ENGINEERING TRIPOS PART IIA 2014

MODULE 3A5 – THERMODYNAMICS AND POWER GENERATION

SOLUTIONS TO TRIPOS QUESTIONS

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1. (a) From the definition $g = h - Ts$ we have $dg = dh - sdT - Tds$.

Combining with $Tds = dh - vdp$ gives $dg = vdp - sdT$. [5%]

(b) Starting from the characteristic equation of state $g = F(T) + RT(\ln p + Bp)$ we obtain,

$$v = \left(\frac{\partial g}{\partial p}\right)_T = \frac{RT}{p}(1 + Bp) \qquad [5\%]$$

$$s = -\left(\frac{\partial g}{\partial T}\right)_p = -\frac{dF}{dT} - R(\ln p + Bp) \qquad [5\%]$$

$$h = g + Ts = F - T\frac{dF}{dT} \qquad [5\%]$$

As $h = u + pv$ is a function of temperature only,

$$c_p - c_v = \left(\frac{\partial h}{\partial T}\right)_p - \left(\frac{\partial u}{\partial T}\right)_v = \frac{dh}{dT} - \frac{dh}{dT} + v\left(\frac{\partial p}{\partial T}\right)_v = v\left(\frac{\partial p}{\partial T}\right)_v$$

The most elegant way to proceed is to take logs of both sides of $pv = RT(1+Bp)$ and then differentiate, keeping v constant, to obtain,

$$\frac{dp}{p} = \frac{dT}{T} + \frac{B\,dp}{(1+Bp)} \quad \rightarrow \quad \left(\frac{\partial p}{\partial T}\right)_v = \frac{p}{T}(1+Bp)$$

Hence,

$$c_p - c_v = v\left(\frac{\partial p}{\partial T}\right)_v = \frac{pv}{T}(1+Bp) = R(1+Bp)^2 \qquad [20\%]$$

(c) (i) As h is a function only of T and $T_2 = T_1$, we have $h_1 = h_2$. The flow in the delivery pipe is adiabatic so, from the SFEE, $h_3 = h_2$ and hence $T_3 = T_2$.

From the steady-flow availability theorem, the ideal shaft power <u>output</u> in taking the gas from state 1 to state 3 is given by,

$$[\dot{W}_X]_{\text{ideal}} = \dot{m}(e_1 - e_3] = \dot{m}\left[(h_1 - T_0 s_1) - (h_3 - T_0 s_3)\right]$$

Hence, the ideal shaft power <u>input</u> (which is the minimum shaft power input) is given by,

$$[-\dot{W}_X]_{\min} = \dot{m}T_0(s_1 - s_3) = -\dot{m}RT_0\left[\ln\left(\frac{p_1}{p_3}\right) + B(p_1 - p_3)\right]$$

$$= -0.5\times 0.3\times 290\left[\ln\left(\frac{2}{9.5}\right) - 0.05\times(2 - 9.5)\right] = 51.47\ \text{kW} \qquad [15\%]$$

(c) (ii) The flow in the delivery pipe is adiabatic so there is no exergy loss due to heat transfer. Hence, the overall exergy equation is,

$$\dot{m}(e_3 - e_1) = [-\dot{W}_X]_{\text{ideal}} = -\dot{W}_X - [\dot{W}_{L,Q}]_{\text{comp}} - [\dot{W}_{L,CR}]_{\text{comp}} - [\dot{W}_{L,CR}]_{\text{pipe}} \qquad [10\%]$$

Actual shaft power <u>output</u> $= -51.47 \times 1.4 = -72.05$ kW

From the SFEE for the compressor,

$$\dot{Q} - \dot{W}_X = \dot{m}(h_2 - h_1) = 0 \quad \rightarrow \quad \dot{Q} = \dot{W}_X = -72.05 \text{ kW}$$

where $\dot{Q}$ is the rate of heat transfer <u>from</u> the cooling water <u>to</u> the gas.

