Version AJW/3

EGT2
ENGINEERING TRIPOS PART IIA

Friday 3 May 2024 9.30 to 11.10

Module 3A5

THERMODYNAMICS AND POWER GENERATION

Answer not more than ***three*** *questions.*

All questions carry the same number of marks.

The ***approximate*** *percentage of marks allocated to each part of a question is indicated in the right margin.*

Write your candidate number ***not*** *your name on the cover sheet.*

STATIONERY REQUIREMENTS
Single-sided script paper

SPECIAL REQUIREMENTS TO BE SUPPLIED FOR THIS EXAM
CUED approved calculator allowed
Engineering Data Book

10 minutes reading time is allowed for this paper at the start of the exam.

You may not start to read the questions printed on the subsequent pages of this question paper until instructed to do so.

You may not remove any stationery from the Examination Room.

1 (a) The specific Helmholtz function is defined as $f = u - Ts$, where u is specific internal energy, T is temperature and s is specific entropy. Starting from this definition, show that

$$\left(\frac{\partial f}{\partial \rho}\right)_T = \frac{p}{\rho^2}$$

where p is pressure and ρ is density. [15%]

(b) The virial equation of state expresses the compressibility, Z, of non-ideal gases as a Taylor-series expansion in density:

$$Z := \frac{p}{\rho RT} = A + B\rho + C\rho^2 + \ldots$$

where R is the specific gas constant and A, B and C are known as the first, second and third virial coefficients respectively.

(i) By considering the limit $\rho \rightarrow 0$, and explaining your reasoning, determine the value of A. [10%]

(ii) For a particular gas the specific Helmholtz function takes the form

$$f = RT \ln \frac{\rho}{\rho_0} - \alpha R(\rho - \rho_0) + c(T - T_0) - cT \ln \frac{T}{T_0}$$

where α and c are constants, T_0 is a reference temperature and ρ_0 is a reference density. Show that the third and higher virial coefficients are zero for this gas and find an expression for the second virial coefficient, B, as a function of temperature. Determine also expressions for the specific entropy, the specific internal energy and the constant-volume specific heat capacity. [40%]

(iii) A sample of the gas described in part (b) (ii), initially at the reference state (i.e., at T_0 and ρ_0), undergoes an adiabatic unrestrained expansion into an evacuated space such that its volume is doubled. Given that $c/R = 5/2$ and $\alpha\rho_0/T_0 = 0.001$, calculate the increase in specific entropy during this expansion in terms of the specific gas constant R. Without further calculation, state whether this is greater than or less than the entropy increase that would occur for an ideal gas with the same molar mass undergoing the same process. Justify your answer. [35%]

2 A steady-flow, adiabatic combustion chamber is to be used to supply hot gas to drive a high-temperature heat engine within a power plant. Hydrogen and air are each supplied to the combustion chamber at $p_0 = 1$ bar and $T_0 = 298.15$ K, and the products leave at 1 bar and 2000 K. The standard enthalpy and standard Gibbs free energy for the combustion reaction are $\Delta\bar{H}^0_{298} = -241.8$ MJ and $\Delta\bar{G}^0_{298} = -228.6$ MJ respectively, both per kmol of H_2. All substances may be treated as perfect gases with isobaric molar heat capacity $\bar{c}_p = 30\,\mathrm{kJ\,kmol^{-1}K^{-1}}$.

(a) Assuming the only species present in the products are O_2, N_2 and H_2O, determine the *molar* air-fuel ratio. [25%]

(b) Calculate the entropy increase per kmol of H_2 due to combustion irreversibility. In undertaking this calculation, evaluate the entropy of each species at p_0 rather than its partial pressure – i.e., ignore the 'entropy of mixing'. [25%]

(c) Determine the maximum rational efficiency of the power plant if the combustion process is the only source of irreversibility. [10%]

(d) Without further calculation, explain whether the maximum rational efficiency would increase or decrease if the air-fuel ratio were to be increased. [10%]

(e) In practice, some other species will be present in the products of combustion. On the basis of data given in the data book for the relevant reactions, suggest which gas (other than O_2, N_2 or H_2O) is likely to be present in the highest concentration and estimate its mole fraction. [30%]

3 Figure 1 shows the layout of a recuperated gas turbine. The working fluid is air (which may be treated as a perfect gas) and the combustion process may be approximated as heat input from an external source.

