{\tau_2}{\tau_1} \frac{c_1}{c_2} \frac{\sqrt{1 - M_{\infty 1}^2}}{\sqrt{1 - M_{\infty 2}^2}} \right) C_{p1} \quad \text{hence } C_p = \frac{C_{p0}}{\sqrt{1 - M_\infty^2}}$$

d) Expect C_u & C_m to scale similarly as these are dominated by pressure but not C_D as also contains viscous terms.

e) Max $M_\infty \sim 0.7$ for typical airfoils beyond which mixed supersonic (hyperbolic) and subsonic (elliptic) flows is likely to occur.

$$6 \text{ a) } \frac{\partial \phi}{\partial t} + F(\phi, t) = 0$$

$$\text{Taylor series } \phi^{n+1} = \phi^n + \Delta t \left. \frac{\partial \phi}{\partial t} \right|^n + \frac{\Delta t^2}{2!} \left. \frac{\partial^2 \phi}{\partial t^2} \right|^n + \frac{\Delta t^3}{3!} \left. \frac{\partial^3 \phi}{\partial t^3} \right|^n + O(\Delta t^4) \quad (1)$$

$$F^{n+1} = F^n - \Delta t \left. \frac{\partial F}{\partial t} \right|^n + \frac{\Delta t^2}{2!} \left. \frac{\partial^2 F}{\partial t^2} \right|^n - \frac{\Delta t^3}{3!} \left. \frac{\partial^3 F}{\partial t^3} \right|^n + O(\Delta t^4)$$

$$\text{Scheme is } \phi^{n+1} = \phi^n - \Delta t \left(\frac{3}{2} F^n - \frac{1}{2} F^{n-1} \right)$$

$$= \phi^n - \Delta t \left[\frac{3}{2} F^n - \frac{1}{2} \left(F^n - \Delta t \left. \frac{\partial F}{\partial t} \right|^n + \frac{\Delta t^2}{2!} \left. \frac{\partial^2 F}{\partial t^2} \right|^n + \dots \right) \right]$$

$$= \phi^n - \Delta t F^n - \frac{\Delta t^2}{2} \left. \frac{\partial F}{\partial t} \right|^n + \frac{\Delta t^3}{2 \cdot 2!} \left. \frac{\partial^2 F}{\partial t^2} \right|^n + \dots \quad (2)$$

but $\left. \frac{\partial \phi}{\partial t} \right|^n = -F^n$ and $\left. \frac{\partial^2 \phi}{\partial t^2} \right|^n = - \left. \frac{\partial F}{\partial t} \right|^n$ from original equation.

\therefore (1) and (2) match to second order : QED.

b) Subtract the series (1)-(2):

$$0 = 0 + \Delta t \left. \frac{\partial \phi}{\partial t} \right|^n + \frac{\Delta t^2}{2!} \left. \frac{\partial^2 \phi}{\partial t^2} \right|^n + \frac{\Delta t^3}{3!} \left. \frac{\partial^3 \phi}{\partial t^3} \right|^n + \Delta t F^n + \frac{\Delta t^2}{2!} \left. \frac{\partial F}{\partial t} \right|^n - \frac{\Delta t^3}{2 \cdot 2!} \left. \frac{\partial^2 F}{\partial t^2} \right|^n + O(\Delta t^4)$$

$$\frac{\partial \phi}{\partial t} \Big|^{(n)} + F^{(n)} = -\frac{\Delta t}{2!} \left(\frac{\partial^2 \phi}{\partial t^2} \Big|^{(n)} + \frac{\partial F}{\partial t} \Big|^{(n)} \right) - \frac{\Delta t^2}{3!} \left(\frac{\partial^3 \phi}{\partial t^3} \Big|^{(n)} - \frac{\partial^2 F}{\partial t^2} \cdot \frac{3!}{2!} + O(\Delta t^3) \right)$$