The compression is isothermal ($T_2 = T_1 = T$), so the lost power due to the heat transfer with the cooling water is given by,

$$[\dot{W}_{L,Q}]_{\text{comp}} = \int_{CS} -\left(1 - \frac{T_0}{T}\right) d\dot{Q} = -\left(1 - \frac{T_0}{T}\right)\dot{Q} = -\left(1 - \frac{290}{320}\right) \times (-72.05) = 6.75 \text{ kW} \qquad [20\%]$$

The flow in the delivery pipe is adiabatic and isothermal. Hence, the lost power due to the pressure drop (due to friction) is given by,

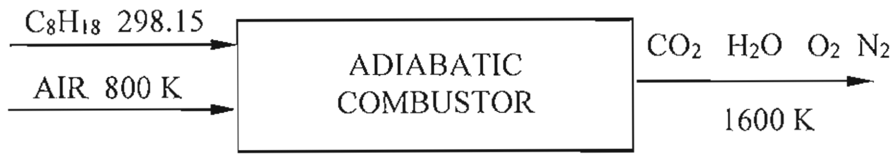
$$[\dot{W}_{L,CR}]_{\text{pipe}} = \dot{m}T_0(s_3 - s_2) = -\dot{m}RT_0\left[\ln\left(\frac{p_3}{p_2}\right) + B(p_3 - p_2)\right]$$

$$= -0.5 \times 0.3 \times 290\left[\ln\left(\frac{9.5}{10}\right) - 0.05 \times (9.5 - 10)\right] = 1.14 \text{ kW} \qquad [10\%]$$

Hence, the lost power due to the internal irreversibility in the compressor is given by,

$$[\dot{W}_{L,CR}]_{\text{comp}} = -51.47 + 72.05 - 6.75 - 1.14 = 12.69 \text{ kW} \qquad [5\%]$$

2. (a)



For stoichiometric combustion of C_8H_{18} with O_2 :

$$C_8H_{18} + 12.5\, O_2 \rightarrow 8\, CO_2 + 9\, H_2O$$

If A is the molar air-fuel ratio, the actual chemical reaction is :

$$C_8H_{18} + A\,(0.21\, O_2 + 0.79\, N_2) \rightarrow 8\, CO_2 + 9\, H_2O + (0.21A - 12.5)\, O_2 + 0.79A\, N_2$$

Writing $T_0 = 298.15$ K, $T_A = 800$ K and $T_P = 1700$ K, the SFEE is,

$$8\bar{h}_{CO_2}(T_P) + 9\,\bar{h}_{H_2O}(T_P) + (0.21A - 12.5)\,\bar{h}_{O_2}(T_P) + 0.79A\,\bar{h}_{N_2}(T_P)$$

$$= \bar{h}_{C_8H_{18}}(T_0) + 0.21A\,\bar{h}_{O_2}(T_A) + 0.79A\,\bar{h}_{N_2}(T_A)$$

From the Thermofluids Data Book, the LCV for liquid C_8H_{18} is 44.430 MJ/kg. Hence,

$$\Delta\bar{H}^0_{298} = 8\bar{h}_{CO_2}(T_0) + 9\bar{h}_{H_2O}(T_0) - \bar{h}_{C_8H_{18}}(T_0) - 12.5\bar{h}_{O_2}(T_0) = -114 \times 44.430$$

$$= -5065.020 \text{ MJ/kmol}$$

Eliminating $\bar{h}_{C_8H_{18}}(T_0)$ we obtain,

$$-\Delta\bar{H}^0_{298} = 8[\bar{h}_{CO_2}(T_P) - \bar{h}_{CO_2}(T_0)] + 9[\bar{h}_{H_2O}(T_P) - \bar{h}_{H_2O}(T_0)] - 12.5[\bar{h}_{O_2}(T_P) - \bar{h}_{O_2}(T_0)]$$

$$+ 0.79A\,[\bar{h}_{N_2}(T_P) - \bar{h}_{N_2}(T_A)] + 0.21A\,[\bar{h}_{O_2}(T_P) - \bar{h}_{O_2}(T_A)]$$

