ENGINEERING TRIPOS PART IIA 2024
MODULE 3A5 – Thermodynamics and Power Generation

Solutions

Examiners' Comments:

Q1 *Virial equation of state.* Part (a) (finding the partial derivative of Helmholtz function with respect to density) was done well by most, though several students spuriously invoked ideal gas relations. Most also saw that compressibility tends to unity at low density but failed to identify this as limiting ideal-gas behaviour. Manipulations to get the second virial coefficient were done well, but many then made a mess of deriving expressions for entropy and internal energy. Most marks were lost on the application of the EoS to adiabatic, unrestrained expansion, with many not seeing this implied constant u (IA material). The majority tried to explain differences in entropy increase (between the real and ideal gas) from a molecular perspective. These explanations were usually vague, incomplete or incorrect. (It is sufficient to note that for an IG the temperature would remain constant.)

Q2. *Hydrogen combustion.* Part (a) involved calculating the air-fuel ratio given the combustor outlet temperature. Only a small fraction of candidates got this completely correct, with many making a mistake with the chemical formula and / or not correctly writing down the SFEE (or indeed not writing it down at all). Most students that got to the last part (estimating the mole fraction of NO) realised they had to use the law of mass action but made the calculation over-complicated by not taking the hint that the mole fraction (of NO) would be very small. (See also hand-written note at the end of the solution.)

Q3. *Recuperated gas turbine.* A popular question but not done well unfortunately, despite having a good overlap with the coursework. Most candidates were able to derive the recuperator effectiveness and efficiency. Many struggled to find an expression relating efficiency to effectiveness. Those that failed to find the correct expression then struggled to determine when the efficiency was independent of effectiveness. Where students were able to battle on, they could usually discuss the effects of pressure ratio for recuperated and simple cycles, although curve sketching was poor. The effect of adding a heat engine to the recuperator was poorly done.

Q4. *Nuclear steam cycle.* Students did this question well. The cycle included feedheating using steam bled from the LPT, but a common mistake was to assume the feedheating was taken at the exhaust of the HPT. Application of energy balance using the properties given to find mass flows and work was done well. It was also good to see that students on the whole could find the rational efficiency of the plant. The final part of the question included the addition of reheat to the cycle and students found this more challenging.

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May 2024

Q1. (a) $f = u - Ts \Rightarrow df = du - Tds - sdT$

$= du - (du + pdv) - sdT$

$= -pdv - sdT$

But $v = 1/\rho \Rightarrow dv = -\frac{1}{\rho^2}d\rho$

$\therefore df = \frac{p}{\rho^2}d\rho - sdT \Rightarrow \left(\frac{\partial f}{\partial \rho}\right)_T = \frac{p}{\rho^2}$ [3]

(b) As $\rho \to 0$ gas behaves as ideal $\Rightarrow Z = 1, \Rightarrow A = 1$ [2]

(c) $\frac{\partial f}{\partial \rho} = \frac{p}{\rho^2} = \frac{RT}{\rho} - \alpha \Rightarrow Z = \frac{p}{\rho RT} = \frac{\rho}{RT}\left(\frac{RT}{\rho} - \alpha R\right)$

$= 1 - \frac{\alpha}{T}\rho$

But $Z \equiv 1 + B\rho + C\rho^2 + \ldots$

$\Rightarrow$ C, D etc $= 0$ & $B = -\frac{\alpha}{T}$ [3]

$s = -\frac{\partial f}{\partial T} = -c + cT \times \frac{1}{T} + c\ln\left(\frac{T}{T_0}\right) - R\ln\left(\frac{\rho}{\rho_0}\right)$

$s = c\ln\left(\frac{T}{T_0}\right) - R\ln\left(\frac{\rho}{\rho_0}\right)$ [2]

$u = f + Ts = c(T - T_0) - \alpha R(\rho - \rho_0)$ [2]

$c_v = \left(\frac{\partial u}{\partial T}\right)_v = \left(\frac{\partial u}{\partial T}\right)_\rho = c$ [1] [8]