Approximate $\frac{\partial \phi}{\partial t} = -F \approx -A \frac{\partial \phi}{\partial x}$

then $\frac{\partial F}{\partial t} = -\frac{\partial^2 \phi}{\partial t^2} \approx A^2 \frac{\partial^2 \phi}{\partial x^2}$ since A is constant,

and $\frac{\partial^2 F}{\partial t^2} = -\frac{\partial^3 \phi}{\partial t^3} \approx -A^3 \frac{\partial^3 \phi}{\partial x^3}$

$$\therefore \frac{\partial \phi}{\partial t} \Big|^{(n)} + F^{(n)} = -\frac{\Delta t}{2!} \left(-A^2 \frac{\partial^2 \phi}{\partial x^2} + A^2 \frac{\partial^2 \phi}{\partial x^2} \right) - \frac{\Delta t^2}{3!} \left(-A^3 \frac{\partial^3 \phi}{\partial x^3} + A^3 \frac{\partial^3 \phi}{\partial x^3} \cdot \frac{3}{2} \right) + O(\Delta t^3)$$

\therefore leading error term is $-\frac{5}{12} A^3 \frac{\partial^3 \phi}{\partial x^3}$

This contains an odd-order derivative

\therefore the error is likely to be dispersive

\therefore false convection.

$$S(a) \quad \frac{\partial \phi}{\partial t} + A \frac{\partial \phi}{\partial x} = 0$$

$$\frac{\phi_i^{n+1} - \phi_i^n}{\Delta t} + A \left(\frac{\phi_{i+1}^n - \phi_{i-1}^n}{2\Delta x} \right) = 0$$

Forward time Centred space (FTCS)

b) The original equation is hyperbolic and information is transferred along the characteristic lines $\left. \frac{\partial x}{\partial t} \right|_n = A$. The FTCS scheme fails to mimic the correct behaviour by transferring information from downstream, as well as from upstream.

$$c) \text{ Use upwinding: } \frac{\phi_i^{n+1} - \phi_i^n}{\Delta t} + A \left(\frac{\phi_i^n - \phi_{i-1}^n}{\Delta x} \right) = 0$$

Forward time Backward space

\Rightarrow only 1st order accurate and suffers from false diffusion, but stable if $\frac{A\Delta t}{\Delta x} \leq 1$.

HALF QUESTION

1/2

$$b) i) T \frac{ds}{dr} = \frac{dh}{dr} - \frac{1}{\rho} \frac{dp}{dr}$$

ISENTROPIC FLOW

 $\Rightarrow \frac{dh}{dr} = \frac{1}{\rho} \frac{dp}{dr}$

FOR STEADY, STATIONARY FRAME, CONSTANT RADIUS ($V_r=0$)
FLOW (ie NO MERIDIONAL CURVATURE):

$$\frac{1}{\rho} \frac{dp}{dr} = \frac{V_\theta^2}{r}$$

$$\Rightarrow \frac{d}{dr} \left(h_0 - \frac{1}{2} (V_x^2 + V_\theta^2) \right) = \frac{1}{\rho} \frac{dp}{dr} = \frac{V_\theta^2}{r}$$

$$\frac{dh_0}{dr} - V_x \frac{dV_x}{dr} - V_\theta \frac{dV_\theta}{dr} = \frac{V_\theta^2}{r}$$

$$\frac{dh_0}{dr} = V_x \frac{dV_x}{dr} + V_\theta \frac{dV_\theta}{dr} + \frac{V_\theta^2}{r}$$

Now: $V_\theta \frac{dV_\theta}{dr} + \frac{V_\theta^2}{r} = \frac{V_\theta}{r} \left(r \frac{dV_\theta}{dr} + V_\theta \frac{dr}{dr} \right) = \frac{V_\theta}{r} \frac{d}{dr} (rV_\theta)$

$$\Rightarrow \text{SRE: } \frac{dh_0}{dr} = V_x \frac{dV_x}{dr} + \frac{V_\theta}{r} \frac{d}{dr} (rV_\theta) \quad \boxed{30\%}$$

b) ii) $\frac{dh_0}{dr}$ = Radial distribution of work transfer.