Taking data from the molar enthalpy tables in the Data Book :

$$5065.020 = 8\times(82.94 - 9.37) + 9\times(67.65 - 9.90) - 12.5\times(56.63 - 8.66)$$
$$+ 0.79A\,(54.12 - 23.72) + 0.21A\,(56.63 - 24.50)$$

$$= 588.560 + 519.750 - 599.625 + 24.016A + 6.747A$$

Hence, the molar air-fuel ratio is : $A = 148.1$ [50 %]

(b) If the degree of dissociation of the CO_2 is α, the actual chemical reaction is :

$$C_8H_{18} + A\,(0.21\,O_2 + 0.79\,N_2) \rightarrow 8(1-\alpha)\,CO_2 + 8\alpha\,CO + 9\,H_2O + a\,O_2 + 0.79A\,N_2$$

Conservation of atomic C :	$8 = 8$
Conservation of atomic H :	$18 = 18$
Conservation of atomic O :	$16 - 16\alpha + 8\alpha + 2b + 9 = 0.42A$
Number of moles of products :	$8 - 8\alpha + 8\alpha + 9 + a + 0.79A = n$

Thus :

$$a = 0.21A - 12.5 + 4\alpha \quad ; \qquad n = 4.5 + A + 4\alpha$$

$-CO - \frac{1}{2}O_2 + CO_2 = 0$ is Data Book reaction (7). The equilibrium equation is :

$$\left(\frac{p_{CO}}{p_0}\right)^{-1}\left(\frac{p_{O_2}}{p_0}\right)^{-0.5}\left(\frac{p_{CO_2}}{p_0}\right) = K_{p7}$$

From the Data Book at 2600 K, $\ln(K_{p7}) = 2.800 \rightarrow K_{p7} = 16.445$

The partial pressures are given by the mole fractions multiplied by the sample pressure p :

$$p_{CO_2} = \frac{8(1-\alpha)}{n}\,p \; ; \qquad p_{CO} = \frac{8\alpha}{n}\,p \; ; \qquad p_{O_2} = \frac{a}{n}\,p$$

Substituting into the equilibrium equation and rearranging gives :

$$\left(\frac{p}{p_0}\right)^{0.5} = \frac{1}{K_{p7}}\left(\frac{1-\alpha}{\alpha}\right)\left(\frac{n}{a}\right)^{0.5}$$

Hence, with $A = 148.11$ as calculated above and $\alpha = 0.05$ as given :

$$\frac{p}{p_0} = \frac{1}{K_{p7}^2}\left(\frac{1-\alpha}{\alpha}\right)^2\left(\frac{4.5 + A + 4\alpha}{0.21A - 12.5 + 4\alpha}\right) =$$

$$\frac{1}{16.445^2}\left(\frac{1-0.05}{0.05}\right)^2\left(\frac{4.5 + 148.11 + 4\times 0.05}{0.21\times 148.11 - 12.5 + 4\times 0.05}\right)$$

With standard pressure $p_0 = 1$ bar : $p = 10.85$ bar. [50 %]

3. (a) Let $\dot{m}_f$ be the fuel mass flowrate to the gas turbine combustor and let LCV be the lower calorific value. From the definition of the gas turbine overall efficiency η_1 :

$$\text{Power output from gas turbine} = \dot{W}_1 = \eta_1 (\dot{m}_f \, LCV)$$

From the definition of the HRSG efficiency η_b :

$$\text{Rate of heat transfer to the steam in the HRSG} = \dot{Q} = \eta_b (1-\eta_1)(\dot{m}_f \, LCV)$$

From the definition of the steam cycle efficiency η_2 :

