EGT2
ENGINEERING TRIPOS PART IIA

Wednesday 2 May 2018   2 to 3.40

Module 3A5

THERMODYNAMICS AND POWER GENERATION

Answer not more than ***three*** *questions.*

All questions carry the same number of marks.

The ***approximate*** *percentage of marks allocated to each part of a question is indicated in the right margin.*

Write your candidate number ***not*** *your name on the cover sheet.*

STATIONERY REQUIREMENTS
Single-sided script paper

SPECIAL REQUIREMENTS TO BE SUPPLIED FOR THIS EXAM
CUED approved calculator allowed
Engineering Data Book

10 minutes reading time is allowed for this paper at the start of the exam.

You may not start to read the questions printed on the subsequent pages of this question paper until instructed to do so.

1 (a) Figure 1 shows an idealised steady-flow fuel cell stack operating isothermally at temperature T and exchanging heat with a reservoir also at temperature T. The stack is supplied with pure H_2 and O_2 in separate streams, each at standard pressure $p_0 = 1$ bar and temperature T. The reaction product H_2O is also exhausted at p_0 and T. Starting from molar forms of the steady flow energy and entropy equations, show that the maximum electrical work output per kmol of H_2 supplied is

$$\left[\overline{W}_E\right]_{\mathrm{MAX}} = -\Delta\overline{G}_T^0$$

where $\Delta\overline{G}_T^0$ is the standard Gibbs function of reaction per kmol for the oxidation of H_2. Derive also an expression for the open-circuit voltage (the Gibbs potential) of a single fuel cell for this idealised arrangement and determine its value when $T = 1200$ K. [30%]

(b) For real fuel cells, the reactants and products are supplied and withdrawn as gaseous mixtures rather than as pure streams. Stating any assumptions, show that the Gibbs function of reaction per kmol under these conditions may be written

$$\Delta\overline{G}_T = \Delta\overline{G}_T^0 + \overline{R}T\sum_i \nu_i \ln\left(\frac{p_i}{p_0}\right)$$

where $\overline{R}$ is the gas constant per kmol, ν_i is the stoichiometric coefficient of species i and p_i is its partial pressure. [30%]

(c) The fuel entering the fuel channels on the anode side of a solid oxide fuel cell stack is a methane-reformed mixture of H_2, H_2O and CO_2 with mole fractions of 0.500, 0.375 and 0.125 respectively. Air flows over the cathodes with a sufficient flow rate for its composition to remain approximately constant. Both streams are at 1200 K and 6 bar.

(i) Starting from the result of part (b), derive an expression for the open-circuit voltage (the Nernst potential) of a single cell and determine its value at the specified conditions. [20%]

(ii) Give a brief description and explanation of what happens to the cell voltages (including any variation that may occur throughout the stack) when the stack is connected to an electrical load. [10%]

(iii) Describe and explain the potential advantages of operating at elevated (i.e., above atmospheric) pressure and explain why the stack is operated at high temperature. [10%]

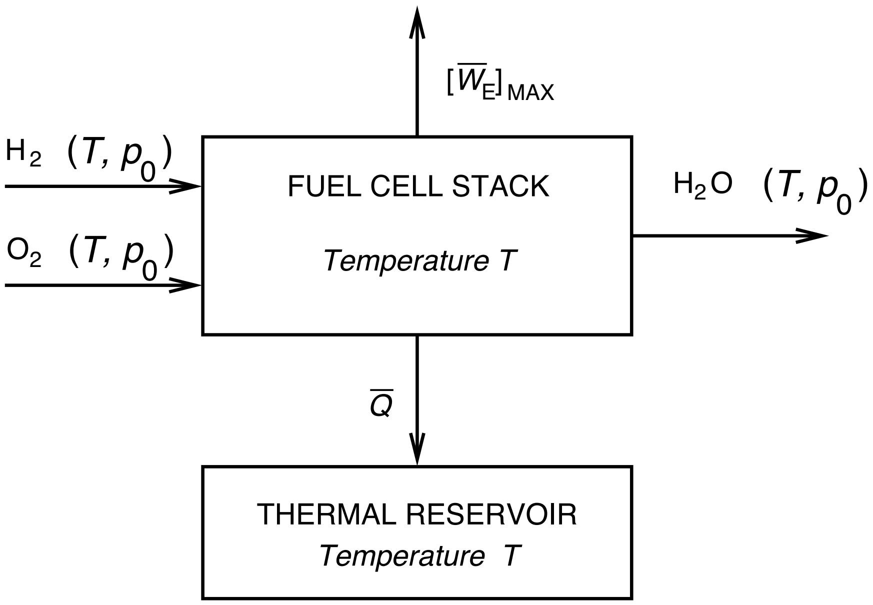
$[\overline{W}_E]_{MAX}$
H_2 (T, p_0)
O_2 (T, p_0)
FUEL CELL STACK
Temperature T
H_2O (T, p_0)
$\overline{Q}$
THERMAL RESERVOIR
Temperature T