(a) Sketch the temperature vs. heat-transfer ($T - Q$) diagram for the recuperator and hence show that the recuperator effectiveness is given by

$$\epsilon = \frac{T_3 - T_2}{T_5 - T_2}$$

where the numbering is as shown in Fig. 1. [15%]

(b) Stating any assumptions, write down an expression for the recuperated gas turbine efficiency, η, in terms of the various cycle temperatures. [10%]

(c) Assuming all processes are reversible except for heat transfer in the recuperator, determine an expression for η as a function of ϵ and the temperature ratios $\tau = T_2/T_1$ and $\theta = T_4/T_1$. Evaluate η for the case when $\tau = 2.5$, $\theta = 5$ and $\epsilon = 0.8$. [20%]

(d) Determine the relationship between τ and θ for which η is independent of the recuperator effectiveness. Provide a physical interpretation of this case and show that the efficiency is then the same as that of an ideal Joule cycle with the same pressure ratio. [15%]

(e) Sketch the variation of η with pressure ratio for fixed θ when *all* processes are reversible. Sketch on the same axes the variation of ideal Joule cycle efficiency. Indicate how these curves would be affected by real processes and suggest how the choice of pressure ratio for a recuperated cycle compares with that for a simple gas turbine. [15%]

(f) The temperature difference between the hot and cold streams of the recuperator is now used to drive a reversible heat engine. Sketch the $T - Q$ diagram for the hot and cold streams in this scenario and explain any difference from your sketch in part (a). Show that the efficiency of the heat engine depends only on the ratio T_3/T_5. Calculate this efficiency when $T_3/T_5 = 0.915$. [25%]

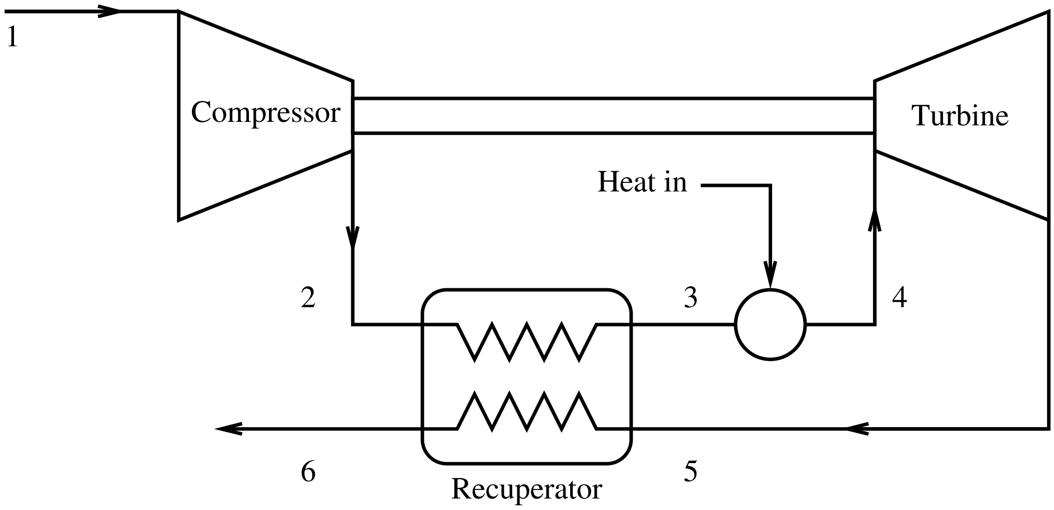

Fig. 1

4 A small pressurised water reactor power plant has an output of 300 MW. The steam cycle comprises a steam generator, high and low-pressure turbines (HPT and LPT), a condenser, feed pumps and a single direct-contact feedheater. Feedpump work and all electrical, mechanical and frictional pressure losses may be neglected. Steam conditions at selected locations are provided in Table 1. For convenience, a section of the $h - s$ (specific enthalpy vs. specific entropy) chart is also provided (Fig. 2) indicating turbine inlet and exit conditions.