$\frac{dV_x}{dr}$ = Radial distribution of axial velocity (flow)

(rV_θ) = Vortex-Law — Very important in determining blade shapes & flow type.

10%

b) (iii) Free-Vortex when $rV_{\theta} = \text{constant}$

Thus: $\frac{dh_0}{dr} = V_{\theta} \frac{dV_{\theta}}{dr}$

Provided uniform work transfer across the radius is $\frac{dh_0}{dr} = 0$ (achieved by appropriate choice of blade shape)

Then: $V_{\theta} \frac{dV_{\theta}}{dr} = 0$ so obtain uniform distribution of axial velocity.

10%

FULL QUESTION

1/3

a) $C_p \Delta T_o = \Delta h_o = U \Delta V_o$ (constant radius)

$$U = \frac{C_p \Delta T_o}{\Delta V_o} = \frac{1149(1500 - 1347.1)}{(413.4 - -145.6)} = \underline{\underline{314.3 \text{ m/s}}} \quad \boxed{5\%}$$

b) $\rho_1 = \frac{P_1}{RT_1} = \frac{1469.9 \times 10^3}{287 \times 1492.4} = \underline{\underline{3.4318 \text{ kg/m}^3}}$

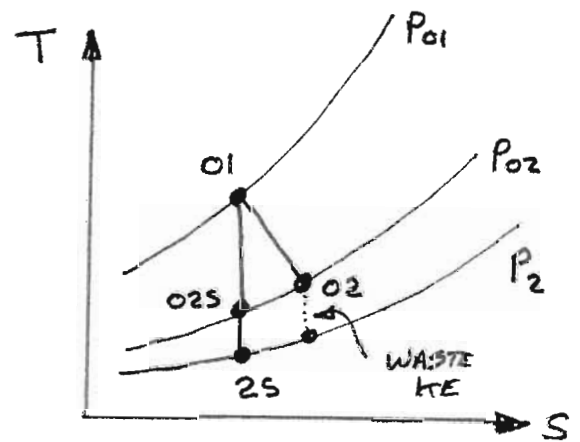
$$\dot{m}_1 = \rho_1 A_1 V_{x1} = 3.4318 \times 0.15 \times 132.0 = \underline{\underline{67.95 \text{ kg/s}}}$$

$$\text{POWER} = \dot{m} \Delta h_o = 67.95 \times 1149 \times (1500 - 1347.1)$$

$$= \underline{\underline{11.94 \text{ MW}}}$$

10%

c) $T_{02s} = T_{01} \left(\frac{P_{02}}{P_{01}} \right)^{\frac{1-\gamma}{\gamma}}$
 $= 1500 \left(\frac{944.0}{1500} \right)^{\frac{0.333}{1.333}} = 1336.1 \text{ K}$



$$\eta_{tt} = \frac{T_{01} - T_{02}}{T_{01} - T_{02s}} = \frac{1500 - 1347.1}{1500 - 1336.1}$$

$\eta_{tt} = 93.29\%$ — Relevant to Turbojet as exit KE used!

$$T_{2s} = T_{01} \left(\frac{P_2}{P_{01}} \right)^{\frac{1-\gamma}{\gamma}} = 1500 \left(\frac{887.4}{1500} \right)^{\frac{0.333}{1.333}} = 1315.7 \text{ K}$$

$$\eta_{ts} = \frac{T_{01} - T_{02}}{T_{01} - T_{2s}} = \frac{1500 - 1347.1}{1500 - 1315.7} = \underline{\underline{82.96\%}}$$

Relevant to land based as exit KE is wasted.

20%

$$d) \frac{\Delta h_o}{U^2} = \frac{C_p \Delta T_o}{U^2} = \frac{1149(1500 - 1347.1)}{314.3^2} = \underline{\underline{1.78}}$$

Measures the amount of work exchange, 1.78 is a sensible value for a turbine.

$$\text{REACTION} = \frac{\Delta h_{\text{rotor}}}{\Delta h_{\text{stage}}} = \frac{1414.9 - 1326.4}{1492.4 - 1326.4} = \underline{\underline{53.3\%}}$$

Reaction is an approximate measure of pressure drop across the rotor compared to the stage. 53.3% says the two blade rows have similar pressure drops.

