EGT2
ENGINEERING TRIPOS PART IIA

Wednesday 4 May 2022 9.30 to 12.40

Module 3A3

FLUID MECHANICS II

Answer not more than ***five*** *questions.*

All questions carry the same number of marks.

The ***approximate*** *percentage of marks allocated to each part of a question is indicated in the right margin.*

Write your candidate number ***not*** *your name on the cover sheet.*

STATIONERY REQUIREMENTS
Single-sided script paper

SPECIAL REQUIREMENTS TO BE SUPPLIED FOR THIS EXAM
CUED approved calculator allowed
Attachment: Compressible Flow Data Book (38 pages)
Engineering Data Book

10 minutes reading time is allowed for this paper at the start of the exam.

You may not start to read the questions printed on the subsequent pages of this question paper until instructed to do so.

You may not remove any stationery from the Examination Room.

1 A rocket test facility consists of a convergent-divergent nozzle connected between a large reservoir upstream and a large vessel downstream. The cross-sectional area of the nozzle at its exit is three times that at its throat. The facility operates using compressed air. Initially the pressure is uniform throughout and there is no flow. Friction and heat transfer can be assumed to be negligible.

(a) The pressure in the downstream vessel is reduced to a level below that in the reservoir.

(i) Describe, with the aid of sketched pressure distributions, the steady flow regimes that occur in the nozzle for different values of the downstream pressure. [15%]

(ii) Under what circumstances does the pressure at the nozzle exit differ from that in the downstream vessel? Sketch representative flow fields for these cases. [15%]

(b) For conditions where the nozzle is choked, calculate the ratio of the exit pressure to the reservoir pressure when:

(i) there are no shock waves present and the flow is subsonic at exit; [10%]

(ii) there are no shock waves present and the flow is supersonic at exit; [10%]

(iii) there is a shock wave just inside the exit of the nozzle. [20%]

(c) At another operating condition, the stagnation pressure at the nozzle exit is found to be 77.2% of the reservoir pressure. Specify and calculate (as a proportion of reservoir pressure) all salient values of the pressure distribution in this case. [30%]

2 (a) Supersonic flow enters a straight pipe of constant cross-sectional area. Heat transfer is negligible, but the pipe wall is rough. Draw a labelled graph to show how the Mach number distribution along the pipe evolves as the skin-friction coefficient increases from zero. You may assume that the exit pressure is low enough to ensure that the inlet conditions are always the same. [30%]

(b) Air flows in a pipe of length 5.9 m and inside diameter 0.2 m. The inlet stagnation pressure is 2.7 bar, and the static pressure at the pipe exit is 1 bar. If the exit is choked, and there are no shocks in the pipe, find:

(i) the two possible values of the Mach number at the inlet; [15%]

(ii) the skin-friction coefficient c_f corresponding to each. [15%]

(c) In practice, the pipe of part (b) is found to have a skin-friction coefficient of 0.003. Show that, for the supersonic inflow condition and with the pipe exit choked, a shock wave occurs between 0.8 m and 1.0 m downstream of the inlet. [40%]

3 (a) Consider the one-dimensional flow of a perfect gas with sound speed a and specific-heat ratio γ. With the velocity (positive rightwards) denoted V, show that the quantity

$$V - \frac{2}{\gamma - 1}a$$

remains constant across a right-running sound wave. [20%]

(b) A storage tank containing compressed air at ambient temperature is fitted with a safety device consisting of a long straight pipe of diameter 0.2 m with a bursting disc covering the end, as sketched in Fig. 1. The disc's outer face is exposed to the atmosphere, and it is designed to rupture when the (absolute) pressure inside the pipe reaches 2.5 bar. The ambient pressure and temperature are 1 bar and 10° C.

(i) Draw an x–t diagram to show the wave pattern that forms after the disc has burst. (Only consider times before any wave reflections occur.) [10%]

(ii) What is the mass flow rate at the pipe exit during this period? [20%]

(c) A stronger tank has a safety device of the same design, but with a higher bursting pressure. The ambient conditions are unchanged.