(d) ADIABATIC + NO WORK (UNRESTRAINED) $\Rightarrow u =$ const $= 0$

$\therefore T = T_0 + \frac{\alpha R}{c}(\rho - \rho_0) \Rightarrow \frac{T}{T_0} = 1 - \frac{\alpha R\rho_0}{2cT_0} = 1 - \frac{\alpha\rho_0}{5T_0}$ $\left[\rho = \frac{\rho_0}{2}\right]$

$\therefore \Delta s = s(T,\rho) = c\ln\left(1 - \frac{\alpha\rho_0}{5T_0}\right) - R\ln\left(\frac{1}{2}\right) \approx R\left(\ln 2 - \frac{1}{2000}\right)$

This is clearly slightly less than the 1/g case for which T is const and therefore $\Delta s = R\ln 2$ [7]

2. (a) $H_2 + A(0.21\,O_2 + 0.79\,N_2) \rightarrow H_2O + (0.21A - \tfrac{1}{2})\,O_2 + 0.79A\,N_2$

$\Rightarrow$ $A + \tfrac{1}{2}$ kmol of products [A = molar AFR]

SFEE: $H_P = H_R$

$\therefore$ $(H_P - H_{P0}) = \cancel{(H_R - H_{R0})}^{\,0} + (H_{R0} - H_{P0})$

$\therefore$ $(A + \tfrac{1}{2})\bar{c}_p(T_2 - T_0) = -\Delta\bar{H}^{\circ}_{T_0} \Rightarrow A = \dfrac{-\Delta\bar{H}^{\circ}_{T_0}}{\bar{c}_p(T - T_0)} - \tfrac{1}{2}$

$= \dfrac{241.8 \times 10^3}{30 \times (2000 - 298.15)} - \tfrac{1}{2} = \underline{\underline{4.24}}$ [5]

(b) ADIABATIC $\Rightarrow$ $\Delta\bar{S}_{irr} = S_P - S_R = (S_P - S_{P0}) + (S_{P0} - S_{R0}) + \cancel{(S_{R0} - S_R)}^{\,0}$

$= (A + \tfrac{1}{2})\bar{c}_p \ln\left(\dfrac{T_2}{T_0}\right) + \Delta\bar{S}^{\circ}_{T_0}$

$= 4.74 \times 30 \times \ln\left(\dfrac{2000}{298.15}\right) + \left\{\dfrac{[-241.8 + 228.6] \times 10^3}{298.15}\right\}$

$= \underline{226.1\ \text{kJ/K.kmol.}}$ [5]

(c) $\eta^{max}_{RAT} = 1 - \dfrac{T_0\Delta\bar{S}_{irr}}{(-\Delta\bar{G}^{\circ}_{T_0})} = 1 - \dfrac{226.1 \times 298.15}{(-228.6 \times 10^3)} \simeq \underline{70.5\ \%}$ [2]

(d) Increasing AFR will decrease T_2, so the same quantity of "Heat" $(-\Delta\bar{H}^{\circ}_{T_0})$ will be supplied, but at a lower temperature. The Carnot efficiency of the heat engine will therefore be lower. Hence η^{MAX}_{RAT} decreases. [2]

(e) Relevant reactions are R2 - R6 in data book; only R4, R5 & R6 have sufficiently low values of K_P to give significant additional species.

R4 : $-2NO + N_2 + O_2 = 0$ $\quad \ln K_P(2000) = 7.824$

R5 : $-H_2 - \tfrac{1}{2}O_2 + H_2O = 0$ $\quad \ln K_P(2000) = 8.145$

R6 : $-\tfrac{1}{2}H_2 - OH + H_2O = 0$ $\quad \ln K_P(2000) = 8.727$

On this basis NO is going to be the most abundant trace gas. To estimate its molar fraction we assume X_{N_2} and X_{O_2} remain unchanged – justified because Kp values are high.

$$n = A + \tfrac{1}{2} = 4.74$$

$$X_{N_2} = \frac{0.79A}{n} = \frac{0.79 \times 4.24}{4.74} = 0.7066$$

$$X_{O_2} = \frac{0.21A - \frac{1}{2}}{n} = \frac{0.21 \times 4.24 - \frac{1}{2}}{4.74} = 0.0823$$

$$X_{H_2O} = \frac{1}{n} = 0.2111$$

$$K_{p4} = \left(\frac{P_{N_2}}{P_0}\right)\left(\frac{P_{O_2}}{P_0}\right) \Big/ \left(\frac{P_{NO}}{P_0}\right)^2 = X_{N_2} X_{O_2} \big/ X_{NO}^2$$

$$\therefore\ X_{NO} = \sqrt{\frac{X_{N_2} X_{O_2}}{K_{p4}}} = \sqrt{\frac{0.7066 \times 0.0823}{e^{7.824}}} = \underline{0.0048} \quad [6]$$

