Version RT/8

EGT2
ENGINEERING TRIPOS PART IIA

Wednesday 23 April 2014 9.30 to 12.30

Module 3A3

FLUID MECHANICS II

*Answer not more than **five** questions.*

All questions carry the same number of marks.

*The **approximate** percentage of marks allocated to each part of a question is indicated in the right margin.*

*Write your candidate number **not** your name on the cover sheet.*

STATIONERY REQUIREMENTS TO BE SUPPLIED FOR THIS EXAM
Single-sided script paper

SPECIAL REQUIREMENTS
Compressible Flow Data Book (38 pages)
CUED approved calculator allowed
Engineering Data Book

You may not start to read the questions printed on the subsequent pages of this question paper until instructed to do so.

1 (a) Draw a carefully-labelled diagram of an ideal continuous-running, closed-circuit supersonic wind tunnel. Explain why it is not possible in practice to operate such a tunnel isentropically. [30%]

(b) The working section of a supersonic wind tunnel is 20 cm square and is required to operate continuously at a Mach number 2.2. At the entry to the nozzle, the stagnation temperature is 288 K and the stagnation pressure is 100 kPa.

(i) What is the area of the nozzle throat? [20%]

(ii) What is the minimum area of the diffuser throat that will allow the tunnel to start? [20%]

(c) During the starting procedure, the stagnation pressure at the diffuser exit is measured at 59 kPa. What is the minimum power required to start the tunnel? [30%]

2 Air flows through a duct which has frictionless walls and constant cross-sectional area. The mass flow rate is constant at 20 kg s^{-1} and a total of 2 MW of heat is added along the length of the tube. Measurements taken at the exit of the tube indicate that the exit static temperature is 660 K and the exit velocity is 330 m s^{-1}.

(a) Calculate

(i) the inlet stagnation temperature [15%]

(ii) the inlet velocity. [30%]

(b) Draw and label a *T-s* diagram to show the complete process. [25%]

(c) How much more heat would have to be added to choke the exit of the pipe? [30%]

3 (a) Explain the hydraulic analogy between waves in a compressible gas flow and waves on the surface of shallow water. Indicate which quantities can be regarded as equivalent within the analogy, and discuss the usefulness and limitations of the analogy. [30%]

(b) A process plant includes a pressure relief system consisting of a long pipe with a thin metal diaphragm at the end. The diaphragm is designed to burst instantaneously when the pressure difference across it exceeds 4 bar. The outside atmosphere is at a pressure of 1.013 bar and a temperature of 15° C. The pipe contains compressed air at the same temperature. The pressure relief system is to be designed for a venting flow rate of 200 kg s^{-1}.

(i) Draw an *x-t* diagram to illustrate the wave pattern shortly after the diaphragm bursts. [20%]

(ii) Calculate the minimum diameter of the pipe required to meet the design venting flow rate. [50%]

4 An aircraft is being designed for long-range flight at supersonic speeds. Figure 1 shows a cross-section through one of its proposed two-dimensional engine intakes. The intake comprises three ramps of different angles as labelled in Fig. 1. It is designed to produce a four-shock system for external compression, focused on the cowl lip at the design cruise Mach number $M_\infty = 2.20$. Conditions may be assumed to be uniform in the direction into the page.

(a) Draw a carefully labelled sketch of the shock system. Assume that the flow upstream of the intake is not affected by the aircraft fuselage. [20%]

(b) Calculate the stagnation pressure loss through the shock system. What modifications would you make to the design to improve this value? [20%]

(c) Partway through the development process it is decided to reduce the design cruise Mach number to $M_\infty = 2.05$ and to simplify the intake by replacing both 6° ramps with a single 12° ramp. Briefly describe any additional geometric changes to the intake required to preserve a focussed shock system. [15%]

(d) Calculate the stagnation pressure loss through the shock system of the new intake and comment on these values compared to those calculated in part (b). [20%]

(e) The aircraft is built with the $M_\infty = 2.05$ intake design. If, in service, the aircraft is inadvertently allowed to accelerate to $M_\infty = 2.20$, carefully sketch the resulting shock system and briefly comment on its desirability. [25%]