(i) Describe, with appropriate sketches, the effect of increasing the bursting pressure on the wave pattern in the pipe after bursting. [10%]

(ii) What is the maximum possible mass flow rate, and at what bursting pressure is it achieved? [25%]

(iii) Explain what happens if the bursting pressure is above the value found in part (ii). [15%]

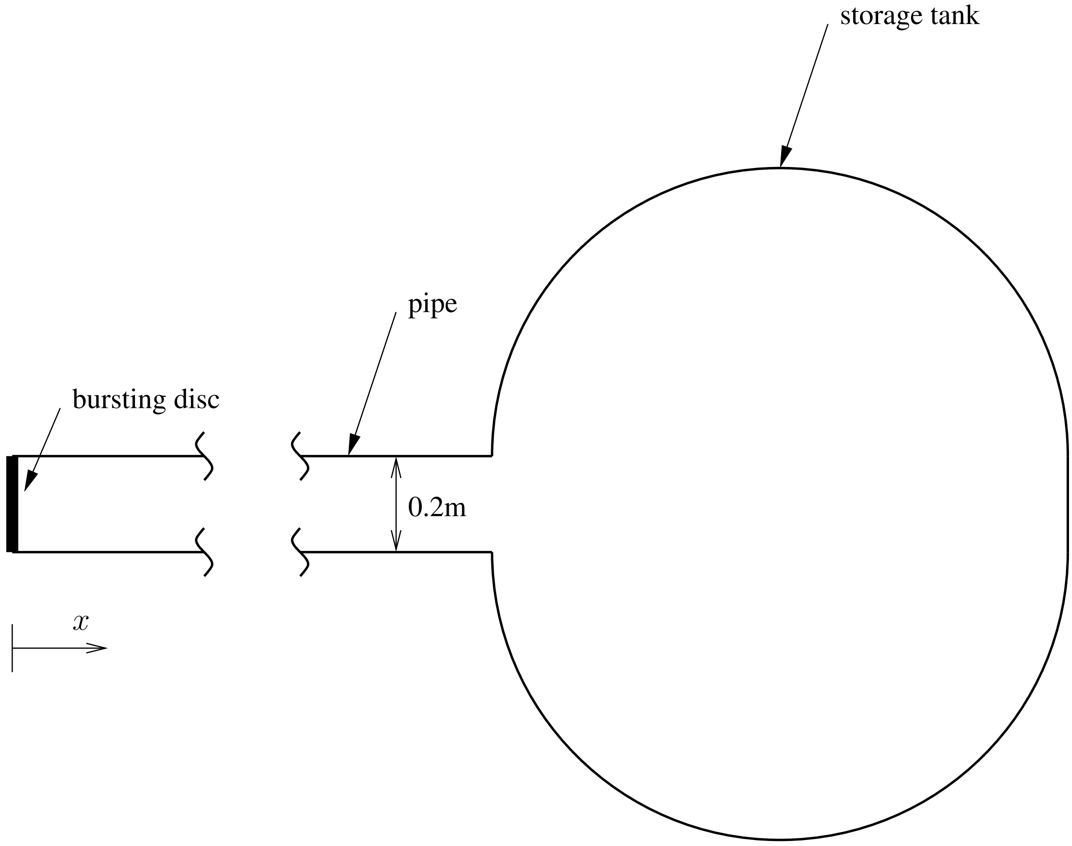

Fig. 1

4 A supersonic jet aircraft is intended to cruise at a Mach number of 2.40. Two engine-intake designs are under initial consideration. The first, sketched in Fig. 2(a), is a conventional external-compression configuration with three planar ramps generating a shock system focused on the cowl lip. The second design has the second and third ramps replaced by a smooth continuous curve that turns the flow through the same overall angle, as shown in Fig. 2(b). In this second design, the principal flow features remain focused on the cowl lip. In each case the lip is sharp, with a wedge angle of 4°.

(a) For each design:

(i) draw carefully labelled sketches of the principal flow features; [30%]

(ii) calculate the static pressure (relative to atmospheric) immediately after the final, normal, shock; [25%]

(iii) calculate the percentage stagnation-pressure loss. [25%]

(b) If the stagnation-pressure loss is to be reduced, which design would you choose to modify? Carefully describe two modifications you would make. [20%]