15%

$$e) \gamma_P|_{\text{STATOR}} = \frac{P_{01} - P_{02}}{P_{02} - P_2} = \frac{1500.0 - 1480.3}{1480.3 - 1171.7} = \underline{\underline{0.0638}}$$

For rotor need P_{02}^{REL} and P_{03}^{REL} .

$$V_{02}^{\text{REL}} = 413.4 - 314.3 = \underline{\underline{99.1 \text{ m/s}}}$$

$$T_{02}^{\text{REL}} = 1414.9 + (157.0^2 + 99.1^2) / (2 \times 1149) = \underline{\underline{1429.9 \text{ K}}}$$

$$P_{02}^{\text{REL}} = P_2 \left(\frac{T_{02}^{\text{REL}}}{T_2} \right)^{\frac{\gamma}{\gamma-1}} = 1171.7 \left(\frac{1429.9}{1414.9} \right)^{\frac{1.333}{0.333}} = \underline{\underline{1222.2 \text{ kPa}}}$$

$$V_{03}^{\text{REL}} = -145.6 - 314.3 = \underline{\underline{-459.9 \text{ m/s}}}$$

$$T_{03}^{\text{REL}} = 1326.4 + (162.0^2 + 459.9^2) / (2 \times 1149) = \underline{\underline{1429.9 \text{ K}}}$$

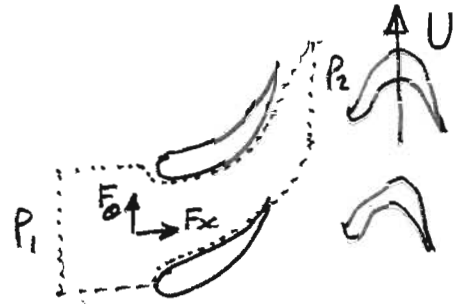
$$P_{03}^{\text{REL}} = 887.4 \left(\frac{1429.9}{1326.4} \right)^{\frac{1.333}{0.333}} = \underline{\underline{1198.8 \text{ kPa}}}$$

(SAME RADIUS = CONSTANT)

$$\gamma_P|_{\text{ROTOR}} = \frac{P_{02}^{\text{REL}} - P_{03}^{\text{REL}}}{P_{03}^{\text{REL}} - P_3} = \frac{1222.2 - 1198.8}{1198.8 - 887.4} = \underline{\underline{0.075}}$$

25%

f) F_x, F_θ Force on flow through the stator passage.



TOTAL FORCE = Δ (MOM. FLUX)

AXIAL $P_1 A_1 + F_{xc} - P_2 A_2 = \dot{m} V_{x2} - \dot{m} V_{x1}$ N.B. $A_1 = A_2$

$F_x = (P_2 - P_1) A_1 + \dot{m} (V_{x2} - V_{x1})$

$= (1171.7 \times 10^3 - 1469.9 \times 10^3) \times 0.15 + 67.95 (157.0 - 132.0)$

$F_x = -43.03 \text{ kN}$ ~~Force~~ Axial component of

force in the flow in inlet upstream direction.

[NOTE: VERY IMPORTANT THAT $A_1 = A_2$ SO CAN IGNORE ANNULUS WALL FORCES.]

$F_\theta = \dot{m} (V_{\theta 2} - V_{\theta 1}) = 67.95 (413.4 - 0) = \underline{\underline{28.09 \text{ kN}}}$

Tangential component of force on the flow is in the same direction as rotor rotation.

ROTOR

Since $A_3 = 0.18 \text{ m}^2 > A_2 = 0.15 \text{ m}^2$ the axial component of the force on the flow has a contribution from the annulus wall pressure distribution. This either has to be approximated $\approx \frac{1}{2} (P_2 + P_3)$ or obtained from CFD etc.

25%