EGT2
ENGINEERING TRIPOS PART IIA

Monday 28 April 2014 9.30 to 11

Module 3A5

THERMODYNAMICS AND POWER GENERATION

Answer not more than ***three*** *questions.*

All questions carry the same number of marks.

The ***approximate*** *percentage of marks allocated to each part of a question is indicated in the right margin.*

Write your candidate number ***not*** *your name on the cover sheet.*

STATIONERY REQUIREMENTS
Single-sided script paper

SPECIAL REQUIREMENTS TO BE SUPPLIED FOR THIS EXAM
CUED approved calculator allowed
Engineering Data Book

You may not start to read the questions printed on the subsequent pages of this question paper until instructed to do so.

1 (a) Derive the thermodynamic relation $dg = vdp - sdT$ where g is the specific Gibbs function and the other symbols have their usual meanings. [5%]

(b) A gaseous pure substance has a characteristic equation of state,

$$g = F(T) + RT\,(\ln p + Bp)$$

where $F(T)$ is a function of temperature only, and R and B are constants. Derive expressions for the specific volume, specific entropy and specific enthalpy of the gas as functions of temperature and pressure. Also, show that the difference in the specific heat capacities can be expressed as

$$c_p - c_v = R\,(1 + Bp)^2$$ [35%]

(c) A particular gas obeying the above equation of state has $R = 0.3$ kJ kg^{-1} K^{-1} and $B = -0.05$ bar^{-1}. The gas enters a compressor at a pressure of 2 bar and a temperature of 320 K (state 1) and is compressed in steady-flow with a mass flowrate of 0.5 kg s^{-1} to a pressure of 10 bar (state 2). During this process the temperature of the gas is maintained constant at 320 K by a flow of cooling water. The cooling water has such a high flowrate that its temperature can be assumed constant at 290 K. After compression the gas flows adiabatically along a pipe where its pressure drops from 10 bar to a final delivery pressure of 9.5 bar (state 3).

(i) Assuming the gas only transfers heat with a heat reservoir at 290 K, calculate the minimum possible shaft power input required to compress the gas in steady-flow from state 1 to the final delivery state 3. [15%]

(ii) In practice, the shaft power input is 40% greater than the minimum possible value. Write down an equation showing how the difference in the exergy flowrates entering at state 1 and leaving at state 3 is related to the actual shaft power and the various 'lost power' terms associated with the compressor and delivery pipe. Calculate the values of these 'lost power' terms. [45%]

2 (a) A combustion chamber operating adiabatically in steady-flow is supplied with liquid octane (C_8H_{18}) at a temperature of 25 °C and air at a temperature of 800 K. The products leave the chamber at 1700 K. Assuming the products comprise the chemical species CO_2, H_2O, O_2 and N_2 only, and neglecting any change of kinetic energy between inlet and outlet, show that the molar air-fuel ratio is about 148.

Treat air as a mixture of 21% O_2 and 79% N_2 by volume. The enthalpy differences of chemical species should be evaluated using the table 'Molar Enthalpies of Common Gases at Low Pressures' in the Thermofluids Data Book. The calorific value of octane can also be found in the Data Book. [50%]

(b) For an experiment, a small sample of the combustion products from the reaction in part (a) is heated to a temperature of 2600 K. At this temperature, it is important to consider the dissociation of CO_2 into CO and O_2 according to the reaction,

$$-\,CO - \frac{1}{2}O_2 + CO_2 = 0$$

Assuming that the CO, O_2 and CO_2 are in chemical equilibrium subject to this reaction, and ignoring other dissociation processes, calculate the pressure at which the sample should be held in order that precisely 5% of the CO_2 is dissociated. [50%]

3 (a) A combined cycle gas turbine (CCGT) plant has a gas turbine with an overall efficiency of η_1 and a steam turbine cycle with a cycle efficiency of η_2. Show that the overall efficiency of the CCGT plant η_{cc} is given by,

$$\eta_{cc} = \eta_1 + \eta_b \eta_2 (1 - \eta_1)$$

where η_b is the actual heat transferred in the heat recovery steam generator (HRSG) divided by the maximum heat that could have been transferred. [25%]

(b) Explain the effect of steam pressure on the performance of the HRSG. Why do most CCGT plants employ dual or triple pressure level steam cycles? [20%]

(c) A schematic diagram of a dual pressure HRSG is shown in Fig. 1. The gas turbine exhaust enters the HRSG at a temperature of 500 °C and the steam pressures are 40 bar and 200 bar. The high pressure feed water enters the HRSG as saturated liquid and leaves as superheated steam at 450 °C. The low pressure feed water is provided by an isentropic feed pump that receives saturated liquid from the condenser at 0.06 bar. The low pressure steam leaves the HRSG as superheated steam at the same temperature as the high pressure feed water enters the HRSG. For both steam pressures, the temperature difference between the gas and the steam at the pinch point (when the steam is saturated liquid) is 20 °C. The gas side of the HRSG is at constant pressure and can be modelled as a perfect gas with c_p = 1.1 kJ kg^{-1} K^{-1}. The temperature of the environment is 25 °C. Neglecting the work input to the feed pump,

(i) evaluate η_b for the HRSG; [25%]

(ii) calculate the lost work due to irreversibilities in the HRSG as a fraction of the maximum heat that could have been transferred. [30%]

(cont.