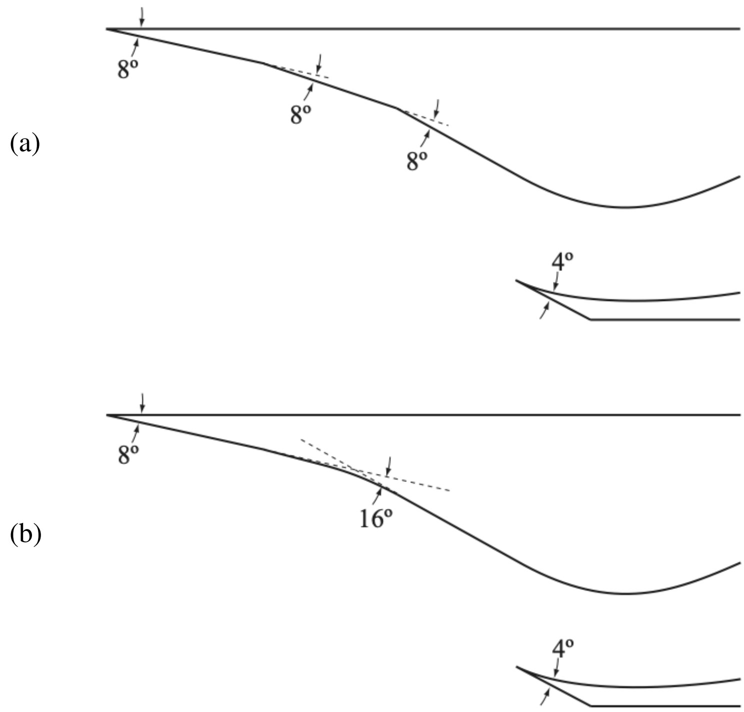

Fig. 2

5 (a) Air flows past a wall at a Mach number $M = 1.25$, and encounters a defect. This may be either a 4° clockwise turn followed by a return to the streamwise direction (as sketched in Fig. 3(a)), or a 4° anticlockwise turn followed by a return to the streamwise direction (as sketched in Fig. 3(b)).

(i) For each geometry, draw carefully labelled sketches of the principal flow features. [20%]

(ii) Calculate the percentage loss in stagnation pressure for each case. [20%]

(iii) If the airflow now slows to $M = 1.15$, which case do you expect to experience the larger change in stagnation-pressure loss? Justify your answer. [20%]

(b) Air flowing along the upper surface of a plate of thickness t at supersonic speed passes over a slot of length h (measured in the streamwise direction). The static pressure below the plate is lower than that above. As a result, the flow turns clockwise through an angle of $\tan^{-1}(t/h)$ at the upstream edge of the slot.

(i) Two possible detail geometries for the slot are shown in Figs. 3(c) and (d). Draw carefully labelled sketches of the principal flow features in each case. [30%]

(ii) Comment on the potential utility of the design in Fig. 3(d). [10%]

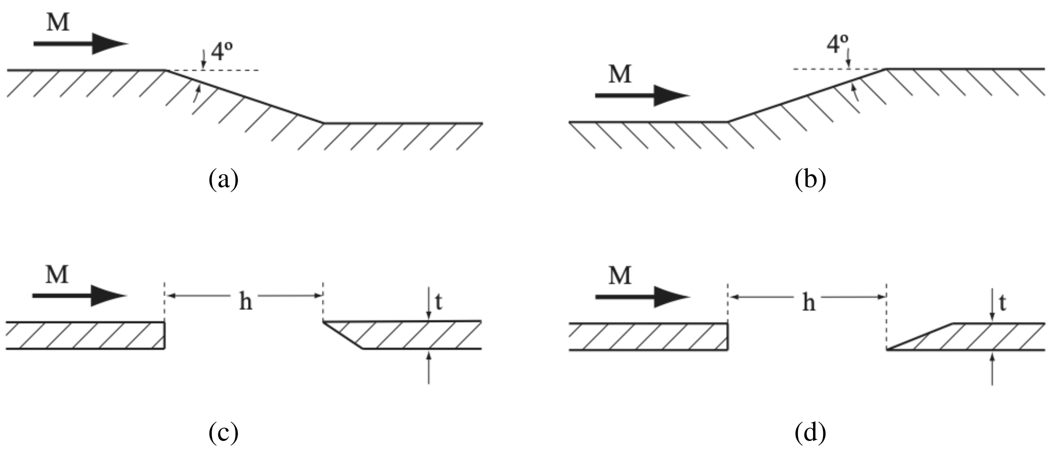

Fig. 3

6 (a) An approximate solution to a partial differential equation (PDE) is produced by the difference equation

$$u_i^{n+1} = u_i^n - a\frac{\Delta t}{2\Delta x}\left(3u_i^n - 4u_{i-1}^n + u_{i-2}^n\right).$$

Here u_i^n denotes the solution at position $x = i\Delta x$ and time $t = n\Delta t$, while a is a constant which can take any real value. Find the equivalent PDE, and hence:

(i) determine the accuracy of the solution in space and in time; [30%]

(ii) discuss the numerical behaviour the scheme might exhibit, and its possible dependence on the parameters a, Δt, and Δx. Ensure that you explain your reasoning. [30%]

(b) An alternative scheme, for the same PDE as in part (a), uses the following difference equation:

$$u_i^{n+1} = u_i^n - a\frac{\Delta t}{2\Delta x}\left(u_{i+1}^n - u_{i-1}^n\right) + \beta\left(u_{i+1}^n - 2u_i^n + u_{i-1}^n\right).$$

Determine β such that the solution is second-order accurate in both time and space. [40%]

7 (a) For each of the following equations or systems of equations, state whether they are hyperbolic, elliptic or parabolic. Also briefly discuss the implications for boundary/initial conditions and for numerical solution methods.