Fig. 1

2 (a) Starting from the definition of specific Helmholtz function $f = u - Ts$, derive the Maxwell relation

$$\left(\frac{\partial s}{\partial v}\right)_T = \left(\frac{\partial p}{\partial T}\right)_v$$

where the symbols have their usual meanings. [10%]

(b) Show that for an ideal gas at fixed temperature the specific internal energy u is independent of the specific volume v. [20%]

(c) Provide brief qualitative explanations from a molecular perspective for:

(i) the result derived in part (b); [5%]

(ii) why the heat capacities of semi-perfect gases depend on temperature. [5%]

For the remainder of this question, CO_2 should be treated as a semi-perfect gas with a specific gas constant of $R = 189$ J kg^{-1} K^{-1} and an isobaric specific heat capacity that varies with Kelvin temperature T according to $c_p = a + bT + cT^2$, where:

$$a = 497 \text{ J kg}^{-1}\text{ K}^{-1} \qquad b = 1.37 \text{ J kg}^{-1}\text{ K}^{-2} \qquad c = -0.67\times 10^{-3} \text{ J kg}^{-1}\text{ K}^{-3}$$

(d) A flow of CO_2, initially at $p_1 = 1$ bar and $T_1 = 298$ K, is compressed adiabatically to $p_2 = 5$ bar by a compressor that has a *polytropic* efficiency of 0.85. The flow may be considered steady, and changes in potential and kinetic energy may be ignored.

(i) Show that the specific entropy change for the polytropic compression process may be written in the form

$$\Delta s = k \ln\left(\frac{p_2}{p_1}\right)$$

and find the value of the constant k. Hence determine the exergetic loss due to irreversibility per kg of CO_2 if the dead state temperature is 298 K. [30%]

(ii) Show that the compressor exit temperature is approximately 442 K. Hence calculate the compression work per kg of CO_2. [30%]

3 (a) With reference to a suitable temperature-entropy diagram, describe and explain the impact of reheat on the performance of a gas turbine. [20%]

(b) An ideal air-standard Joule cycle is augmented by the addition of reheat. The temperature at the inlet of both turbines is the same and is equal to T_3. The ratio of turbine entry temperature to compressor entry temperature $T_3/T_1 = \theta$. The pressure ratio of the compressor is r_p and that of the first turbine is r_{p1}. Find an expression for the value of r_{p1} that yields the maximum specific net work output. [20%]

(c) A recuperator is added to the exhaust of the second turbine of the cycle in part (b). The compressor and turbines are now all assumed to have the same polytropic efficiency η and the working fluid is a perfect gas with constant ratio of specific heats, γ.

(i) For recuperation to be possible, what condition on cycle temperatures must be satisfied. [5%]

(ii) What is the required relationship between r_p, r_{p1}, θ, η and γ for recuperation to be possible? Compare this to the equivalent result without reheat. [30%]

(d) Discuss, briefly, the challenges associated with reheat and recuperation in gas turbines for power generation. [25%]

4 (a) Derive an expression for the overall efficiency η_{cc} of a combined cycle gas turbine power plant where the gas turbine has an overall efficiency η_{ov}, the steam cycle efficiency is η_{st} and the heat recovery steam generator (HRSG) 'boiler efficiency' is η_b. [15%]

(b) The steam cycle of a combined cycle plant has a boiler pressure of 150 bar, a turbine entry temperature of 550°C, a condenser pressure of 0.04 bar, no reheat and no feedheaters. Stating your assumptions, evaluate the maximum specific work output from the turbine and the maximum steam cycle efficiency. [15%]

(c) The temperature of the gas turbine exhaust entering the HRSG that supplies heat to the steam cycle of part (b) is 600°C. The pressure drop in the HRSG may be neglected and the pinch point temperature difference is 10 K. Assuming that the gas turbine exhaust can be modelled as a perfect gas, evaluate, stating any additional assumptions:

(i) the HRSG boiler efficiency; [25%]

(ii) the exergetic lost work due to irreversibilities in the HRSG, expressed as a fraction of the maximum work output from the steam turbine, and state the cause of this loss. [20%]

(d) Why is a 'preheater loop' often used in the HRSG? Explain, with diagrams as appropriate, how it works. [25%]

END OF PAPER