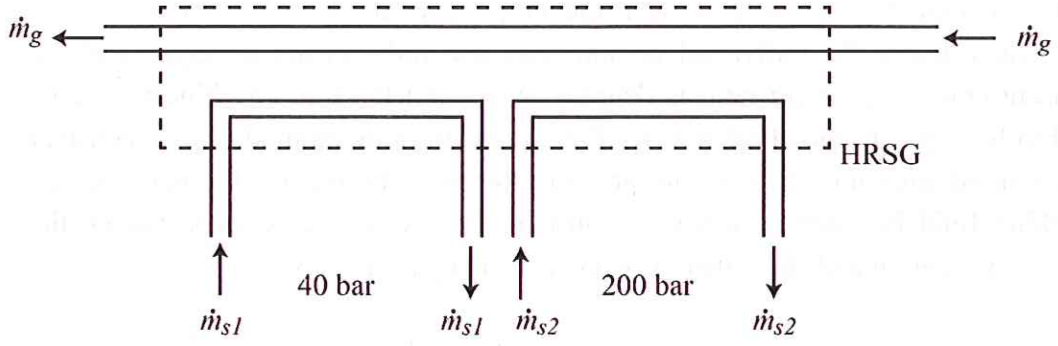

Fig. 1

4 (a) Explain, qualitatively, the effect of intercooling and reheating on the performance of a gas turbine cycle. [15%]

(b) A gas turbine cycle uses an intercooler to split the compression process into two parts and a reheater to split the expansion process into two parts. The temperature at inlet to both compressors is the same, as is the temperature at inlet to both turbines. The pressure ratios across the individual compressors and turbines are all equal and the overall isentropic temperature ratio is denoted by r_t . All the turbomachinery may be assumed to have polytropic efficiency η . The combustor may be modelled as constant pressure heat addition and there are no pressure losses in the intercooler and reheater. The working fluid is a perfect gas with constant specific heat capacities. Sketch the cycle on a *T-s* diagram and show that the cycle efficiency is given by,

$$\eta_c = \frac{2\theta[1-r_t^{-\eta/2}] + 2[1-r_t^{1/(2\eta)}]}{\theta[2-r_t^{-\eta/2}] - r_t^{1/(2\eta)}}$$

where θ is the ratio of turbine inlet temperature to compressor inlet temperature. [30%]

(c) A recuperator is fitted to the cycle described above. The recuperator can be considered ideal in that there are no pressure losses and it has a heat exchanger effectiveness of unity. Evaluate the cycle efficiency when the overall pressure ratio is 20, the temperature ratio θ is 6, and the polytropic efficiency η of the turbomachinery is 0.9 . The ratio of the specific heat capacities for the gas is 1.4 . [20%]

(d) The recuperated cycle is modified so that it now has n intercoolers and n reheaters. The temperature at the inlet to each of the $(n + 1)$ compressors is the same, as is the temperature at inlet to each of the $(n + 1)$ turbines. The pressure ratios across the individual compressors and turbines are all equal and the overall isentropic temperature ratio is denoted by r_t as before. Find an expression for the cycle efficiency η_c in terms of r_t, θ and η and show that,

$$\eta_c \rightarrow 1 - \frac{1}{\eta^2\theta} \quad \text{as} \quad n \rightarrow \infty$$

Sketch the variation of η_c with n. [35%]

END OF PAPER

ENGINEERING TRIPOS PART IIA 2014

MODULE 3A5 – THERMODYNAMICS AND POWER GENERATION

ANSWERS

1. (a) $v = \dfrac{RT}{p}(1 + Bp)\,, \qquad s = -\dfrac{dF}{dT} - R(\ln p + Bp)\,, \qquad h = F - T\dfrac{dF}{dT}$

   (b) $[\dot{W}_X]_{\min} = 51.47$ kW,
   $[\dot{W}_{L,CR}]_{\text{comp}} = 12.69$ kW, $[\dot{W}_{L,Q}]_{\text{comp}} = 6.75$ kW, $[\dot{W}_{L,CR}]_{\text{pipe}} = 1.14$ kW

2. (b) 10.85 bar

3. (c) (i) 0.59 (ii) 0.0415

4. (c) 0.683

G. Pullan & J.B. Young