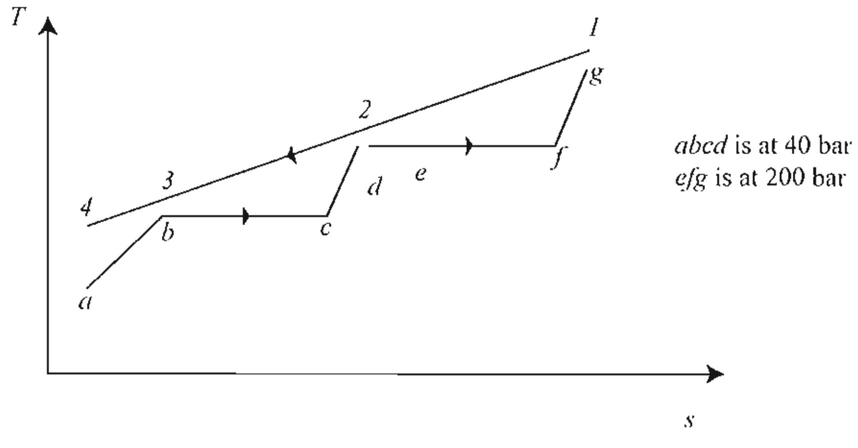
$$\text{Power output from the steam cycle} = \dot{W}_2 = \eta_2 \dot{Q} = \eta_2 \eta_b (1-\eta_1)(\dot{m}_f LCV)$$

The overall efficiency of the CCGT is thus,

$$\eta_{cc} = \frac{\dot{W}_1 + \dot{W}_2}{\dot{m}_f LCV} = \eta_1 + \eta_2 \eta_b (1-\eta_1)$$ [25%]

(b) The designer of the HRSG aims to maximise heat transfer (high η_b) and minimise exergy loss (minimise entropy creation due to irreversibilities). The former is achieved by achieving a gas temperature at HRSG outlet (stack temperature) that is as close to the environment temperature as possible; the latter is achieved by minimising the temperature difference between the gas side and steam side of the HRSG. A multiple pressure HRSG is designed to achieve this. Compared to a low steam pressure, a high steam pressure will tend to have a smaller average temperature difference between gas and steam in the HRSG, but have a higher stack temperature. [20%]

(c)



Saturation temperature at 200 bar is 365.75 °C.
Saturation temperature at 40 bar is 250.35 °C.

Gas temperatures: T_1 = 500 °C, T_2 = 365.75 + 20 = 385.75 °C, T_3 = 250.35 + 20 = 275.35 °C

Steam properties:

Location	h (kJ/kg)	s (kJ/ kg K)	Comment
a	151.5	0.521	Saturated liquid at 0.06 bar (neglect feed pump work, isentropic pump)
b	1087.5	-	Saturated liquid at 40 bar
d	3132.0	6.645	Interpolated from tables at 40 bar, 365.75 °C
e	1826.6	-	Saturated liquid at 200 bar
g	3061.7	5.904	From tables at 200 bar and 450 °C

(c) (i) SFEE for "hot portion" of 40 bar steam pass:

$$\dot{m}_g c_p (T_2 - T_3) = \dot{m}_{s1}(h_d - h_b) \quad \rightarrow \quad \frac{\dot{m}_{s1}}{\dot{m}_g} = \frac{c_p(385.75 - 270.35)}{(3132.0 - 1087.5)} = 0.0621$$

SFEE for "cold portion" of 40 bar steam pass:

$$\dot{m}_g c_p (T_3 - T_4) = \dot{m}_{s1}(h_b - h_a)$$

$$T_4 = T_3 - \frac{\dot{m}_{s1}}{\dot{m}_g}\frac{(h_b - h_a)}{c_p} = 270.35 - 0.0621\frac{(1087.5 - 151.5)}{1.1} = 217.5°\text{C}$$

$$\eta_b = \frac{q_{act}}{q_{\max}} = \frac{c_p(T_1 - T_4)}{c_p(T_1 - T_0)} = \frac{500 - 217.5}{500 - 25} = 0.59$$ [25%]