(a) Draw a layout diagram indicating the arrangement of components and sketch the cycle on a $T - s$ (temperature vs. entropy) diagram. [15%]

(b) Stating your assumptions, estimate the mass flow of bled steam required for feedheating, expressed as a fraction of steam generator flow. [15%]

(c) Determine the steam generator flow rate and the power output from each turbine. [20%]

(d) Calculate the cycle efficiency and rational efficiency. (Take the environment temperature as 15°C.) What are the main factors limiting the cycle efficiency? [25%]

(e) The cycle is now modified to include reheat. 30% of the mass flow from the steam generator is throttled to 10 bar and mixed with the flow leaving the HPT. (Note that there is no external supply of heat to the reheater.) As a result, the specific entropy of the steam leaving the LPT increases to 6.6 kJ $\text{kg}^{-1}\text{K}^{-1}$ and the bled steam mass flow for feedheating changes to 0.169 kg per kg of flow through the steam generator. Pressures around the cycle and conditions at the inlet and exit of the steam generator do not change.

Calculate the new cycle efficiency. Comment on the result and explain the purpose of using reheat for this application. [25%]

Table 1

	p, bar	T, °C	h, kJ/kg	s, kJ/kg K
condenser exit	0.04	29	121.4	0.422
feedheater exit	2	120	504.7	1.530
HPT inlet	60	276	2784.6	5.890
LPT inlet	10	180	2510.6	5.997
LPT exit	0.04	29	1946.5	6.464

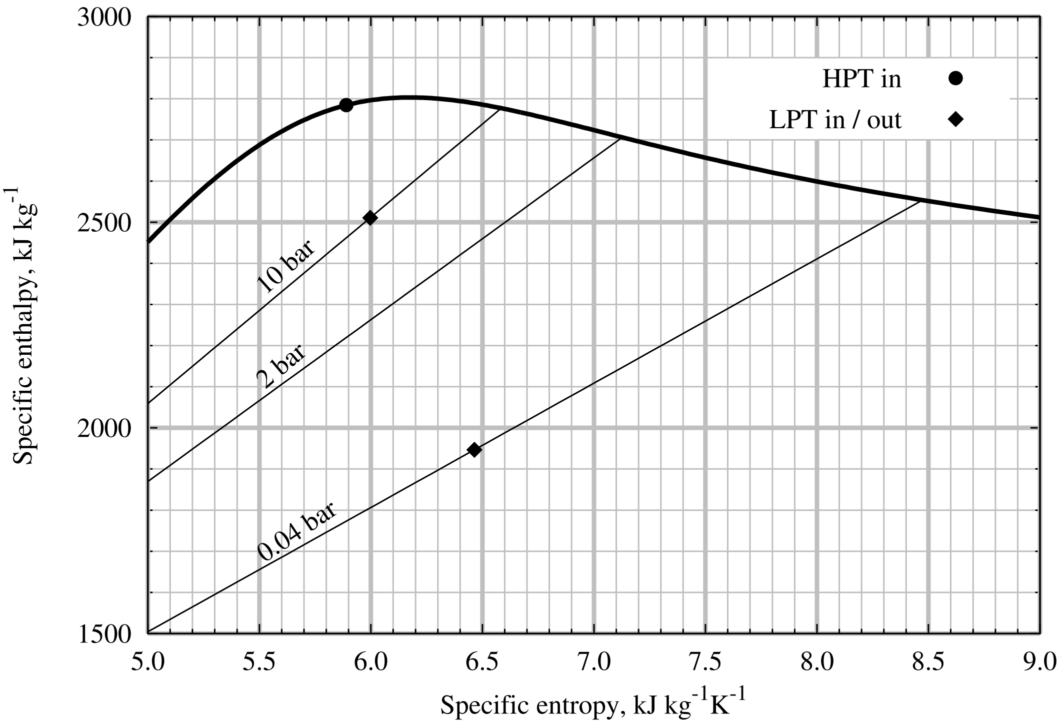

Fig. 2

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ANSWERS

1

(a)
(b)
(c)
(d) $R(\ln 2 - 1/2000)$

2

(a) 4.24
(b) 226.1 $\mathrm{kJ\,K^{-1}kmol^{-1}}$
(c) 70.5%
(d)
(e) $X_{\mathrm{NO}} \approx 0.005$

3

(a)
(b)
(c) 51.7%
(d) 55.3%
(e)
(f) 8.5%

4

(a)
(b) 0.174
(c) 388.1 $\mathrm{kg\,s^{-1}}$; 106.3 MW; 193.7 MW
(d) 33.9%; 75.5%
(e) 32.0%