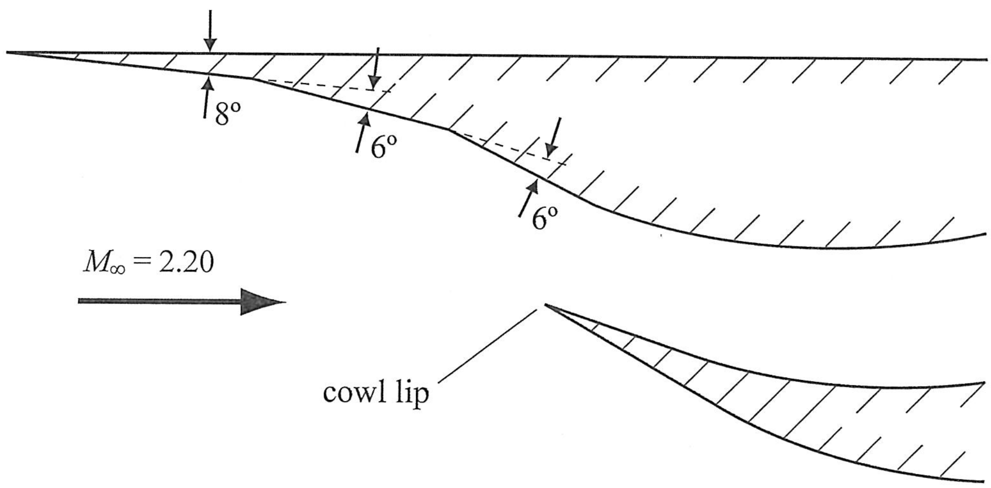

Fig. 1.

5 Steady, isentropic, two dimensional flow with a free stream Mach number of M_∞ that is slightly disturbed from uniform by a thin two-dimensional aerofoil may be described by a flow potential, ϕ , in cartesian coordinates:

$$\left(1-M_\infty^2\right)\frac{\partial^2\phi}{\partial x^2}+\frac{\partial^2\phi}{\partial y^2}=0$$

$$\text{subject to: } \frac{\partial\phi}{\partial y}=U_\infty\frac{\tau}{c}g'\left(\frac{x}{c}\right) \quad \text{on } y=0,\ 0<x<c$$

where g' is a shape function for the aerofoil, τ is a measure of its thickness, c is its chord and the free stream velocity is U_∞.

(a) Briefly describe the behaviour of this equation when:

(i) $M_\infty < 1$
(ii) $M_\infty > 1$ [15%]

(b) By making appropriate substitutions and assumptions, show that it is possible to re-write the potential equation and its boundary condition as:

$$\frac{\partial^2\tilde{\phi}}{\partial\tilde{x}^2}+\frac{\partial^2\tilde{\phi}}{\partial\tilde{y}^2}=0$$

$$\text{subject to: } \frac{\partial\tilde{\phi}}{\partial\tilde{y}}=g'(\tilde{x}) \quad \text{on } \tilde{y}=0,\ 0<\tilde{x}<1$$

[25%]

(c) By considering the x-component of momentum, show that it is possible to express a pressure coefficient, c_p , for the airfoil at an arbitrary subsonic Mach number, M_∞ , in terms of the equivalent incompressible value, c_{p0} , by:

$$c_p=\frac{c_{p0}}{\sqrt{1-M_\infty^2}}$$

[30%]

(d) Which gross aerofoil performance parameters of interest to the designer would you expect to scale in a similar way to c_p ? Briefly justify your choices. [15%]

(e) Suggest a practical upper limit for M_∞ as used in part (c). Give brief reasons why this relationship will not hold above this value. [15%]

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6 The differential equation

$$\frac{\partial \phi}{\partial t} + F(x,t) = 0$$

is to be solved numerically using an explicit Adams-Bashforth time-marching scheme

$$\phi^{n+1} = \phi^{n} - \Delta t\left(\frac{3}{2}F^{n} - \frac{1}{2}F^{n-1}\right)$$

with a time step Δt .