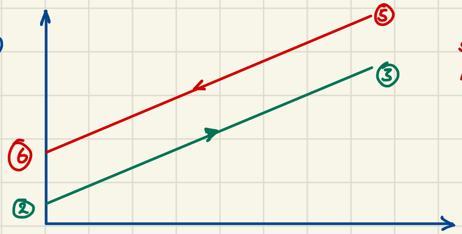
NOTE: Most credit also given if different trace species identified (esp. H_2) but correct method applied to estimate mole fraction.

For H_2:
$$X_{H_2} \simeq \frac{X_{H_2O}}{K_{p5}\, X_{O_2}^{1/2}} = \frac{0.2111}{e^{8.145} \times 0.0823^{1/2}} = 0.00021$$

OR//
$$K_{p6} = \frac{X_{H_2O}}{X_{H_2}^{1/2} X_{OH}} = \frac{X_{H_2O}}{X_{H_2}^{1/2} (2X_{H_2})}$$

$$\Rightarrow\ X_{H_2}^{3/2} = \frac{X_{H_2O}}{K_{p6} \times 2} \Rightarrow X_{H_2} \simeq 0.00066$$

Q3. (a)



$$\varepsilon = \frac{Q_{act}}{Q_{max}} = \frac{\dot{m}c_p(T_3-T_2)}{\dot{m}c_p(T_5-T_2)} = \frac{T_3-T_2}{T_5-T_2}$$ [3]

(b) $$\eta = \frac{w_T - w_C}{q} = \frac{(T_4-T_5)-(T_2-T_1)}{T_4-T_3}$$ [2]

(c) Need to eliminate T_5 and T_3 : $T_5 = T_4/\tau$

$T_3 = T_2 + \varepsilon(T_5 - T_2) = T_2 + \varepsilon\left(\frac{T_4}{\tau} - T_2\right)$

$$\therefore \eta = \frac{T_4(1-1/\tau) - T_1(\tau-1)}{T_4 - \left\{T_2 + \varepsilon\left(\frac{T_4}{\tau} - T_2\right)\right\}}$$

$$\boxed{\eta = \frac{(\theta/\tau - 1)(\tau-1)}{(\theta-\tau) + \varepsilon(\tau - \theta/\tau)}}$$

@ $\theta = 5$, $\tau = 5/2$; $\varepsilon = 4/5$

$$\eta = \frac{1 \times 3/2}{5/2 + 4/5 \times 1/2} = \frac{1.5}{2.9} = \underline{0.517}$$ [4]

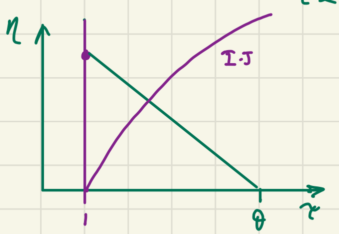
(d) Dependence on ε disappears when $\tau = \theta/\tau \Rightarrow \tau^2 = \theta$

This is because the turbine and compressor exit temperatures are then equal, so the recuperator is doing nothing.

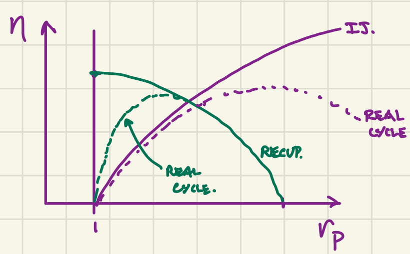
Then $$\eta = \frac{(\tau-1)^2}{\tau(\tau-1)} = \frac{\tau-1}{\tau} = 1 - 1/\tau = \eta_{IJ}$$

Thus $\underline{\tau = \sqrt{5}}$ when $\theta = 5$ and $\eta = 1 - \frac{1}{\sqrt{5}} = \underline{0.553}$ [HIGHER η @ LOWER τ than for (c)] [4]