(c) (ii) For the whole HRSG, there is no heat flow into the control volume and hence :

$$\dot{S}_{irrev} = \dot{m}_g (s_4 - s_1) + \dot{m}_{s1}(s_d - s_a) + \dot{m}_{s2}(s_g - s_e)$$

Applying the SFEE to the 200 bar steam pass gives:

$$\dot{m}_g c_p (T_1 - T_2) = \dot{m}_{s2}(h_g - h_e) \quad \rightarrow \quad \frac{\dot{m}_{s2}}{\dot{m}_g} = \frac{c_p(500 - 385.75)}{(3061.7 - 1826.6)} = 0.102$$

Hence,

$$\frac{\dot{S}_{irrev}}{\dot{m}_g} = c_p \ln\left(\frac{T_4}{T_1}\right) + \frac{\dot{m}_{s1}}{\dot{m}_g}(s_d - s_a) + \frac{\dot{m}_{s2}}{\dot{m}_g}(s_g - s_e)$$

$$= 1.1 \times \ln\left(\frac{490.65}{773.15}\right) + 0.0621(6.645 - 0.521) + 0.102(5.904 - 4.015)$$

$$= -0.500 + 0.383 + 0.193 = 0.0727 \text{ kJ/kgK}$$

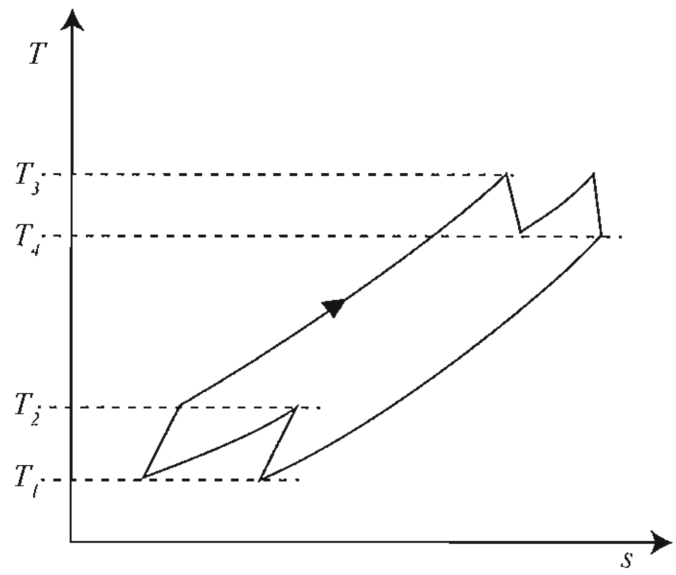
$$\frac{T_0 \dot{S}_{irrev}}{\dot{m}_g c_p (T_1 - T_0)} = \frac{298.15 \times 0.0727}{1.1(500 - 25)} = 0.0415$$ [30%]

4. (a) Intercooling and reheating allow the compression and expansion processes, respectively, to become more isothermal. Isothermal compression requires minimum work input and isothermal expansion produces the maximum work output since, for a reversible process:

$$dw = -vdp = -RT\frac{dp}{p} \qquad \text{(for an ideal gas)}$$

A sketch of the Joule cycle on a *T-s* diagram, with either an intercooler or reheater added, shows that the area enclosed (equal to the net work output per unit mass flow for a reversible cycle) has increased. However the efficiency has decreased because the intercooler (or reheater) has effectively added an extra Joule cycle with a lower pressure ratio. [15%]

(b)



[5%]

Temperatures : $T_2 = T_1 r_t^{1/(2\eta)}$; $T_3 = \theta T_1$; $T_4 = T_3 r_t^{-\eta/2} = \theta T_1 r_t^{-\eta/2}$