(a) Show that the scheme is second order accurate. [40%]

(b) Making the approximation

$$F(x,t) \approx A\frac{\partial \phi}{\partial x}$$

where A is a positive constant, determine whether this scheme will suffer from false convection or false diffusion. [60%]

7 Note that part (a) of this question is not related to part (b).

(a) (i) Discretise the scalar convection equation

$$\frac{\partial \phi}{\partial t} + A\frac{\partial \phi}{\partial x} = 0$$

Where A is a positive constant, using first-order forward time differencing and second-order centered spacing. [25%]

(ii) Explain on physical grounds why the resulting scheme is unconditionally unstable. [15%]

(iii) Suggest a simple change to make the scheme stable. [10%]

(b) Carefully explaining all the assumptions made, derive the Simple Radial Equilibrium equation:

$$\frac{dh_0}{dr} = V_x\frac{dV_x}{dr} + \frac{V_\theta}{r}\frac{d}{dr}(rV_\theta)$$

where r is the radial distance, h_0 the stagnation enthalpy and V_x , V_θ are the axial and tangential velocity components respectively. [30%]

(ii) Give a physical interpretation of each of the terms in the Simple Radial Equilibrium equation. [10%]

(ii) From the Simple Radial Equilibrium equation, obtain the "free-vortex" distribution and state the conditions required to achieve it. [10%]

(TURN OVER

8 The mean radius, annulus cross-sectional areas and aerodynamic operating conditions of a single stage axial flow turbine are listed in Table 1.

(a) By considering the specific work, determine the turbine blade speed at the mean radius. [5%]

(b) Calculate the total output power of the turbine. [10%]

(c) Calculate both the total-to-total and total-to-static isentropic efficiencies for the turbine and include a suitable *T-s* diagram. Explain which measure would be most appropriate for:

(i) an aeroengine turbine;

(ii) a land based, industrial gas turbine. [25%]

(d) Calculate the stage loading and the degree of reaction for the turbine. Give a physical interpretation for both of these quantities. [20%]

(e) Calculate the stagnation pressure loss coefficients for the stator blade row. [10%]

(f) Calculate the axial and tangential forces *on the total flow through* the stator blade row, and draw a diagram to clearly show the direction of these forces. Explain, without any calculations, which of the axial and tangential forces *on the total flow through* the rotor blade row would be difficult to calculate. [30%]

You may assume that the working fluid has:

$$R = 287\ \mathrm{J\,kg^{-1}\,K^{-1}},\ \gamma = 1.333\ \text{and}\ c_p = 1149\ \mathrm{J\,kg^{-1}\,K^{-1}}.$$

		Stator inlet	Stator exit, rotor inlet	Rotor exit
Mean radius	(m)	0.3	0.3	0.3
Cross-sectional area	(m^2)	0.15	0.15	0.18
Stagnation temperature	(K)	1500.0	1500.0	1347.1
Static temperature	(K)	1492.4	1414.9	1326.4
Stagnation pressure	(kPa)	1500.0	1480.3	944.0
Static pressure	(kPa)	1469.9	1171.7	887.4
Axial velocity	($m\ s^{-1}$)	132.0	157.0	162.0
Tangential velocity	($m\ s^{-1}$)	0.0	413.4	− 145.6

Table 1.

END OF PAPER

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ENGINEERING TRIPOS PART IIA 2014

MODULE 3A3: FLUID MECHANICS II
Double Module

List of numerical answers

Q1: b) i) $A=0.01996m^2$ ii) $Ad/An > 0.0318m^2$
c) m = 4.753 kg/s Compressor work: 224 kW

Q2: a) i) $T_{01} = 614.5$ K ii) $u_1 = 258.5$ m/s
c) Q = 4.32 MW

Q3: c) $A=0.29m^2$, (54cm square)

Q4: b) 8.1% loss
d) 5.5% loss

Q5: a) 314 m/s
b) 11.94 MW
c) total-to-total: 93.3%, total-to-static: 83%
d) stage loading: 1.78, reactiom: 53.3%
e) 0.0638
f) $F_x = -43.03$ kN, $F_{theta} = 28.09$ kN