(e) $$\varepsilon = 1 \Rightarrow \eta = \frac{(\theta/\tau - 1)\cancel{(\tau-1)}}{\frac{\theta}{\tau}\cancel{(\tau-1)}} = 1 - \frac{\tau}{\theta} \; ; \; \eta_J = 1 - \frac{1}{\tau}$$

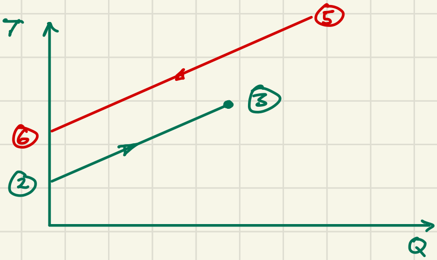


$\tau = r_p^{\frac{\gamma-1}{\gamma}}$



OPTIMUM PERFORMANCE AT LOWER r_p FOR RECUPERATED CYCLE. [3]

(f)



Work is now being extracted, so the enthalpy increase of the cold stream must be less than the enthalpy decrease of the hot stream. (Streams still // but cold stream stops earlier.)

If the process is ~~I~~reversible $\quad s_3 - s_2 = s_5 - s_6 \quad$ (globally adiabatic)

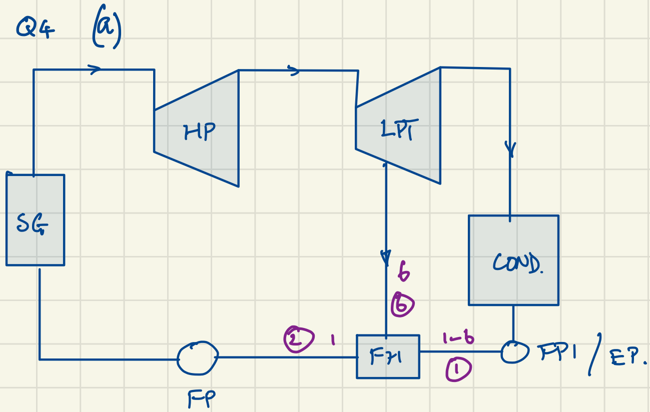
$$\therefore \quad c_p \ln\left(\frac{T_3}{T_2}\right) = c_p \ln\left(\frac{T_5}{T_6}\right) \Rightarrow \frac{T_3}{T_2} = \frac{T_5}{T_6} \Rightarrow \frac{T_6}{T_2} = \frac{T_5}{T_3}$$

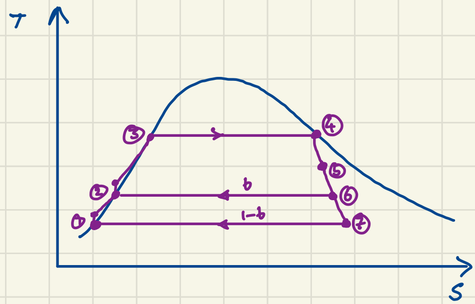
The temperature ratio $\frac{T_H}{T_C}$ is thus constant along the stream.

$$\therefore \quad \eta_{HE} = 1 - \frac{T_C}{T_H} = 1 - 0.915 = \underline{8.5\ \%}$$

[4]

Q4 (a)





[3]

(b) Interpolate condition on h-s chart $\Rightarrow h_6 = 2320$

SFEE applied to FH : $h_2 = b h_6 + (1-b) h_1 \Rightarrow b = \dfrac{h_2 - h_1}{h_6 - h_1}$

$$\therefore\ b \simeq \frac{504.7 - 121.4}{2320 - 121.4} \simeq \underline{0.174}$$ [3]

(c) $w_{HPT} = h_4 - h_5 = 2784.6 - 2510.6 = \underline{274 \text{ kJ/kg}}$

$w_{LPT} = h_5 - h_7 - b(h_6 - h_7) = (2510 - 1946.5) - 0.174(2320 - 1946.5)$

$= \underline{499 \text{ kJ/kg}}$

$$\dot{W} = \dot{m}(w_{HPT} + w_{LPT}) \Rightarrow \dot{m} = \frac{300 \times 10^3}{499 + 274} = \underline{388.1 \text{ kg/s}}$$