(i)

$$\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = 0.$$

[15%]

(ii)

$$u\frac{\partial u}{\partial x} = \alpha\frac{\partial^2 u}{\partial y^2},$$

where α is a positive constant. [15%]

(iii) The Euler equations in one dimension, i.e.

$$\frac{\partial}{\partial t}\begin{bmatrix}\rho \\ \rho u \\ E\end{bmatrix} + \frac{\partial}{\partial x}\begin{bmatrix}\rho u \\ \rho u^2 + p \\ u(E+p)\end{bmatrix} = 0,$$

where the symbols have their usual meanings. [20%]

(b) For steady adiabatic flow through a radial turbine, the stagnation-enthalpy change across the rotor is given by:

$$h_{03} - h_{02} = \tfrac{1}{2}\left(U_3^2 - U_2^2\right) - \tfrac{1}{2}\left(V_{3,\mathrm{rel}}^2 - V_{2,\mathrm{rel}}^2\right) + \tfrac{1}{2}\left(V_3^2 - V_2^2\right),$$

where U is the local blade speed, V is the local (absolute) flow speed, and the subscript 'rel' denotes a local relative flow speed.

An optimal radial turbine will have the largest possible power output. On this basis:

(i) what can be concluded from the $\frac{1}{2}\left(U_3^2 - U_2^2\right)$ term? [10%]

(ii) what can be concluded from the $-\frac{1}{2}\left(V_{3,\mathrm{rel}}^2 - V_{2,\mathrm{rel}}^2\right)$ term? [10%]

(iii) what can be concluded from the $\frac{1}{2}\left(V_3^2 - V_2^2\right)$ term? [10%]

(iv) what stagnation-enthalpy change should an optimal radial turbine achieve? [20%]

Ensure that you explain your reasoning in your answers to parts (i)–(iii).

8 The stagnation temperature and stagnation pressure at the inlet of an axial-flow compressor rotor are 288 K and 1 bar respectively. The flow through the rotor can be assumed to be at a constant radius of 0.3 m, and the inlet swirl angle is 20°. For a particular operating point: the relative flow angle at the rotor inlet is −60° with a relative Mach number of 0.7; the relative-stagnation-pressure loss coefficient for the rotor is 0.04; and the relative flow angle at the rotor exit is −30°.

(a) Draw, and carefully label, the velocity triangle at the rotor inlet. [10%]

(b) Determine the inlet flow coefficient. [5%]

(c) Explain how the inlet Mach triangle is related to the inlet velocity triangle that you drew in part (a), and determine the absolute inlet Mach number. [5%]

(d) Determine the static temperature, static pressure, and the axial and tangential velocities of the flow upstream of the rotor. [20%]

(e) Determine the angular velocity of the compressor rotor. [10%]

(f) Assuming that the flow through the compressor has constant axial velocity, determine the static-pressure ratio across the rotor. [30%]

(g) Determine the stage-loading coefficient for the rotor row, and comment on its value. [20%]

Notes:

1) You may assume that the working fluid is a perfect gas with the properties of air.

2) For a compressor rotor with relative stagnation pressures $p_{01,\mathrm{rel}}$ at inlet and $p_{02,\mathrm{rel}}$ at outlet, the relative-stagnation-pressure loss coefficient is

$$\frac{p_{01,\mathrm{rel}} - p_{02,\mathrm{rel}}}{p_{01,\mathrm{rel}} - p_1},$$

where p_1 is the inlet static pressure.

END OF PAPER

ENGINEERING TRIPOS PART IIA

Module 3A3: FLUID MECHANICS II

ANSWERS

1(b) (i) 0.974 (ii) 0.0473 (iii) 0.377

(c) (ii) 0.528 throat; 0.152 before shock; 0.608 after shock; 0.737 exit

2(b) (i) 0.459, 1.788 (ii) 0.0124, 0.0020

3(b) (ii) 10.4 kg/s

(c) (ii) 15.6 kg/s, 3.58 bar

4(a) (ii) 9.50, 9.65 (iii) 9.9%, 8.9%

5(a) (ii) 0.08%, 0.12%

6(a) (i) Second-order space; first-order time

(b) $\frac{1}{2}\left(\frac{a\Delta t}{\Delta x}\right)^2$

7(b) (iv) $-U_2^2$

8(b) 0.477

(c) 0.372

(d) 280 K, 0.908 bar, 117 m/s, 42.7 m/s

(e) 820 rad/s

(f) 1.23

(g) 0.551