Total work input to compressors $= w_c = 2c_p T_1 [r_t^{1/(2\eta)} - 1]$

Total work output from turbines $= w_t = 2c_p T_1 \theta [1 - r_t^{-\eta/2}]$

Heat input in combustor $= q_{com} = c_p T_1 [\theta - r_t^{1/(2\eta)}]$

Heat input in reheater $= q_{rh} = c_p T_1 \theta [1 - r_t^{-\eta/2}]$

The cycle efficiency is given by,

$$\eta_c = \frac{w_t - w_c}{q_{com} + q_{rh}} = \frac{2\theta[1 - r_t^{-\eta/2}] + 2[1 - r_t^{1/(2\eta)}]}{[\theta - r_t^{1/(2\eta)}] + [\theta - \theta r_t^{-\eta/2}]} = \frac{2\theta[1 - r_t^{-\eta/2}] + 2[1 - r_t^{1/(2\eta)}]}{\theta[2 - r_t^{-\eta/2}] - r_t^{1/(2\eta)}}$$ [25%]

(c) The recuperator will reduce the heat input required from the combustor by,

$$q_{recup} = c_p(T_4 - T_2) = c_p T_1 [\theta r_t^{-\eta/2} - r_t^{1/(2\eta)}]$$

Hence,

$$\eta_c = \frac{w_t - w_c}{q_{com} - q_{recup} + q_{rh}} = \frac{2\theta[1 - r_t^{-\eta/2}] + 2[1 - r_t^{1/(2\eta)}]}{2\theta[1 - r_t^{-\eta/2}]} = 1 - \frac{[r_t^{1/(2\eta)} - 1]}{\theta[1 - r_t^{-\eta/2}]}$$

For the parameters given $[r_t = r_p^{(\gamma-1)/\gamma} = 2.354, \ \theta = 6, \ \eta = 0.9]$ we obtain,

$$\eta_c = 1 - \frac{0.609}{1.918} = 0.683$$ [20%]

(d) With n intercoolers and n reheaters, the cycle efficiency becomes :

$$\eta_c = \frac{m\theta[1 - r_t^{-\eta/m}] + m[1 - r_t^{1/(m\eta)}]}{m\theta[1 - r_t^{-\eta/m}]} = 1 - \frac{[r_t^{1/(m\eta)} - 1]}{\theta[1 - r_t^{-\eta/m}]}$$ [10%]

where $m = n + 1$.
As $m \to \infty$, we have $r_t^{1/m\eta} \to 1$ and $r_t^{-\eta/m} \to 1$, *i.e.*, a 0/0 situation. We therefore write,

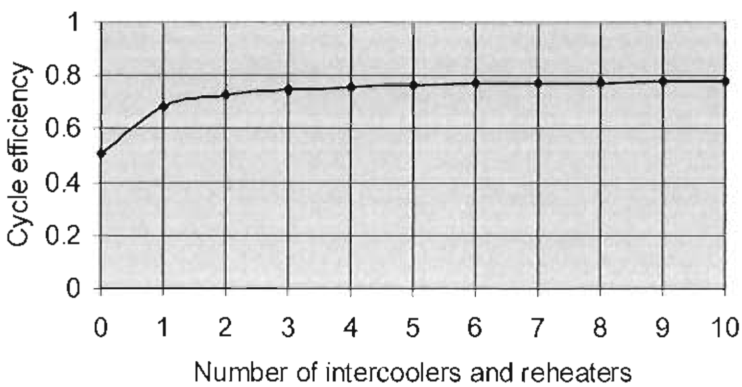
$$r_t^{1/m\eta} = 1 + \varepsilon_1 \quad \to \quad \frac{1}{m\eta}\ln(r_t) = \ln(1 + \varepsilon_1) \cong \varepsilon_1$$

$$r_t^{-\eta/m} = 1 + \varepsilon_2 \quad \to \quad -\frac{\eta}{m}\ln(r_t) = \ln(1 + \varepsilon_2) \cong \varepsilon_2$$

Hence, as $n \to \infty$,

$$\eta_c \to 1 - \frac{(1/m\eta)\ln(r_t)}{\theta(\eta/m)\ln(r_t)} = 1 - \frac{1}{\eta^2\theta}$$ [15%]

As n increases, η_c asymptotes to a value of $1 - 1/(0.81 \times 6) = 0.794$



[10%]