$\dot{W}_{HPT} = 388.1 \times 274 = \underline{106.3 \text{ MW}}$

$\dot{W}_{LPT} = 388.1 \times 499 = \underline{193.7 \text{ MW}}$ [4]

(d) $\eta_{CYCLE} = \dfrac{w_{HPT} + w_{LPT}\ (-w_{FP})}{q_{in}}$ $\qquad q_{in} = h_4 - h_2 = 2279.9 \text{ kJ/kg}$

$$\therefore\ \eta_{CYCLE} = \frac{274 + 499}{2279.9} = \underline{33.9\ \%}$$

$$\eta_{RAT} = \frac{w_{net}}{b_4 - b_2} = \frac{w_{net}}{q_{in} - T_0(s_4 - s_2)} = \frac{274 + 599}{2279.9 - 288(5.89 - 1.53)} = \underline{\underline{75.5\%}}$$

MOST LOSS is because heat input is at too temperature

AFTER THAT COMES TURBINE LOSSES, THEN HEAT REJECTION above T_0 from condenser. [5]

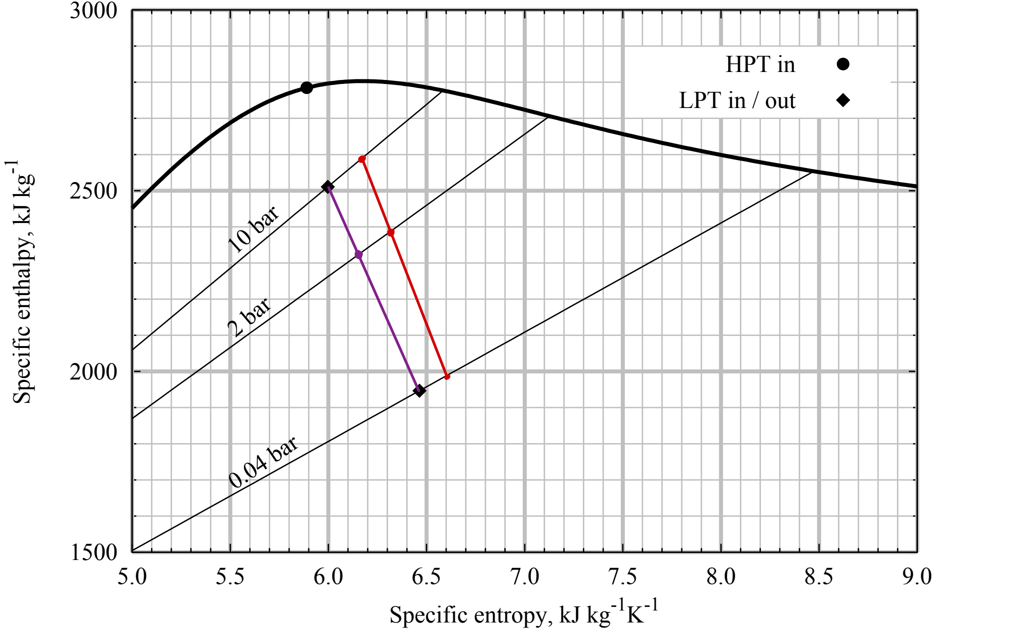
(e) $\eta_{cyc} = 1 - \frac{q_{COND}}{q}$; $q_{COND} = (1-b) \times T_c \times (s_7 - s_1)$

$$\therefore \eta_{cyc} = 1 - \frac{(1-0.169) \times 302.15 \times (6.6 - 0.422)}{2279.9}$$

$$= 32.0\%$$

The efficiency actually decreases. The purpose of feedheat in this case is to reduced the wetness fraction in the LP turbine in order to avoid erosion damage. In fact the cycle efficiency is almost bound to increase because lower exit wetness ⇒ higher exit enthalpy & entropy.

[5]



3000
2500
2000
1500
5.0
5.5
6.0
6.5
7.0
7.5
8.0
8.5
9.0
Specific enthalpy, kJ kg-1
Specific entropy, kJ kg-1K-1
HPT in
LPT in / out
10 bar
2 bar
0.04 bar